Mock CET-IV

Keys

1.	(1)	2.	(1)	3.	(3)	4.	(2)	5.	(4)	6.	(3)
7.	(1)	8.	(2)	9.	(3)	10.	(1)	11.	(3)	12.	(4)
13.	(2)	14.	(2)	15.	(4)	16.	(1)	17.	(2)	18.	(4)
19.	(2)	20.	(4)	21.	(3)	22.	(2)	23.	(2)	24.	(1)
25.	(2)	26.	(4)	27.	(3)	28.	(1)	29.	(2)	30.	(1)
31.	(3)	32.	(4)	33.	(2)	34.	(4)	35.	(3)	36.	(1)
37.	(3)	38.	(3)	39.	(4)	40.	(4)	41.	(2)	42.	(4)
43.	(1)	44.	(4)	45.	(2)	46.	(3)	47.	(4)	48.	(2)
49.	(3)	50.	(2)	51.	(4)	52.	(2)	53.	(2)	54.	(4)
55.	(3)	56.	(1)	57.	(3)	58.	(4)	59.	(3)	60.	(3)

Solutions

1. given $\Rightarrow \left|\dfrac{(x-2)+iy}{(x-1)+i(y-1)}\right| = 1$

$$\Rightarrow (x-2)^2 + y^2 = (x-1)^2 + (y-1)^2 \Rightarrow x - y + 1 = 0$$

2 given $\Rightarrow \tan^{-1}\left[\dfrac{(x-1)+x+(x+1)-x(x-1)(x+1)}{1-(x-1)x-i(x+1)(x-1)}\right]$

$$\Rightarrow 3x - x(x^2-1) = 0 \quad \therefore x = 0,\ \ 2 - x^2 = 0$$

$$\therefore\ x = 0,\ \ \pm\sqrt{2}$$

3. $given \Rightarrow 1 - \tan 2x \tan 3x = 0$

$$\Rightarrow \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x} = \infty \ \ \therefore \tan 5x = \infty$$

$$\therefore 5x = n\pi + \pi/2 \quad \therefore n = \frac{n\pi}{5} + \pi/10$$

4. $given \Rightarrow \sqrt{2}\, e^{-i\pi/4} \quad < \sqrt{2}e^{i\theta} \quad < \sqrt{2}e^{-i\pi/4}$

$$\Rightarrow e^{-i\pi/4} \quad < e^{i\theta} \quad < e^{+i\pi/4} \quad \Rightarrow -\pi/4 < \theta \quad < +\pi/4 \quad \therefore \theta \in (-\pi/4,\ \pi/4)$$

5. $given \Rightarrow 2\sin 4x\cos 4x = 1 \Rightarrow \sin 8x = 1$

$$\therefore 8x = n\pi + (-1)^n \frac{\pi}{2} \quad \therefore\ x = \frac{n\pi}{8} + (-1)^n \frac{\pi}{16}$$

6. condition of existence of inverse trigonometric functions.

7. $given \Rightarrow 2\cos 2\theta \cos\theta + \cos 2\theta = 0$

$$\therefore \cos 2\theta\ (2\cos\theta + 1) = 0 \quad \therefore \cos 2\theta = 0, \cos\theta = -1/2$$

$$\therefore\ 2\theta = 2n\pi \pm \pi/2 \ \therefore \theta = n\pi \pm \frac{\pi}{4} \quad \therefore \theta = 2n\pi \pm 2\pi/3$$

8. $given = 1 + \dfrac{1}{\left(\dfrac{1-i+1}{1-i}\right)} = \dfrac{1}{1} + \dfrac{1-i}{-i} \quad = \dfrac{-i+1-i}{-i} \ = \ \dfrac{1-2i}{-i} \ = 2 - \dfrac{1}{i} \ = 2 + i$

9. $x_1.x_2.x_3........ = cis\dfrac{\pi}{3}.cis\dfrac{\pi}{3^2}.\ ....$

$$= cis\,\pi\left(\frac{1}{3} + \frac{1}{3^2} + ...\right) = cis\left[\pi\left(\frac{1/3}{1-1/3}\right)\right] \quad [by\ S_\infty\ of\ a\ g.p. with\ r < 1]$$

$$= cis\left(\frac{\pi/3}{2/3}\right) \ = cis\,\pi/2 = i$$

10. $given \Rightarrow \cos 12\theta - \cos\theta = 20\theta - \cos 2\theta$

$$\therefore \cos 20\theta = \cos 12\theta \ \Rightarrow -2\sin 16\theta \sin 4\theta = 0$$

$$\therefore \sin 16\theta = 0\ \ or\ \sin 4\theta = 0 \quad \therefore 16\theta = n\pi,\ 4\theta = n\pi \quad \therefore \theta = \frac{n\pi}{16},\quad \theta = \frac{n\pi}{4}$$

