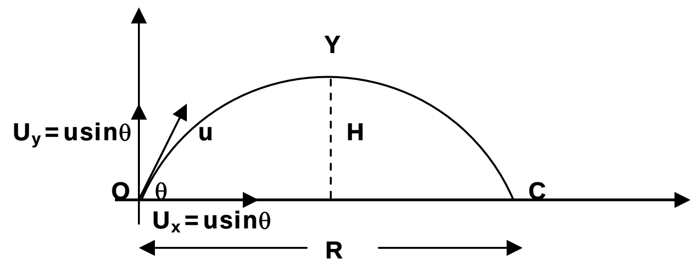
MOTION IN TWO AND THREE DIMENSOIONS

Summary

Projectile: When a body is projected in air in any direction, then the body is called projectile.
The angle which with the direction of projection makes with the horizontal is known as **angle of projection.**
The path traced by the body in its journey is called as **trajectory.**



Equation of projectile motion

$$Y = (\tan\theta)\, x - \quad g / 2U^2\cos\theta^2$$

Time of flight: The time taken by a projectile from start to fall back to ground is called time of flight.

$$T = \frac{2U\sin\theta}{g}$$

Maximum height reached (H):

$$H = \frac{(U\sin\theta)^2}{2g}$$

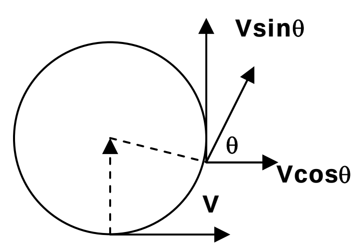
Horizontal Range (R): It is the total horizontal distance covered by a projectile during time of fileght.

$$R = \frac{U^2\sin 2\theta}{g}$$

Uniform Circular Motion:
When a particle moves in a circle with constant speed then the motion is known as uniform circular motion.

Centripetal Force
It is the force required to give circular motion to the particle. When a particle moves in a circular path of radius r, its speed is constant but the direction of velocity is continuously changing. Thus the particle experiences acceleration. The magnitude of the acceleration is given by $\mathbf{a = v^2 / r}$

The centripetal force is given by $\mathbf{F = mv^2 / r}$



Banking of Tracks:
When a vehicle goes around a curve on the road with greater speed, then there is a tendency for the vehicle to overturn outwards. To avoid this, the road is given a slope rising outwards. This is known as 'banking of roads'.
The expression for angle of banking is given by

$$\tan\theta = v^2 / rg$$

Solved problems:

1) The maximum range of body is 100 m. The maximum height reached by it is.
a) 100 m b) 50 m c) 25 m d) 10 m

Solution: The maximum range is $R_{max} = u^2/g$----------- (1)
The maximum height is $H=u^2\sin^2\theta/2g$ -------- (2) for $\theta=90^0$ H is maximum
i.e. $H_{max} = u^2/2g$--------(3) therefore $H_{max} = R_{max}/2 = 50$ m
the correct ans is (b)

2) A body is thrown with a velocity of 908 m/s making an angle 30^0 with the horizontal. It reaches the ground after
a) 3 s b) 2 s c) 1.5 s d) 1 s
Solution: Time of flight is given by
$T = 2U\sin\theta/2 = 2\times 9.8\times\sin 30^0/9.8 = 1$ s
The correct answer is d)

3) The horizontal range is 4 times the maximum height reached by projectile. The angle of projection is
a) 90^0 b) 45^0 c) 60^0 d) 30^0
Solution: here R = 4H
$U^2\sin 2\theta/g = 4\ u^2\sin^2\theta/2g$
$2\sin\theta co\theta = \sin^2\theta$
$\theta = 45^0$
The correct answer is (b)

4) Radius of the curved path is r. width of the path is b m. The outer edge is raised by h w.r.t. inner edge so that a car with speed v can pass safe over it. The value of h is

a) v^2b/rg b) v/rgb c) v^2b/gr d) v^2b/r

Solution: we have banking angle is

$\tan\theta = v^2/rg = h/b$

$h = v^2b/rg$ there fore correct answer is (a).

