-SOLUTIONS TO COMMON ENTRANCE TEST -MODEL PAPER -6

1) 4	**2). 2**	**3)3**	**4)3**	**5)3**	**6)3**	**7)1**	**8)3**	**9)1**	**10)2**	**11)1**	**12)3**	**13)1**
14)3	**15) 2**	**16)4**	**17)1**	**18)1**	**19)3**	**20)2**	**21)3**	**22)3**	**23)3**	**24)1**	**25)2**	**26)3**
27) 1	**28)3**	**29)2**	**30)3**	**31)1**	**32)2**	**33)4**	**34)1**	**35)2**	**36)3**	**37)3**	**38)4**	**39)1**
40) 4	**41)1**	**42)4**	**43)1**	**44)2**	**45)1**	**46) 3**	**47)4**	**48)3**	**49)4**	**50)1**	**51)2**	**52)3**
53)3	**54)2**	**55)2**	**56) 2**	**57)2**	**58)1**	**59)2**	**60)1**					

Explanatory Notes:

1. $(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b})\times(\vec{a}+\vec{c}) = [\vec{a}+\vec{b}+\vec{c}\ \ \vec{a}+\vec{b}\ \ \vec{a}+\vec{c}]$

$= [\vec{a}+\vec{b}\ \ \vec{a}+\vec{b}\ \ \vec{a}+\vec{c}]+[\vec{c}\ \ \vec{a}+\vec{b}\ \ \vec{a}+\vec{c}]$ $=0+$ $[\vec{c}\ \ \vec{a}+\vec{b}\ \ \vec{a}+\vec{c}] = \vec{c}.[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$

$\vec{c}.[\vec{a}\times(\vec{a}+\vec{c})+\vec{b}\times(\vec{a}+\vec{c})] = \vec{c}.[\vec{a}\times\vec{a}+\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}] = \vec{c}.[0+\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}]$

$\vec{c}.(\vec{a}\times\vec{c})+\vec{c}.(\vec{b}\times\vec{a})+\vec{c}.(\vec{b}\times\vec{c})$ $=0+$ $\vec{c}.(\vec{b}\times\vec{a})$ $+0=$ $-c.(\vec{a}\times\vec{b})=-[a\,b\,c]$

2. Given |a|=3, |b| =4, |c|=5; $\vec{a}\perp(\vec{b}+\vec{c})=0\Rightarrow\vec{a}.\vec{b}+\vec{a}.\vec{c}=0$ ....(1)

Similarly $\vec{b}.(\vec{c}+\vec{a})=0\Rightarrow\vec{b}.\vec{c}+\vec{b}.\vec{a}=0$ ....(2)

$\vec{c}.(\vec{a}+\vec{b})=0\Rightarrow\vec{c}.\vec{a}+\vec{c}.\vec{b}=0$ ....(3)

Adding (1), (2) and (3) we get $2(\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{c})=0$ ....(4)

$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}.\vec{b}+2\vec{b}.\vec{c}+2\vec{c}.\vec{a} = |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+0$ =9+16+25 =50

$|\vec{a}+\vec{b}+\vec{c}|=\sqrt{50}=5\sqrt{2}=\frac{10}{\sqrt{2}}$ choice 2 is correct

3. D.r's of the line joining the points (3,4,5) and (4,5,6) are 1, 1, 1 therefore d.r.s of line are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

4. The diagonals of parallelogram are $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ i.e. 4i +j-k abd -2i+3i -5k . Angle between these diagonals is given by $\cos\theta=\left|\frac{(4i+j-k).(-2i+3i-5k)}{\sqrt{16+1+1}\sqrt{4+9+25}}\right|$ =0 Angle $=90^0$

5. Let $x=1^{1/n}$ $\Rightarrow$ $x^n-1=0$; $(x-1)(x-w^2)..........(x-w^{n-1})=x^n-1$

$(x-w)(x-w^2)...........(x-w^{n-1}) = \frac{x^n-1}{x-1}$ $\therefore$ $(16-w)(16-w^2)..........(16-w^{n-1}) = \frac{16^n-1}{15}$

