SOLUTION TO CET MOCK TEST 7

01.Solution:Let a=AR^{p-1}, $b = AR^{q-1}, c = AR^{r-1}$

$LHS = [(q-r)+(1)r-p)+(p-q)]\log A + [(p-1)(q-r)$

$+(q-1)(r-p)+(r-1)(p-q)]\log R$

$=0$

$Ans:(a)$

02.Solution:0.1353535..... =0.1+0.035+0.00035+...

$$=0.1+\left[\frac{35}{10^3}+\frac{35}{10^5}+........\right]$$

$$=0.1+\frac{35/10^3}{1-\frac{1}{10^2}} = \frac{1}{10}+\frac{35}{990} = 67/495$$

Ans:(c)

03. Solution:put x=$\sqrt{6+\sqrt{6}+..........} \Rightarrow x^2$=6+x

x^2-x-6=0 we get x=3 or x=-2,x $\neq$ -2 $\therefore$ x=3

Ans:a)

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04.Solution: put n=1 we get $\frac{3!}{0!}.Thus, it$ is divisible by 6.

Ans:(c)

05.$Solution: T_{r+1} = {}^nC_r.x^{n-r}.a^r$

$$T_{12} = {}^{13}C_{11}x^{13-11}\left(\frac{1}{x}\right)^{11} = \frac{13!}{2!11!}.x^2.\frac{1}{x^{11}} = 13.6.x^{-9} = 78x^{-9}$$

$Ans: c)$

06.$Solution: T_{r+1} = {}^nC_r.x^{n-r}.a^r$

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$Ans: c)$

07.Solution:It is obvious that (d) is not the answer, if (b)is correct, consider

f(x_1) = $f(x_2) \Rightarrow x_1^{\,2} = x^2{}_1$, this does not imply that $x_1 = x_2$ always.

Hence, (b) is not the answer. Correct option is (a)

08. Answer(c)

09.$Solution : Mid$ point formula

$$\left(\frac{5+x}{2},\frac{3+y}{2}\right)=(7,2)$$

$Ans:(9,1)$ (c)

10.Solution:$\sqrt{(x-2)^2+(y-3)^2}$=0

we get, Ans:(a)

11.Solution: x+y-3=0 &2x-3y+8=0 then we get $(\frac{1}{5},\frac{14}{5})$

Ans:(c)

12.$Solution : m, 3m$ be the slopes. Then, m+3m=-$\frac{2h}{b}, m.3m=\frac{a}{b}$

m=-$\frac{h}{2b}\Rightarrow 3\left(-\frac{h}{2b}\right)^2=\frac{a}{b}\Rightarrow 3h^2=4ab$

$Ans:(b)$)

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13.$Solution : A.P : a-d, a, a+d \Rightarrow a-d+a+a+d=180^0 \Rightarrow a=60^0$

$\Rightarrow (60-d)^0, 60^0, (60+d)^0$

By hypothesis, $\frac{60^0-d}{\frac{\pi}{180}(60^0+d)}=\frac{60}{\pi}$

$Simplifing$ we get d=30^0

The angles of the traingle are $30^0, 60^0, 90^0$

$Ans:a)$

14.$Solution : Taking$ $\frac{1+n}{1-n}=\frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta}=\frac{2\sin(\theta+2\alpha)\cos\alpha}{2\cos(\theta+2\alpha)\sin\alpha}$

$\Rightarrow$ (1-n).tan(θ+α)=(1-n).tanα

$Ans:c)$

15.Solution:using $a^2=b^2+c^2-2bc.CosA \Rightarrow a=\sqrt{2}$

$Then, B$ can be found from the cosine rule:CosB=$\frac{c^2+a^2-b^2}{2ca}$

$\Rightarrow B=45^0$ then C=180-75=105^0

$Ans:a)$

16.Answer (b)

17.Ans (a)

18.Ans(d)

19.$Solution: 13^{130} \equiv 9^{65} \equiv (-1)^{65} \equiv 9(\text{mod}10) \therefore The$ last digit is 9

Ans:c)

20.Solution: $n(n^2-1) = (n-1).n(n+1)$, a product of these consecutive integers and hence it is divisible by $3\dagger = 6$. Now (n-1) and (n+1) are both consecutive even numbers and hence one of them is a multiple of 4. $\therefore$ $n(n^2-1)$ is divisible by 24.