5) An electron in an hydrogen atom revolving with a linear velocity 2.18×10^6 m/s or radius 0.528 A^0. The acceleration of the electron is

a) zero b) 9×10^{22} c) 10^{22} d) none

Solution: centripetal acceleration is $a = v^2/r = 9\times10^{22}$ m/s^2

The correct answer is (b).

Exercise problems

ANSWERS:

1.d	2.c	3.c	4.d	5.d	6.d	7.d	8.b	9.b	10.a
11.b	12.a	13.d	14.a	15.a	16.b	17.a	18.c	19.a	20.c
21.a	22.c	23.d	24.a	25.c	26.d	27.a	28.c	29.a	30.d

NEWTON'S LAWS OF MOTION

SUMMARY:

FIRST LAW OF MOTION: According to first law of motion, every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by external forces to change that state.

SECOND LAW OF MOTION: According to second law of motion, the rate of change of momentum of a body is proportional to the impressed force and it takes place in the direction of force.

$$F \propto [dP/dt] \text{ or } F = [dP/dt] \quad \text{or } F = ma$$

The unit of force is newton or N.

THIRD LAW OF MOTION: According to third law of motion, every action has equal and opposite reaction. When two bodies A and B exert force on each other, the force of A on B is equal to the force of B on A.

$$F_{AB} = -F_{BA}$$

MOMENTUM: The product of mass and velocity of a body is called linear momentum. **$P = mv$.**

IMPULSE: If a force Facts on a body for a duration dt, then impulse is defined as the product of force and its time of action, thus

Impulse = Force × short time = F × dt

LAW OF CONSERVATION OF LINEAR MOMENTU

"The total momentum before collision is equal to the total momentum after collision or the total momentum of an isolated system is conserved"

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Weight of a body in a lift

Earth attracts every body towards its center. The force or attraction exerted by the earth on the body is called gravitational force. If m be the mass of the body then the force of gravity on it will be W=mg (weight of the body) When the body is on the moving platform, the new weight is called as apparent weigh.

Special cases:

1) The lift is unaccelerated (i.e. a=0 or a=constant) then reaction R=mg=0.
 Here apparent weight w is actual weight i.e. w=mg.
2) When the lift is accelerated upward. In this case apparent weight W=mg+ma=m(g+a).
3) When the lift is accelerated downward. In this case apparent weight W= mg-ma= m (g-a).
4) In the case of free fall of a body a=g. the apparent weight W=0.

FRICTION

"The force which opposes the relative motion between two surfaces in contact and which is always acting along the tangential direction to the surfaces in contact in contact is called FRICTION."

Note: The force of friction may increase with increase in roughness of interacting surfaces.

Static Friction: It is that frictional force which appears between interacting surfaces to two bodies before the instant of actual movement of one body over other.

Coefficient of static friction: It is the ratio of maximum force of friction and the normal reaction due to two surfaces in contact.

$$\mu = F_S / R$$

Kinetic Friction: It is that frictional force which appears between the interacting surfaces of two bodies after the instant of actual movement of one body over anther.

Coefficient of kinetic friction: It is the ratio of kinetic friction to the normal reaction due to the interacting surfaces.

$$\mu_k = F_K / R$$

Limiting Friction: The maximum static frictional force which appears when a body just starts moving over the surface of another body is called limiting friction.

Coefficient of limiting friction: It is the ratio of limiting friction to the normal reaction

$$\mu_s = F_s / R$$

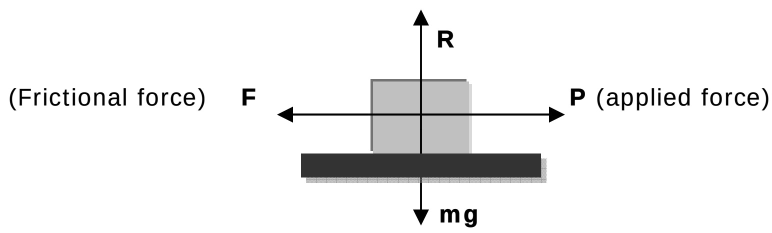
Rolling friction: It is the frictional force which appears when a body rolls over the surface of anther body.