11. $given \Rightarrow \cos^{-1}x = \cos^{-1}y = \cos^{-1}z = \pi$

$$\Rightarrow x = y = z = \cos\pi = -1$$

$$\therefore x + y + z = -1 - 1 - 1 = -3\ \&\ \frac{1}{x+y+z} = \frac{1}{-3} = -\frac{1}{3}$$

$$\therefore\ x + y + z + \frac{1}{x+y+z} = -\frac{3}{1} - \frac{1}{3} = \frac{-10}{3}$$

12. $given \Rightarrow \left(\sin^{-1}x+\cos^{-1}x\right)+\cos^{-1}x=\dfrac{\pi}{3}$

$\Rightarrow \dfrac{\pi}{2}+\cos^{-1}x=\dfrac{\pi}{3}\ \therefore \cos^{-1}x=-\dfrac{\pi}{6}\ \therefore \cos\dfrac{\pi}{6}=\dfrac{\sqrt{3}}{2}$

13. $use\ C_1(-g,-f), C_2(-g_1,-f_1)\ and\ origion\ are\ collinear.$

$\therefore slope\ of\ C_1C_2 = slope\ OC_1\ and\ simplify$

14. Equation of circle passing through A(2, 0), B(0, 2) & (0, 0) is

$x^2+y^2-2x-2y=0$ Since O(a, 2a) lies on this, $a^2+4a^2-2a-4a=0$

$\Rightarrow 5a^2-6a=0\quad \therefore\ \pi(5a-6)=0\ \therefore\ a=0,\ 6/5$

15. condition $\Rightarrow$ 2(1) (3/2) + 2(2) (-3) = ~~3~~+k $\Rightarrow$ ~~3~~ -12 = 3+k $\therefore$ k=-12

16. $R_1 : S_1 - S_2 = 0$ i.e. $-y + 1 = 0\ \therefore\ y = 1$

$R_2 : S_2 - S_3 = 0$ i.e. $x-2y+1=0\ \therefore\ x-2+1=0,\ x=1$

17. Center & radius of given circle are C(1, 1) & r=1. Required locus is a circle with C(1, 1) & r = 1 + 4 =5.

$\therefore$ Its equation is $(x-1)^2+(y-1)^2=5^2 \Rightarrow x^2+y^2-2x-2y-23=0.$

18. $given\ g_1 = 1 = f_1 = c_1\ :\ g_2 = \dfrac{1}{2} = \dfrac{1}{2} : c_2 = 1$

$2g_1g_2 + 2f_1f_2 = 2(1)\dfrac{(1)}{2}+2(1)\left(\dfrac{1}{2}\right)=2$

$also,\ c_1+c_2 = 1+1 = 2\ \therefore the\ condition\ is\ satisfied\ \therefore\ they\ cut\ orthogonally$

19. $given \Rightarrow c=0\ \&\ -g-f-1=0\ \therefore g+f=-1$

$l=\sqrt{s}\Rightarrow 1=1+4+2g+4f+c........(1)$

$\therefore 2g+4f=-4\ \Rightarrow g+2f=-2...........(2)$

$(1)\&(2)\Rightarrow -f=1\ \therefore f=-1 : g=-1-f=-1+1=0$

$\therefore R.E.\ is\ x^2+y^2-2y=0$

20. given $2y^2-3x+6y-1=0 \rightarrow (1)$

$\Rightarrow 4y+4=0\ \therefore\ y=-1$ (by diff. wrt y treating x as constant)

$\therefore$ (1) $\Rightarrow$ 2-3x-4-1 = 0 $\therefore$ 3x = -3 $\therefore$ x = -1

21. $given \quad h = 2,\ k = 1 : h + a = 3 \Rightarrow a = 3 - 2 = 1$

$\because VS \ // OX \ \therefore axis \ // to \ x - axis$

$\therefore R.E.\ is (y-k)^2 = 4a(x-h)$

$\Rightarrow (y-1)^2 = 4(x-2) \Rightarrow y^2 - 4x - 2y + 9 = 0.$

22. $given, a^2 = 25,\ b^2 = 9,\ h = 1 = k$

$ae = \sqrt{a^2 - b^2} = \sqrt{16} = 4$

$focii\ are\ (h \pm ae,\ k) = (1 \pm 4,\ 1) \therefore (5,1)\ \&\ (-3,1)$

23. $l:\ y = \frac{-2x}{3} + \frac{k}{3} \ \therefore m = -2/3 : C = K/3 : a^2 = 9, b^2 = 4$

$condition\ is\ C^2 = a^2m^2 + b^2 \ \Rightarrow \frac{k^2}{9} = 9\left(\frac{4}{9}\right) + 4 = 8$

$\therefore k = \sqrt{72} \ \therefore k = \pm 6\sqrt{2}$

24. $e_1 = \sqrt{2}$ Since it is a rectangular hyperbola.