Ans:c)

21.$Solution: X = 3C - B - 2A = \begin{pmatrix} -8 & -3 \\ -2 & -21 \end{pmatrix}$

$Ans:(a)$

22.Solution: $\frac{\pi}{12} = 15^0$ expanding we get

$= \left(\frac{1}{2}\right)^2 \left(-\frac{1}{2}\right)^2 = \frac{1}{16}$

$Ans: d)$

23.$Solution: Operating$ $R_1 + (\sin\beta)R_2 + (\cos\beta)R_3$

$$\Delta = \begin{vmatrix} 0 & 0 & 1+\cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$$

$= (1+\cos 2\beta)(\sin^2\alpha + \cos^2\alpha) = 1 + \cos 2\beta$

which is independent of α

$Ans: a)$

24.$Solution := \begin{pmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{pmatrix}$

$|A - \lambda I| = 0 \Rightarrow \lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\Rightarrow \lambda = 1,1,5$

$Ans: d)$

25.Solution: $\omega.\omega^2 = \omega^3 \quad \therefore \omega^{-1} = \omega^2$ which is of (d). Observe that $1/\omega=\omega^2$ *but* (c) is not the correct alternative because ω^2 is already present in (d). Moreover, in the given system one can also say that $1/\omega$ is meaningless because the binary operation in the group is multiplication and not division.

26. In $Z_6, 5^{-1} = \textit{the}$ inverse of 5=1 $\because$ 5+1=0,the identity

Now, $3+5^{-1} = 3+1 = 4 \quad \because (3+5^{-1})^{-1} = 4^{-1} = 2 \because 4+2=0$

Ans:(c)

27. Any subgroup of a group must contain the identity element. If you look at the alternatives Only(a) i.e. $\{0,3\}$ contains the identity 0 and none of these is not present . $\therefore$ (a) is the correct Answer:

28.*Solution* : Pr*ojection* of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|} = 19/9$

Ans : *c*)

29.*Solution* : $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}.\vec{b}) = 1+1|\vec{a}||\vec{b}|\cos\theta$

$= 2(1-\cos\theta) = 4\sin^2\dfrac{\theta}{2} \Rightarrow |\vec{a}-\vec{b}| = 2\sin\dfrac{\theta}{2}$

Ans : *a*)

30.*Solution* : We have $\tan^{-1}\left[\tan^{-1}\dfrac{d}{1+a_1a_2} + \tan^{-1}\dfrac{d}{1+a_2a_3} + \ldots\ldots\ldots\ldots\ldots\tan^{-1}\dfrac{d}{1+a_{n-1}a_n}\right]$

$= \tan^{-1}\left(\dfrac{a_2-a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\dfrac{a_3-a_2}{1+a_2a_3}\right) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \tan^{-1}\left(\dfrac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)$

$= (\tan^{-1}a_2 - \tan^{-1}a_1) + (\tan^{-1}a_3 - \tan^{-1}a_2) + \ldots\ldots\ldots\ldots\ldots + (\tan^{-1}a_n - \tan^{-1}a_{n-1})$

$= (\tan^{-1}a_n - \tan^{-1}a_1) = \tan\left(\dfrac{a_n-a_1}{1+a_na_1}\right) = \tan^{-1}\left(\dfrac{(n-1)d}{1+a_1a_n}\right)$

$G.E = \tan\left(\tan^{-1}\left(\dfrac{(n-1)d}{1+a_1a_n}\right)\right) = \dfrac{(n-1)d}{1+a_1a_n}$

Ans : *a*)

31.*Solution* : $\left(\dfrac{1}{4}\sin^{-1}x\right) = \cos^{-1}\dfrac{\sqrt{3}+1}{2\sqrt{2}} = 15^0 \Rightarrow \sin^{-1}x = 60^0$

$\Rightarrow x = \sqrt{3}/2$

Ans : *d*)

32.Solution:$S_\infty = \dfrac{\sin^2 x}{1-\sin^2 x} = \dfrac{\sin^2 x}{\cos^2 x} = \tan^2 x$