The rolling friction depends on the normal reaction R and radius of the wheel

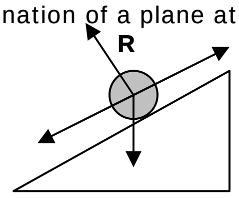
$$F_r = \mu_r (R / r)$$

Laws of friction

1) The limiting friction depends upon the nature of the surfaces in contact.
2) Friction does not depend on the area of the surfaces in contact.
3) The direction of the friction is opposite to the direction of motion of the body.
4) Limiting friction is directly proportional to the normal reaction between the two surfaces in contact.



Angle of Repose: It is the angle of inclination of a plane at which a body placed on it just begins to slide down.



$$\tan\theta = \frac{mg\sin\theta}{mg\cos\theta}$$

$$\tan\theta = F/R = \mu$$

$$\tan\theta = \mu$$

Solved problems:

1). A bullet of mass 200 gm acquires a speed of 400 m/s in a gun barrel of length 1 m. The average force exerted on the bullet is

a) 32×10^3 N b) 16×10^3 N c) 8×10^3 N d) zero

Solution: Acceleration of the bullet $a = \frac{v^2-u^2}{2S}$

$a = 8 \times 10^4\ m/s^2$ and force on the bullet is F = ma

$F=16 \times 1110^3$ N.

2). The weight of a person in a lift moving up with acceleration is 620 N and in a lift coming down with same acceleration is 420 N. The real weight of the person is

a) 1040N b) 500N c) 520N d) 600N

Solution: when lift is moving upwards $W_1=mg+ma$------- (1)

When lift is moving downwards $W_2=mg-ma$------ (2)

Adding above two equations we get mg=W=520 N.

3) Two stones of masses in the ration 4:5 fall freely from heights in the ratio 9:25. When they reach ground the ratio of their momenta is

a) 3:4 b) 25:12 c) 12:25 d) 5:4

solution: ratio of their momenta $P_1/P_2 = m_1v_1 / m_2v_2 = m_1/m_2[\sqrt{(h_1/h_2)}] = 12/25$

correct answer is c).

4) In a head on collision, a sphere of mass 2 kg moving to right with a velocity of 12 m/s collides with another sphere of mass 4 kg moving with same velocity in opposite direction. After collision they stick together and travel with common velocity___________m/s.

a) 4 b) -4 c) 2 d) 8

Solution: from law of conservation of momentum

$m_1u_1+m_2u_2 = m_1v_1+m_2v_2$

$2\times12 - 4\times12 = (2+4)\ V$

OR V = -4 m/s. Correct ans. Is b).

5) A constant force acts on a body of mass 0.9 kg at rest for 10 s. If the body moves a distance of 250 m, the magnitude of the force is

a) 3N b) 3.5 N c) 4 N d) 4.5 N

Solution; Force F= ma and distance S= ut +1/2 at^2 by substituting the given values we get, F=4.5 N

The correct ans. Is d).

6) A mass m kg is rests on a rough surface having coefficient of between the surfaces as μ. A force F at 30^0 with horizontal pulls the mass. Limiting friction between surfaces and mass is

a) F/2 b) μ (mg-F/2) c) μmg d) μmg-F/2

Solution: net reaction = mg-Fsin30^0 = mg-F/2

Frictional force $F_S = \mu R = \mu$ (mg-F/2)

Correct ans. Is (b)

7) A block of mass 20 kg is placed on a rough horizontal plane and a force of 12 N is applied. If coefficient of friction is 0.1 the frictional force acting on it is

a) 20 N b) 12 N c) 8 N d) 28 N

Solution: F = μmg =$0.1\times20\times10$ = 20N

The correct ans. Is (a)

8) A body of weight 10 N is pressed against a vertical wall with a horizontal force of 12 N. If μ = 0.6 the frictional force acting on the body is

a) 8 N b) 5 N c) 7.2 N d) 11 N

Solution: frictional force is self adjusting force, there fore F = μR = 12×0.6 = 7.2 N