$e_1^2 = 2 \qquad e_2^2 = \frac{a^2 + b^2}{a^2} = \frac{12}{4} = 3 \quad \therefore e_1^2 - e_2^2 = 2 - 3 = -1$

25. $\theta = 2\tan^{-1}\left(\frac{b}{a}\right) = 2\tan^{-1}\left(\frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{2(3/4)}{1 - 9/16}\right) = \tan^{-1}\left(\frac{3/2}{7/16}\right) \ \therefore \theta = \tan^{-1}\left(\frac{24}{7}\right)$

26. 222 = 2.111 $= 2^1 . 3^1 . 37^1$

$\therefore \alpha_1 = \alpha_2 = \alpha_3 = 1 \quad \therefore N = (1+\alpha_1).(1+\alpha_2).(1+\alpha_3) = 2.2.2 = 8$

27. by a theorem p / ab $\Rightarrow$ p/a or p/b, p is prime.

28. 8/72 $\therefore$ 8/72 . 182 . 363 $\therefore$ required remainder =0.

29. use | A . B| = |A| . |B| =0 ($\because$ |A|= 0)

30. use $C_2^1 \rightarrow C_2 + C_3$ and then C_2 & C_1 are proportional. $\therefore$ value of determinant =0.

31. $a_{11}=2/3 \quad a_{12}=3/3=1$

$a_{21}=3/3=1 \quad a_{21}=4/3 \qquad \therefore A=\begin{bmatrix} 2/3 & 1 \\ 1 & 4/3 \end{bmatrix} \quad \therefore |A|=\frac{8}{9}-1=\frac{-1}{9}$

32. $A\ adj\ A=|A|\ I\ ;\ |A|=\begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}$

$=1-2(3)+1(1)=-4\ \therefore A\ adj\ A=-4I$

33. In an abelian group, $o(ab^{-1}c)=o(aa^{-1}b)=o(eb)=o(b)$

34. $(3^{-1}+4+5^{-1})=(4+4+2)^{-1}=(10)^{-1}=3^{-1}=4$

35. given $3*x*2^{-1}=1 \Rightarrow 3*x=1*2$

$\Rightarrow 7\left(\frac{3x}{3}\right)=7\left(\frac{1.2}{3}\right) \Rightarrow 3x=2 \therefore x=2/3$

36. $\vec{a}+\vec{b}+\vec{c}=\vec{o} \quad \Rightarrow \vec{a}+\vec{b}=-\vec{c} \ \Rightarrow a^2+b^2+2\vec{a}.\vec{b}=c^2$

$\Rightarrow 4+9+2(2.3)\cos\theta=16 \quad \therefore \cos\theta=\frac{16-13}{12}=\frac{3}{12}=\frac{1}{4}$

37. $\Rightarrow(\vec{a}\times\vec{b})\times\vec{c} \ =\{\vec{c}\times(\vec{a}\times\vec{b})\}=\{(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}\}=\vec{b}$

$\therefore |(\vec{a}\times\vec{b})\times\vec{c}|=|\vec{b}|=\sqrt{2}$

38. $\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 1 & 1 & 2 \end{vmatrix}=-\hat{i}-7\hat{j}+4\hat{k} \quad A=|\vec{a}\times\vec{b}|\sqrt{1+49+16}=\sqrt{66}$

39. given any homogenous equation of 2nd degree $y_2=0$

40. $put\ t=\cos\theta \quad \therefore x=2\cos^{-1}(\cos 3\theta)=2(3\theta)=6\theta$

$\therefore x=6\cos^{-1}t \quad put\ t=\sin\theta\ \therefore y=3\sin^{-1}(\sin 3\theta)=9\theta$

$=9\sin^{-1}t \quad \therefore \frac{x}{6}+\frac{y}{9}=\cos^{-1}t+\sin^{-1}t=\pi/2$