6. $1+i\sqrt{3}$ =2 cis $\frac{\pi}{3}$ $\therefore$ $(1+i\sqrt{3}i)^{n/2}=2^{\frac{n}{2}}(\cos\frac{n\pi}{6}+i\sin\frac{n\pi}{6})$ $\therefore$ $\sin\frac{n\pi}{6}=0\Rightarrow n=6$

lease value is 6

7. $i^i = \left(e^{\frac{i\pi}{2}}\right)^i = e^{\frac{i^2\pi}{2}} = e^{\frac{-\pi}{2}} = \frac{1}{e^{\frac{\pi}{2}}}$ =(+ve real)

8. $f(\log_e x)=\log_e(\log_e x)$ $\frac{d}{dx}(\log(logx))=\frac{1}{logx}.\frac{1}{x}$

9. $2y\frac{dy}{dx}=3px^2$; $\frac{dy}{dx}_{(2,3)}=2p$; equating slopes 2p=4 $\Rightarrow$ p=2; since (2,3) lies on the curve $y^2=px^3+q$;$(3)^2=p(2)^3+q$ $\Rightarrow$ q =8p +q $\Rightarrow$ 9=16 +q $\Rightarrow$ q=-7

10. By short cut Required area is $\frac{16ab}{3}$

11. By short cut $\int_0^{\frac{\pi}{2}}\frac{dx}{a^2\cos^{2x}+b^2\sin^2 x} = \frac{\pi}{2ab}$ Required integral = $\frac{\pi}{2.4.3}=\frac{\pi}{24}$

12. $\int\frac{acosx+bsinx}{c\,oosx+d\,sinx} = \frac{ac+bd}{c^2+d^2}x+\frac{ad-bc}{c^2+d^2}\log(c\,cosx+d\,sinx)$ (Short cut)

Required integral = $\int\frac{-2\cos x+3\sin x}{4\,oosx+5\,sinx} = \frac{-8+15}{16+25}x+\frac{-10-12}{16+25}\log(4\cos x+5\,sinx)$ Ans : 3

13. A standard problem Ans: 1

14. At x=0 LHL = $\lim_{x\to0-}1-2x$ =1; RHL = $\lim_{x\to0+}x^2+2$ =2; f(0)=2 ; LHL #RHL =f(o)

f is continuous from right but discontinuous from left

15. At x=1, $(x^3-1)]_{x=1}=0$ and $(x-1)]_{x=1}=0$ $\therefore$ values of both branches match at x=1 $\therefore$ f(x) is continuous at x=1

$(x^3-1)']_{x=1}=3x^2]_{x=1}=3$ and $(x-1)']_{x=1}=1$ $\therefore$ Slopes of both branches don't match at x=1 $\therefore$ f(x) is not differentiable

16. 4

17. For ellipse or parabola $e^2+e'^2 \leq 2\#3$; For Hyperbola it is possible (e>1 and e'>) Ans : 1

18. by short cut e= $\sqrt{\frac{B-A}{B}}=\sqrt{\frac{4-3}{4}}=\frac{1}{2}$ Ans : 1

19. Diameter of circle =2r=Distance between parallel lines 2x+3y-9=0 and 2x+3y+19/2=0

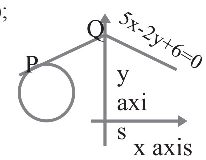
$= \left|\frac{-9-19/2}{\sqrt{4+9}}\right|=\left|\frac{-37}{2\sqrt{13}}\right|$ $\therefore$ r= $\frac{37}{4\sqrt{13}}$

20. For finding the last digit always use congruence with mod 10 ; 43 $43 \equiv 3(mod10)$;
$(43)^{17} \equiv 3^{17}$ (mod 10) i.e. last digit of $(43)^{17}$ is same as 3^{17}
$\therefore$ $3^{17} = 3^{4x4+1} \equiv 1 \times 1 \times 1 \times 1 \times 3$ (mod10) ; $3^{17} \equiv 3$(mod 10) ; Number in unit place is 3.