The correct ans. Is (c)

9) A body is moving with a velocity of 36 kmph on horizontal surface having coefficient of friction 0.2. The distance traveled by it before coming to rest is

a) 20 m b) 15 m c) 30 m d) 25 m

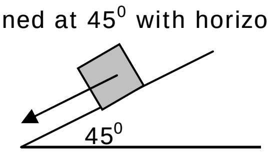
Solution: $1/2mv^2 = \mu mgs$ or $s = v^2/\mu g$ =25m

The correct ans. (d)

10) A mass of 10 kg is lying on a table top. If the table top is inclined at 45^0 with horizontal, the mass juist begins to slide. The coefficient of static friction is

a) 1 b) ½ c) 2 d) none

solution: $\mu = \tan\theta = \tan 45^0 = 1$



MULTIPLE CHOICE QUESTIONS

ANSWERS:

1. (a)	2.(c)	3. (c)	4.(b)	5.(b)	6. (c)	7.(c)	8.(c)	9.(c)	10.(d)
11.(c)	12.(c)	13.(b)	14.(a)	15(b)	16.(a)	17.(a)	18.(b)	19.(a)	20.(d)
21.(b)	22.(c)	23.(c)	24.(a)	25.(c)	26.(d)	27.(c)	28.(b)	29.(c)	30.(b)
31.(d)	32.(c)	33.(b)	34.(c)	35.(c)	36.(b)	37.(d)	38.(d)	39.(d)	40.(a)
41.(b)	42.(c)	43.(c)	44.(d)	45.(c)	46.(a)	47.(a)	48.(d)	49.(c)	50.(d)
51.(d)	52.(a)	53.(d)	54.(b)	55.(c)	56.(c)	57.(b)	58.(b)	59.(b)	60.(d)

WORK-ENERGY-POWER

SUMMARY

WORK: Work is defined as scalar product of vectors force and displacement W=F.S=FScosθ
If force and displacement are in same direction, then W=FS
When $\theta=90^o$ W=O no work is done.
Work is measured in joule.

POWER: It is rate of doing work **P=W/t**
Unit of power is Watt.

ENERGY: It is defined as capacity of doing work and is measured in joule.

POTENTIAL ENERGY: It is defined as energy possessed by a body due to its position
V=mgh
KINETIC ENERGY: Energy possessed by a body due to its motion
$E=1/2Mv^2$ (or) $E=P^2/2m$

Work-Energy theorem: Work done by an unbalanced force on a body is equal to the change in kinetic energy.
$W=1/2mv^2-1/2mu^2$
Law of conservation of Energy: Energy can neither be created nor destroyed but it can only be converted. or Energy can be converted by one form to the other form. The total energy is remains constant

CONSERVATIVE AND NON-CONSERVATIVE FORCES

Conservative force

A force is said to be conservative when the work done by it in moving a body is independent of the path fallowed between initial and final points.
E.g. Gravitational force, electrostatic force, magnetic force,

Non-conservative force

A force is said to be non-conservative when work done by it in moving a body depends on the actual path fallowed between initial and final points.
E.g. Friction, electromagnetic induction

COLLISIONS

When a body strikes anther body, collision is said to occur. However in physics, collisions not necessarily involve one body striking against another.
"**Collision between two bodies is said to occur when they actually strike against each other or path of motion one body is affected by the other"**
Ex.: When an approaching alpha particle deviated by heavy nucleus.

Collisions are classified into two types based on the conservation of energy.
(1) Elastic collision and (2) Inelastic collision

(1) Elastic collision

A collision in which there is no loss of K.E. is known as elastic collision. i.e. in a collision initial K.E. is equal to final K.E.
E.g. Collision between atomic and sub atomic particles

(2) Inelastic collision

A collision in which there is some loss of kinetic energy is called inelastic collision.
E.g. A bullet hitting the wooden plank, a ball bouncing on ground, the two bodies stick together after collision etc.