$\therefore \frac{1}{6}+\frac{1}{9}\frac{dy}{dx}=0 \quad \therefore \frac{dy}{dx}=\frac{-9}{6}=-\frac{3}{2}$

41. $l_n y = l_n \cos x + l_n \cos 2x + l_n \tan 3x,\ l_n x = \log_e^x$

$$\frac{1}{y}.y^1 = -\tan x - 2\tan 2x - 3\tan 3x$$

$at\ x = \pi/3,\ \ y = \cos\pi/3\ \ \cos 2\pi/3.\ \cos 3\pi/3$

$$= \frac{1}{2}\left(-\frac{1}{2}\right)(-1) = \frac{1}{4}\ \therefore y^1 = -\frac{1}{4}\left[\sqrt{3} - 2\sqrt{3} - 0\right] = \frac{\sqrt{3}}{4}$$

42. A right angled triangle of given hypotenuse has maximum area when it is isosceles.

$$\therefore x = y \quad \therefore x^2 + y^2 = a^2 \quad \Rightarrow 2x^2 = a^2$$

$$\therefore x = a/\sqrt{2} = y \quad \therefore \Delta = \frac{1}{2}xy = \frac{\left(a/\sqrt{2}\right)^2}{2} = \frac{a^2}{4}$$

43. $y^1 = x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0 \ \ \therefore x = 1, 2; \quad at\ x = 1,\ y = -7/6.$

44. $S_t = y/y^1 \qquad S_n = y.y^1 \qquad \therefore Z = S_t.S_n = y^2 = \dfrac{10}{x^3} \therefore 2\ \ \alpha\ \ 1/x^3$

45. $I = -\int (1+\cos x)^9\ (-\sin x)dx \qquad -\int\ (1+\cos x)^9\, d(1+\cos x) = -\dfrac{(1+\cos x)^{10}}{10}$

46. $I = \int\ e^x \left(\dfrac{1 - 2\sin x/2 \cos x/2}{2\sin^2 x/2}\right)dx$

$$= \int\ e^x\left(\frac{1}{2}\cos ec\, x/2 - \cot x/2\right)dx \qquad = -\int\ e^x\left(\frac{1}{2}\cot x/2 - \frac{1}{2}\cos ec^2\ x/2\right)dx$$

$$= -e^x \cot x/2 + c$$

47. $use \int_0^{\pi/2} \log\tan\theta\, d\theta = \int_0^{\pi/2} \log\cot\theta\, d\theta = 0$ (standard problem)

48. given $dy = \left[(1+x^2).1 + y^2(1+x^2)\right]dx = (1+y^2)(1+x^2)dx$

$$\therefore \int\ \frac{dy}{1+y^2} = \int\ (1+x^2)dx \Rightarrow \tan^{-1} y = x + x^3/3 + c$$

49. $\alpha + \beta = -b/a\ :\ \ \alpha\ \beta\ = c/a \quad$ &

$$\alpha + \beta = -c/b\ :\ \ \alpha\ \beta\ = a/b$$

$$\therefore -\frac{b}{a} = -\frac{c}{a}\ \ \&\ \ -\frac{c}{a} = \frac{a}{b} \Rightarrow b^2 = ac : a^2 = bc$$

$$\Rightarrow b^4 = a^2c^2 = bc.c^2 = bc^3 \Rightarrow b^3 = c^3$$

50. $T_{r+1} = {}^{25}C_r\, x^r \;\therefore T_{12} = {}^{25}C_{11}\, x^{11} \;:\; T_{10} = 25C_9.x^9$

$$\therefore \frac{T_{12}}{T_{10}} = \frac{{}^{25}C_{11}x^{11}}{{}^{25}C_9 x^9} = \frac{\frac{25!}{(14)!\,11!}x^2}{\frac{25!}{(16)!9!}} = \frac{16.15.(14)!9!}{(14)!11.10.9!}x^2 = \frac{16.15.x^2}{11.10} = \frac{24x^2}{11}$$

51. given $\Rightarrow -1 \le \log\left(\frac{1+x}{1-x}\right) \le 1 \;\Rightarrow e^{-1} \le e^{\log\left(\frac{1+x}{1-x}\right)} \le e^1$

$$\Rightarrow e^{-1} \le \frac{1+x}{1-x} \le e \Rightarrow (1-x)/e \le 1+x \le e(1-x)$$

$$\Rightarrow \frac{1}{e} - \frac{x}{e} \le 1+x \;\; \le e - ex \;\Rightarrow\; \frac{1}{e} - 1 \le x + x/e$$

$$\Rightarrow \frac{1-e}{e} \le \frac{(1+e)}{e} \Rightarrow \frac{1-e}{1+e} \le x \text{ -----} (i)$$

$$1 + x \le e - ex \;\Rightarrow x + ex \le e - 1 \Rightarrow x(1+e) \le e - 1 \therefore x \le \frac{e-1}{1+e} \text{ ---} (ii)$$

$$(i)\,\&\,(ii) \Rightarrow \frac{1-e}{1+e} \le x \le \frac{e-1}{1+e} \therefore \in \left[\frac{1-e}{1+e}, \frac{e-1}{1+e}\right]$$