21. Given equation of hyperbola $\frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$ a= $\frac{12}{5}$ b= $\frac{9}{5}$ $\Rightarrow$ $e^2 = 1 + \frac{b^2}{a^2}$
$= 1 + \frac{81}{144} = \frac{225}{144}$ $\therefore$ e= $\frac{15}{12} = \frac{5}{4}$ $\therefore$ focus is (ae, 0) =(3,0) ; For ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$,
$a^2 = 16$, $b^2 = b^2$, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{b^2}{16}$ $\Rightarrow$ e= $\sqrt{\frac{16-b^2}{16}} = \frac{\sqrt{16-b^2}}{4}$
Focus is (ae, 0) = $(\sqrt{16-b^2}, 0)$ By data foci coincides , hence (3,0) and $(\sqrt{16-b^2}, 0)$ coincide;
$\sqrt{16-b^2} = 3$; $16 - b^2 = 9$ $\Rightarrow$ $b^2 = 7$

22. Given Focal distance x+a =4 , 4a =8 $\therefore$ a=2 , x=4-2=2; $y^2 = 8 \times 2 = 16$ y= ± 4 Ans :3

23. Tangent at a point P on a circle meets 5x-2y+6=0 at a point Q on y axis where Q(0,3);
$\therefore$ PQ =length of tangnet from Q to given circle = $\sqrt{0 + 3^2 + 6.0 + 6.3.2} = 5$



24. $G.E = \lim_{x \to 0} \left(\frac{2^x - 1}{2^x}\right)\left(\frac{\sqrt{tanx + 4} + 2}{tanx + 4 - 4}\right) =$
$\lim_{x \to 0} \left(\frac{2^x - 1}{x}\right) \frac{x}{tanx} \left(\frac{\sqrt{tanx + 4} + 2}{2^x}\right)$ =log2. 1. 4 =4 log2 =log 16

25. Required equation is $(y-k)^2 = 4a(x-h)$, Three arbitrary constants are involved here . so we will differentiate 3 times to eliminate these constants. clearly (1) and (3) cannot be correct options to check (2) differentiate equation w.r.t y we get 2(y-k) =4a $\frac{dx}{dy}$ $\Rightarrow$ y-k =2a $\frac{dx}{dy}$. Differntiating w.r.t y we get
1=2a $\frac{d^2x}{dy^2}$ $\Rightarrow$ $\frac{d^2x}{dy^2} = \frac{1}{2a}$ Again differentiating w.r.t y we get $\frac{d^3x}{dy^3} = 0$

26. $\int \frac{dx}{x(x+1)} = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$ =logx -log(x+1) = $\log \frac{x}{x+1}$
$\int \frac{dx}{x(x+1)} + \int \frac{dx}{y(y+1)} = \int 0\, dx$ $\Rightarrow$ $\log \frac{x}{x+1} + \log \frac{y}{y+1}$ =logc $\Rightarrow$
$\log\left(\frac{xy}{(x+1)(y+1)}\right)$ =log c $\Rightarrow$ cxy =(x+1)(y+1)

27. Here General term $T_{r+1} = {}^9C_r \left(\sqrt[6]{x}\right)^{9-r} \left(\frac{1}{\sqrt[3]{x}}\right)^r = {}^9C_r (-1)^r x^{\frac{9-3r}{r}}$ For term independent of x
$\frac{9-3r}{r} = 0$ $\Rightarrow$ r=3 , required value ${}^{-9}C_3$

28. $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{1}{1-\frac{1}{2}} + \frac{2(\frac{1}{2})}{\frac{1}{4}}$ =2+4=6