Coefficient of restitution (e)

The degree of elasticity of collision is determined by a quantity known as coefficient of restitution (e).
"It is the ratio of relative velocity of separation after collision to the relative velocity of approach before collision"

$$e = \frac{\text{Relative velocity of separation of collision}}{\text{Relative velocity of approach before collision}}$$

(OR)

$$e = \frac{V_2 - V1}{U_1 - U_2}$$

Where u_1 and u_2 are initial velocities of the two colliding bodies and v_1 and v_2 are the velocities their final velocities after collision.
For a perfectly elastic collision, e=1.
For a perfectly inelastic collision, e=0.
For all collisions e lies between 0 and1. i.e. $0<e<1$.

Note: When ball is rebouncing the coefficient of restitution is $e = \sqrt{h_2/h_1}$ where h_1 is the height from which the ball is dropped and h_2 is the height of rebound.

SOLVED PROBLEMS:

1) Calculate the work done to take a cement bag of 40kg to top of a building of height at 100m ($g=9.8m/s^2$)
a) 39,200J b) 3,920J c) 3.920J d) 390J

Solution: m=40kg, h=100m and $g=9.8m/s^2$
V=mgh=40×100×9.8
V=39,200J

2) If man weighing 60 kg climbs up a staircase of height 5 m in 10 s, then power used is ($g=9.8\ ms^2$)
a) 200J b) 250J c) 294j d) 350J

Solution: P=W/t= mgh/t =60×9.8×10 = 294J

3) A crane can lift 5000 kg of coal in 50 minute from a mine of 200 m depth. If the crane is 75% efficient. What is the power used ($g=9.8\ ms^2$).
a) 3000J b) 3500J C) 4356J d) 4000J

Solution: POWER P= mgh/t = 5000×9.8×200/60 = 3267J
But efficiency is 75% P= 3267×75% = 4356 J

4) The velocity of a train is increased from 10 m/s to 20 m/s in 5 minutes by the application of some force by a machine of power 1 MW. What is the mass of train if there is no friction?
a) 500 tones b) 2000 tones c) 7000 tones d) 8000 tones

Solution: work done is change in K.E.
$W= 1/2\ mv^2-1/2mu^2 = 1/2m\ (v^2-u^2) =150m$----------- (1)
And work done W = Power×time
$1\times10^6\times s\times60 = 3\times10^8 J$--------------------- (2)
From 1 & 2
$m= 2\times10^6$ kg=2000 tones.

MULTIPLE CHOICE QUESTIONS

ANSWERS:

1. (d)	2.(b)	3. (d)	4.(d)	5.(d)
6. (a)	7.(b)	8.(c)	9.(a)	10.(a)
11.(d)	12.(d)	13.(a)	14.(a)	15(c)
16.(c)	17.(b)	18.(b)	19.(a)	20.(d)
21.(c)	22.(c)	23.(a)	24.(d)	25.(a)
26.(b)	27.(d)	28.()a	29.(a)	30.(a)
31.(a)	32.(a)	33.(b)	34.(b)	35.(a)

GRAVITAION

NEWTON'S LAW OF GRAVITAION:

According to Newton's law of gravitation, the force of attraction between the two point masses m_1 and m_2 is directly proportional to the product of two masses and inversely proportional to square of the distance between them.

$$F \propto m_1 m_2 / r^2 \quad \text{or} \quad F = G m_1 m_2 / r^2$$

Where G is constant of proportionality and is called universal gravitational constant.
Its value is 6.67×10^{-11} N-m^2/kg.

Relation between G and g:

When a body falls freely, it is attracted by the earth with a force given by Newton's law of gravitation. This force known as gravity. Thus the force of gravity produces acceleration in a freely falling body. This acceleration is called acceleration due to gravity.

$g = GM/R^2$ Where R is the radius of the earth and M is its mass.

Variation of acceleration due to gravity:

1. Due to altitude: $g' = g\,[1-2h/R]$ Where h is height from a surface of the earth, R is radius of the Earth.

2. Due to depth [below earth's surface]

$g' = g\,[1-d/R]$ Where d is the depth of inside the earth. R is the radius of the earth.