$L = e^{\tan^2 x \text{ x } \log 2} = e^{\log 2^{\tan^2 x}} = 2^{\tan^2 x}$..........(1)

and the roots of $x^2 - 9x + 8 = 0$ are 1 & 8

$\therefore 2^{\tan^2 x} = 1 = 2^0, 2^{\tan^2 x} = 8 = 2^3$

$\tan^2 x = 0, \tan^2 x = 3 \Rightarrow \tan x = 0, \tan x = \pm\sqrt{3}$

$\therefore x = \pi / 3$ is the only value of x such that $0 \prec x \prec \pi/2$

$$\Rightarrow \frac{\cos x}{\cos x + \sin x} = \frac{1}{1+\tan x} = \frac{1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{3-1} = \frac{1}{2}(\sqrt{3}-1)$$

Ans : (*b*)

33.Solution:we have $e^{i\theta}.e^{2i\theta}.............e^{ni\theta} = 1$

$$\Rightarrow e^{i(1+2+.......+n)\theta} = 1 \Rightarrow e^{i.\frac{n(n+1)}{2}\theta} = 1$$

$$\Rightarrow \cos\frac{n(n+1)}{2}\theta + i\sin\frac{n(n+1)}{2}\theta = 1$$

$$\Rightarrow \cos\frac{n(n+1)}{2}\theta = 1, \sin\frac{n(n+1)}{2}\theta = 0$$

$$\Rightarrow \frac{n(n+1)}{2}\theta = 2m\pi, m \in z$$

$$\theta = \frac{4m\pi}{n(n+1)}, m \in z$$

Ans : (*c*)

34.*Solution* : $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix}$

operating : $R_1 \to R_1 + R_3$

$= 0(R_1$ & R_2 *are* same)

Ans:a)

35.$Solution: Two$ circles:$x^2+y^2-2ax+c^2=0$ & $x^2+y^2-2by+c^2=0$

$\Rightarrow C_1(a,0), C_2(o,b) \quad \therefore r_1=\sqrt{a^2-c^2}, r_2=\sqrt{b^2-c^2}$

Since, the two circles touch each other externally $\Rightarrow C_1C_2=r_1+r_2$

$\Rightarrow \sqrt{a^2+b^2}=\sqrt{a^2-c^2}+\sqrt{b^2+c^2}$

Squaring both sides,simplication we get,

$c^4=a^2b^2-c^2(a^2+b^2)+c^4$

$\Rightarrow a^2b^2=c^2(a^2+b^2)$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

$Ans:(c)$

36.$Solution: Circles$ cut orthogonally is $2gg'+2ff'=c+c'$

$\Rightarrow 0+2k^2=6+k$

$\Rightarrow 2k^2-k-6=0$

$\Rightarrow k=2$ or k=-3/2

Ans:(a)

37.$Solution: y^2-2y=x-2$

$\Rightarrow (y-1)^2=1.(x-1)$

$\Rightarrow 4a=1$, *focus* is X=A,Y=0

or x-1=1/4 ,y-1=0 $\therefore$ (5/4,1)

Ans:d)

38.$Solution: \frac{2b^2}{a}=\frac{1}{2}.2b \Rightarrow 2b=a \Rightarrow 4b^2=a^2$

$\Rightarrow a^2=4a^2(1-e^2) \Rightarrow e=\frac{\sqrt{3}}{2}$

Ans:c

39.Solution.:$25x^2-16y^2=400 \Rightarrow \frac{x^2}{16}-\frac{y^2}{25}=1$

Chord bisected at (6,2)

$\Rightarrow \frac{6x}{16}-\frac{2y}{25}=\frac{6^2}{16}-\frac{2^2}{25}=1$

$\Rightarrow 150x-32y=900-64$

$\Rightarrow 75x-16y=418$

$Ans: b)$

40.Solution:f(x)= $\frac{\tan x+\sec x-(\tan^2 x-\sec^2 x)}{\tan x-\sec x+1}=\frac{(\tan x+\sec x)(\tan x-\sec x+1)}{(\tan x-\sec x+1)}$