29. sinx +siny =a $\Rightarrow$ 2sin $\frac{x+y}{2}$ cos $\frac{x-y}{2}$ =a ; cosx +cosy =b $\Rightarrow$ 2cos $\frac{x+y}{2}$ cos $\frac{x-y}{2}$ =b
$\frac{a}{b}$ =tan $\frac{x+y}{2}$

30. $\tan(\tan^{-1} \frac{\sqrt{1-x^2}}{x}) = \sin(\sin^{-1} \frac{2}{\sqrt{5}})$ $\Rightarrow$ $\frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$ $\Rightarrow$ $\frac{1-x^2}{x} = \frac{4}{5}$ $x^2 = 5/9$
x= $\pm \frac{\sqrt{5}}{3}$ But x= $-\frac{\sqrt{5}}{3}$ does not satisfy the given equation $\therefore$ x= $\frac{\sqrt{5}}{3}$ Ans :3

31. G.E. =bcosA +c cosA +c cosB + a cos B + a cosC + b cos C (Using projcetion formula and arranging)
=c +a+b=a+b+c

32. $\sum \tan^{-1} \frac{1}{1+n+n^2} = \sum \tan^{-1}\left[\frac{n+1-n}{1+(n-1)n}\right] = \sum\left[\tan^{-1}(n+1) - \tan^{-1} n\right]$
$= (\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + \ldots\ldots\ldots\ldots = -\tan^{-1} 1 + \tan^{-1}\infty = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$ Ans :2

33. Ans : b^2 (From direct property)

34. Ans : 1 (From direct property)

35. $4 - x^2 > 0$ $\therefore$ $-2 < x < 2$, note log o is not defined

36. sin2x =tanx $\Rightarrow$ 2 sinx cosx = $\frac{sinx}{cosx}$ either sinx =0 or 2cosx = $\frac{1}{cosx}$ $\Rightarrow$ cosx = $\pm \frac{1}{\sqrt{2}}$
solutions are x= $n\pi$ or $2n\pi \pm \frac{\pi}{4}$ or $2n\pi \pm \frac{3\pi}{4}\left[cosx = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}\right]$

37. cosx cosec2x =cot2x ⇒ $\frac{cosx}{2\sin x\, cosx}$ ⇒ cosx =cos2x ⇒ 2x = $2n\pi \pm x$;

x= $\frac{2n\pi}{2\pm1}=\frac{2n\pi}{3}$ or $2n\pi$

38. A-B={a, b} ; B-A={e, f} ; (A-B)x (B-A) ={(a, e), (a, f) , (b, e) (b, f)} , 4 elements.

39. $P\wedge q$ is false (one of them is false) $P\vee q$ is true (one of them is true) and $False \rightarrow true$ is true

40. G.E = $\log_{10^{-2}} 10^{-4}$ = $\frac{-4\log 10}{-2\log 10}$ =2

41. Let a*b =a then ab=a ⇒ b=1

42. f is a function , as it is defined for all 1,2 and 3. It is one-one because 1 and 3 map into 'a' , it is onto. Ans : 4

43. Ans ; 1; Symmetric matrix has diagonally opposite elements equal; and skew symmetric matrix has diagonal elements zero and diagonally oppoiste elements have opposite sign. thus only zero matrix can have both the properties.

44. Ans: a ; Let y= $\frac{x-1}{3}$; x=3y+1; ∴ g(x) =3x-1

45. 960 =2^6 x 3 x 5 ∴ number of divisors =(6+1)(1+1)(1+1) =28

46. |adj A| =$|A|^{n-1}$ (By property) where n is the number of rows or order of square matrix;

∴ 125=5^3 $\Rightarrow n=4$

47. $x^{1/x}$ is maximum at x=e ∴ maximum value =$e^{1/e}$

48. V= $\frac{4}{3}\pi r^3$; $\frac{dv}{dt}$ = $\frac{4}{3}\pi . 3r^2\frac{dr}{dt}$ = $4\pi\times2.5\times3$ =75 π

49. When x=0, y=b $\frac{dy}{dx}=\frac{-b}{a}e^{\frac{-x}{a}}=-\frac{b}{a}$; y-b = $-\frac{b}{a}$ (x-0) ⇒ ay-ab = -bx; ∴ bx +ay =ab