52. use $\sim (p \to q) \equiv p \Lambda \sim q$:

53. solving given equations, x= y = 1

required equation is $y - y_1 = m(x - x_1) \Rightarrow y - 1 = m(x-1) = mx - m$

$$\therefore mx - y + (1-m) = 0 \;\; Its\ y - \text{int}ercept = \frac{1-m}{m} = -3$$

$$\therefore m = -1/2 \;\therefore equation\ is\ x + 2y - 3 = 0$$

54. $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & 1/2 & -1 \\ 1/2 & 1 & -2 \\ -1 & -2 & k \end{vmatrix} = 0$

$$\Rightarrow 2(k-4) - 1/2\left(\frac{k}{2} - 2\right) - 1(-1+1) = 0 \;\Rightarrow\; 2k - 8 - \frac{k}{4} + 1 = 0$$

$$\Rightarrow 2k - \frac{k}{4} = 7 \Rightarrow \frac{7k}{4} = 7 \Rightarrow k = 4$$

55. $given \Rightarrow A = 180 - (30 + 45) = 105$

$by\, S.R. \frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{2}{\sin 105} = \frac{b}{\sin 30} \Rightarrow b = \frac{2.1/2}{\sqrt{3}+1/2\sqrt{2}}$

$= \frac{\sqrt{2}}{\sqrt{3}+1} \;\therefore b = \frac{\sqrt{2}(\sqrt{3}-1)}{3-1} = \frac{\sqrt{3}-1}{\sqrt{2}}$

56. given $\Rightarrow a\frac{s(s-c)}{ab} + c\frac{s(s-c)}{bc} = \frac{3b}{2}$

$\Rightarrow \frac{s}{b}(s-c+s-a) = \frac{3b}{2} \Rightarrow 2s(a+b+c-a+c) = 3b^2$

$\Rightarrow 2s = 3b \Rightarrow a+b+c = 3b \therefore 2s = 3b \Rightarrow a+b+c = 3b$

$\therefore 2 = a+c \therefore a,b,c \; are\; in\; A.P.$

57. $\lim_{x\to\infty}\left[\frac{n(n^2+1-n^2)}{\sqrt{n^2+1}+n}\right] = \lim_{x\to\infty}\frac{n}{n\left(\sqrt{\frac{n^2+1}{n^2}}+1\right)}$

$= \lim_{x\to\infty}\frac{1}{\left(\sqrt{\frac{n^2+1}{n^2}}+1\right)} = \lim_{x\to\infty}\frac{1}{\left(\sqrt{1+\frac{1}{n^2}}+1\right)} = \frac{1}{2}$

58. By L.H. Rule, given = $\lim_{x\to 0}\left[\frac{\frac{1}{3+x} - \frac{1}{3-x}}{1}\right] = \frac{1}{3}+\frac{1}{3} = \frac{2}{3}$

59. $f(1) = 1 \quad L.L. = \lim_{x\to 1}(1-x) = 0 : R.L. = \lim_{x\to 1}(1+x) = 2$

$L.L \neq R.L. \therefore \lim_{x\to 1} f(x)\; does\; not\; exist\; \therefore (3)\; is\; the\; answer.$

60. since f is continuous at $x = \pi,\; f(\pi) = \lim_{x\to\pi}\left[\frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}\right]$

$= \lim_{x\to\pi}\left[\frac{1(-\sin x)}{2\sqrt{2+\cos x}} \Big/ 2(\pi-x)(-1)\right] \; (by\; L.H.\; Rule)$

$= \lim_{x\to\pi}\left[\frac{\sin(x)}{4\sqrt{2+\cos x}}\left(\frac{1}{\pi-x}\right)\right] = \frac{1}{4}.\frac{1}{\sqrt{2-1}}.\; \lim_{x\to\pi}\left(\frac{\sin(\pi-x)}{(\pi-x)}\right) (by\; L.H.\; Rule)$

$= \frac{1}{4}.(1) = \frac{1}{4}$