$f(x)=\sec x+\tan x \Rightarrow f'(x)=\sec x(\sec x+\tan x)$

$Ans : c)$

41.$Solution : y=\cos 3x.\cos 4x=\frac{1}{2}(\cos 7x+\cos x)$

$\Rightarrow y_n=\frac{1}{2}\left(\frac{d^n}{dx^n}(\cos 7x)+\frac{d^n}{dx^n}(\cos x)\right)$

$=\frac{1}{2}\left[7^n\cos\left(\frac{n\pi}{2}+7x\right)+\cos\left(\frac{n\pi}{2}+x\right)\right]$

$Ans : (c)$

42.$Solution :$ let u=$\sec^{-1}\left(\frac{1}{2x^2-1}\right).v=\sqrt{1-x^2}$

put x=cos$\theta \Rightarrow$ u=2θ,v=sinθ

$\left[\frac{du}{dv}\right]=\frac{du/d\theta}{dv/d\theta}=\frac{2}{\cos\theta}=\frac{2}{x}$

$\left[\frac{du}{dv}\right]_{x=1/2}=4$

$Ans : b)$

43.Solution:siny=x.sin(a+y) $\Rightarrow$ siny=x(sina.cosy+cosa.siny)

$\Rightarrow$ tany=x(sina+cosa.tany) $\Rightarrow$ (tany)(1-x.cosa)=x.sina

$\Rightarrow \tan y=\frac{x.\sin a}{1-x.\cos a}\Rightarrow y=\tan^{-1}\left(\frac{x.\sin a}{1-x.\cos a}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{1}{1+\frac{x^2.\sin a}{(1-x.\cos a)^2}}$ x $\frac{(1-x.\cos a).\sin a-x.\sin a(-\cos a)}{(1-x.\cos a)^2}$

$\Rightarrow \frac{dy}{dx}=\frac{\sin a}{1+x^2-2x.\cos a}\Rightarrow \lambda=\sin a$

$Ans : a)$

44.$Solution : y_1=\cos(m\sin^{-1}x).\frac{m}{\sqrt{1-x^2}}$

$\Rightarrow (1-x^2)y_1^2=m^2(1-y^2)$: *Difftn.* w.r.t.x.again we get

$\Rightarrow (1-x^2)y_2-xy_1+m^2y=0$

$Ans : c)$

45.Solution: $\frac{dy}{dx}=6-2x\ \ \therefore \frac{dy}{dx}=0\Rightarrow x=3, y=9$

$Ans: d)$

46.Solution: $\frac{dy}{dx}$=$a^{1-n}nx^{n-1} \Rightarrow$ *Length* of SN=y.$\frac{dy}{dx}=n(a^{1-n})^2 x^{2n-1}$

$\Rightarrow$ Since length of SN is of constant length,therefore 2n-1=0 $\Rightarrow$ n=1/2

Ans:a)

47.*Solution* : *Solving* y=4-x^2 & $y=x^2 \Rightarrow$ *po*int of intersection($\sqrt{2}$, 2)

m_1 = *slope* of tangent=-$2\sqrt{2}$

m_2 = slope of tangent=$2\sqrt{2}$

$$\therefore \tan\theta=\left|\frac{4\sqrt{2}}{1-8}\right|=\frac{4\sqrt{2}}{7}\Rightarrow \theta=\tan^{-1}(4\sqrt{2}/7)$$

$Ans: c)$

48.*Solution* : *Let.l = length of the sides of the equilateral triagnle*

$$Area = A=\frac{\sqrt{3}}{4}l^2 \Rightarrow \frac{dA}{dt}=\frac{\sqrt{3}}{2}.l.\frac{dl}{dt}$$

$$\Rightarrow \frac{dA}{dt}=\frac{\sqrt{3}}{2}.l.2=\sqrt{3}l$$

$$\Rightarrow \left(\frac{dA}{dt}\right)_{l=10}=10\sqrt{3}sq.units$$

$Ans: c)$

49.*Solution* : $y=\frac{\log x}{x}$

$$\frac{dy}{dx}=\log x.\left(-\frac{1}{x^2}\right)+\frac{1}{x}.\frac{1}{x}=-\frac{1}{x^2}(\log x-1)$$

$$\frac{dy}{dx}=0\Rightarrow x=e\ \&\ \frac{d^2y}{dx^2}\angle 0$$

$$Max.Value=\frac{\log e}{e}=1/e$$

$Ans: c)$

$50.Solution: put.x=\cos\theta.we.get$

$=\cos\left[2\cot^{-1}(\tan\frac{\theta}{2})\right]=\cos\left[2(\frac{\pi}{2}-\frac{\theta}{2})\right]=\cos(\pi-\theta)=-\cos\theta$