3. Variation of g due to rotation of the earth of latitude.

$g' = g - R\omega^2\cos^2\theta$ where θ is latitude

a) **at equator**

$\theta = 0$, $g' = g - R\omega^2$

b) **at poles**

$\theta = 90^0$, $g' = g$

The value of g is maximum at poles and minimum at equator.

KEOKER'S LAW OF PLANATORY MOTION

First law: Each planet revolves around the sun in an elliptical orbit with the sun as one of the focus of the ellipse.

Second law: The line joining the sun and the planet sweeps out equal areas in equal intervals of time.

Third law: The squares of the periodic times of the planets are proportional to the cubes of the semi-major axis of their orbits.

$$T^2 \propto R^3$$

Orbital Velocity of a Satellite: It is the velocity required to put the satellite into its orbit around the earth .

$$V_o = \sqrt{gR} = \sqrt{[GM/R]}$$

Period of revolution: Time period of satellite is the time taken by it to complete one revolution around earth.

$$T = \frac{2\pi}{R}\{(R+h)^3/g\}^{1/2}$$

Height of satellite:

$$H = \left|\frac{T^2R^2g}{4\pi^2}\right|^{1/3} - R$$

Geostationary satellite: It is an artificial satellite revolving around the earth such that it appears stationary with respect to earth. i.e. it has time period and sense of rotation equal to that of earth.

Escape velocity: It is the velocity with which a body flies away from a planet to infinity not comes back to the planet.

$$V_e = (2gr)^{1/2} \quad \text{or} \quad V_e = \sqrt{2} \times V_0$$

Where V_0 is the orbital velocity

SOLVED PROBLEMS:

1) The value of g on earth's surface is 9.9 m/s^2. The value at a height of 64 km from the surface of the earth is__m/s^2.

a) 9.6 b) 9.849 c) 9.8245 d) 9.7755

Solution: using $g' = g \times R^2/(R+h)^2 = 9.6$ m/s^2
Correct answer is (a)

2) The escape velocity from the surface of the earth is11.2 km/s. If a planet has twice the radius that of the earth and on which acceleration due to gravity on this planet will be ___________km/s

a) 11.2 b) 5.6 c) 22.4 d) 33.6

solution: escape velocity is given by

$V_e = (2gR^1)^{1/2} = (2 \times 2g \times 2R)^{1/2} = 2 \times (2gR)^{1/2} = 2V_e = 2 \times 11.2$

$V_e = 22.4$ km/s there fore correct answer is (c).

3) Weight of body at earth's surface is W. At a depth half way to the centre of earth it will be
a) W/8 b) W/4 c) W/2 d) W
Solution: we have $g' = g[1-d/R] = g/2$ [because $d=R/2$]
$mg' = mg/2$
$W' = W/2$
The correct answer is (c).

4) The rotation period of an earth satellite close to the surface of earth is 83 minute. The satellite in an orbit at a distance of 3 times earth radii from its surface will be
a) 83 minute b) 249 minute c) $83\sqrt{8}$ minute d) 664 minute

Kepler's relation, we have $T_1^2 \propto R_1^3$ and $T_2^2 \propto R_2^3$
$T_2 = T_1 \times [R_2/R_1]^{3/2}$
$T_2 = 83 \times [R+3R/R]^{3/2}$
$T_2 = 83 \times 8 = 664$ minutes.
There fore correct answer is (d)

5) The weight of the body decreases to 1/3rd of its value at a depth of
a) 3R/2 b) 2R/3 c) 4R/3 d) 3R
Solution: the variation of g with depth is given by
$g' = g[1-d/R] = g/3 = g[1-d/R]$
or $d = 2R/3$
The correct answer is (b)

Exercise Problems

Answers

1. d	2.a	3. b	4. c	5. c
6. c	7. c	8. d	9. b	10. c
11. d	12.a	13.c	14.d	15.c
16.d	17.b	18.b	19.d	29.b
21.c	22.b	23.d	24.c	25.d
26.c	27.c	28.d	29.b	30.d
31.d	32.d	33.c	34.d	35.b