50. Ans :1

51. Ans: 2

52. Ans: 3 Note that it is not a upper or lower triangular matrix to choose option 2;

Characteristic equation is $\begin{vmatrix}-\lambda & 0 & -1\\ 0 & 2-\lambda & 0\\ -4 & 2 & 3-\lambda\end{vmatrix}=0$ ⇒ $-\lambda(2-\lambda)(3-\lambda)-[0+4(2-\lambda)]$ =0

$(2-\lambda)[(-\lambda(3-\lambda))-4]$ =0 ⇒ $(2-\lambda)[\lambda^2-3\lambda-4]=0$; $(2-\lambda)[(\lambda-4)(\lambda+1)]=0\Rightarrow$ $\lambda=2,\ ,4\ ,-1$

53. If a_1 , a_2 , a_3 ....a_n are in G.P. then loga_1 , loga_2 , loga_3 .... loga_n are in A.P. Let loga_1 =a, log a_2 =a+d etc then

G.E. = $\begin{vmatrix}a & a+d & a+2d\\ a+3d & a+4d & a+5d\\ a+6d & a+7d & a+8d\end{vmatrix}$ = $\begin{vmatrix}a & d & 2d\\ a+3d & d & 2d\\ a+6d & d & 2d\end{vmatrix}$ =0 by c_2-c_1 and c_3 -c_1

54. detA=-1(1-0) =-1 adjA = $\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$ $\frac{adjA}{detA}$ = $\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$ =A'

55. Using fact (short cut) the image of (h, k) in the line ax+by+c=0 is given by

$\frac{x-h}{a}=\frac{y-k}{b}=-\frac{2(ah+bk+c)}{a^2+b^2}$ ∴ $\frac{x-2}{1}=\frac{y-3}{3}=-\frac{2(1)+3(3)+4}{10}$ x=-1, y=-6

56. m_1 +m_2 = $\frac{-2h}{4}$ ....(1) m_1 m_2 = $\frac{3}{4}$ ....(2) ; m_1 =3m_2 (By data) , From (1) 3m_2 = $\frac{-2h}{4}$ and m_2 = $\frac{-2h}{12}$ = $-\frac{h}{6}$; From (2) 3m_2^2 = $\frac{3}{4}$ ⇒ m_2 = $\pm\frac{1}{2}$; m_1 =4 or -4

Ans: both 2 and 3

57. Since 2 and 3 are the roots of f(x)=0 ⇒ f(2) =4m +n-10 =0 and f(3) =9m +n+15=0 ; Solving these equations we get m=-5, n=30 ∴ choice 2 is correct

58. 10^n +3.4^{n+2} +5 ; Put n=1 =10^1 +3.4^{1+2} +5 =10+3.64+5 =207 which is divisible by 9 ∴ choice 1 correct

59. G.E=($\cos^2 5^0$ +$\cos^2 85^0$) +($\cos^2 10^0$ +$\cos^2 80^0$)+...... ($\cos^2 40^0$ +$\cos^2 50^0$)+($\cos^2 45^0$ +$\cos^2 90^0$)

=($\cos^2 5^0$ +$\sin^2 5^0$) +($\cos^2 10^0$ +$\sin^2 10^0$) +.........($\cos^2 40^0$ + $\sin^2 40^0$)+ $\frac{1}{2}$ +0

=8+ $\frac{1}{2}$ = $\frac{17}{2}$

60. I = $\int_0^a f(x)g(x)dx$ = $\int_0^a f(a-x)g(a-x)dx$ = $\int_0^a f(x)g(a-x)dx$ [f(a-x)=f(x)]

⇒ 2I = $\int_0^a f(x)\{g(x)+g(a-x)\}dx$ = $\int_0^a 5f(x)$ ∴ I = $\frac{5}{2}\int_0^a f(x)dx$