$\Rightarrow -x$

$\Rightarrow I=-\frac{x^2}{2}+c$

$Ans: c)$

$51.Solution: I=\sqrt{2}\int\sqrt{1-\cos\left(\frac{\pi}{2}+x\right)}dx=\sqrt{2}\int\sqrt{2\sin^2\frac{1}{2}\left(\frac{\pi}{2}+x\right)}dx$

$=2\int\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx=\frac{-2.\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{1/2}+c$

$\Rightarrow a=1/2, b=\pi/4$

$Ans: a)$

$52.Solution: I=\int(fog)(x)\cos xdx=\int f(g(x)).\cos xdx$

$=\int f(\sin x)\cos xdx=\int\frac{\sin^2 x}{1+\sin^2 x}.\cos xdx=\int\frac{t^2}{1+t^2}dt$

$where.\sin x=t\Rightarrow\cos xdx=dt$

$\int(1-\frac{1}{1+t^2})dt=t-\tan^{-1}t+c$

$=\sin x-\tan^{-1}(\sin x)+c$

$Ans: a)$

53.Solution:3cosx+2sinx=k(4sinx+5cosx)+$\frac{2}{41}\log(4\cos x-5\sin x)$

$\Rightarrow 3=5k+\frac{8}{41}\Rightarrow k=\frac{23}{41}\therefore\ f(x)=\int kdx=\frac{23}{41}x$

$Ans: b)$

$54.Solution: I=\int\left(\log x+\frac{1}{x}\right)e^x dx=e^x\log x+c$

$Ans: c)$

$55.Solution: -1 \leq x \leq 1 \Rightarrow 1-x \geq 0$

$$I = \int_{-1}^{1} |1-x| dx = \left[x - \frac{x^2}{2} \right]_{-1}^{1} = 2$$

$Ans: c)$

$$56.\text{Solution:} I_8 + I_6 = \int_0^{\pi/4} (\tan^8\theta + \tan^6\theta).d\theta = \int_0^{\pi/4} (\tan^6\theta.\sec^2\theta).d\theta$$

$$= \left[\frac{\tan^7\theta}{7} \right]_0^{\pi/4} = 1/7$$

$Ans: d)$

$57.Solution: y = x - x^2 \;\&\; y = mx \Rightarrow x = 0, 1-m$

$$Area = \int_0^{1-m} (x - x^2 - mx)dx = \left[(1-m)\frac{x^2}{2} - \frac{x^3}{3} \right]_0^{1-m}$$

$$= \left[(1-m)\frac{x^2}{2} - \frac{x^3}{3} \right]_0^{1-m} = (1-m)^3 \left(\frac{1}{2} - \frac{1}{3} \right) = \pm\frac{9}{2}$$

$Taking + sign, (1-m)^3 = 27 \Rightarrow m = -2$

$Taking - sign, \text{(1-m)}^3 = -27 \Rightarrow m = 4$

$Ans: d)$

58.Solution:$(x-a)^2 + y^2 = a^2$.Circle with radius a

Ans:a)

$$59.\text{Solution:} \left[1 + 2\left(\frac{dy}{dx} \right)^2 \right]^3 = 25 \left(\frac{d^2y}{dx^3} \right)^2$$

$Degree = 2$

$Ans: b)$

60.*Solution* : *put* x-y=z $\Rightarrow 1-\frac{dy}{dx}=\frac{dz}{dx}$

$\therefore$ *The* given equation becomes $1-\frac{dz}{dx}=\cos z$

$$\frac{dz}{dx}=1-\cos z \Rightarrow \int \frac{dz}{1-\cos z}=x+c$$

$$\int \frac{1}{2}\operatorname{cosec}^2\frac{z}{2}dz=x+c \Rightarrow -\cot\frac{z}{2}=x+c$$

$\Rightarrow$ x+cot $\frac{x-y}{2}$ =c

Ans:b)