MOCK TEST – 03
SUBJECT: MATHEMATICS
ANSWERS: COMMON ENTRANCE TEST 2012

ANSWERS

1. (1)

Take $\cos^{-1}(-\frac{1}{2})=\frac{2\pi}{3}$ then $\sin\left[\frac{1}{4}\cos^{-1}(-\frac{1}{2})\right]=\sin\left[\frac{1}{4}\frac{2\pi}{3}\right]=1/2$

2. (2)

Since $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = -\frac{3\pi}{2}$,

$\Rightarrow x = -1;\ y = -1,\ z = -1$

$\therefore x^{50} + y^{50} + z^{50} - \frac{3}{x^{49}+y^{49}+z^{49}} = 1+1+1-\frac{3}{-1-1-1} = 4$

3. (4)

$\tan\left(\frac{p\pi}{4}\right)=\cot\left(\frac{q\pi}{4}\right) \quad \Rightarrow \tan\left(\frac{p\pi}{4}\right)=\tan\left(\frac{\pi}{2}-\frac{q\pi}{4}\right) \Rightarrow \frac{p\pi}{4}=n\pi+\frac{\pi}{2}-\frac{q\pi}{4}$

$\Rightarrow p\pi = 4n\pi + 2\pi - q\pi \Rightarrow (p+q)\pi = 2(2n+1)\pi \quad \Rightarrow p+q = 2(2n+1)$

4. (2)

$\frac{\sqrt{3}}{2}-\frac{i}{2}=1\left(\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)\right)=1e^{\frac{-\pi}{6}i} \quad \therefore \log\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)=\log 1+i\left(\frac{-\pi}{6}\right)$

$\therefore$Principle amplitude $= \tan^{-1}\left(\frac{\frac{-\pi}{6}}{0}\right) = \tan^{-1}(-\infty) = \frac{-\pi}{2}$

5. (3)

$z_1\, z_2\, z_3 \ldots\ldots \text{to } \infty = (x_1\, x_2\, x_3 \ldots\ldots \text{to } \infty)(y_1\, y_2\, y_3 \ldots\ldots \text{to } \infty)^4 \quad (\text{CiS}\left(\frac{\pi}{4}\right)$

$\text{CiS}\left(\frac{\pi}{4^2}\right)\text{CiS}\left(\frac{\pi}{4^3}\right)\ldots\ldots \text{to } \infty)(\text{CiS}\left(\frac{\pi}{3}\right)\text{CiS}\left(\frac{\pi}{3^2}\right)\text{CiS}\left(\frac{\pi}{3^3}\right)\ldots\ldots \text{to } \infty)^4$

$= \text{CiS}\left(\frac{\pi}{4}+\frac{\pi}{4^2}+\frac{\pi}{4^3}+\ldots\ldots to\,\infty\right)\left[cis\left(\frac{\pi}{3}+\frac{\pi}{3^2}+\frac{\pi}{3^3}+\ldots\ldots to\,\infty\right)\right]^4$

$= cis\pi\left(\frac{\frac{1}{4}}{1-\frac{1}{4}}\right)\left[cis\pi\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)\right]^4 = \text{CiS}\left(\frac{\pi}{3}\right)i^4 = \text{CiS}\left(\frac{\pi}{3}\right)$

6. (3) The distance s moved by a particle in time t is given by s = a cos 2t + b sin 2t.

vel $= \frac{ds}{dt} = -2a\sin 2t + 2b\cos 2t$;

Its acceleration is ; $\frac{d^2s}{dt^2} = -4a\cos 2t - 4b\sin 2t = -4(a\cos 2t + b\sin 2t) = -4s$

7. (4) Let the given line divided into parts of lengths x and y and let x + y = 2k, given .

Area of rectangle is $A = xy = x(2k - x) = 2kx - x^2$

$\frac{dA}{dx} = 2k - 2x = 0 \Rightarrow x = k$ and $y = 2k - x = 2k - k = k$

or x = y = k . Therefore ratio in which a given line be divided into two parts so that the rectangle contained by them is maximum is 1 : 1

8. (4) $y = x^2 + \frac{\sin 2x}{2}$

At (0, 0) $\frac{dy}{dx} = 2x + \cos 2x = 2.0 + \cos 0 = 1$

The equation of normal is (y-0) = -1(x-0) or x + y = 0

9. (2) xy = 2 ------- (1) and $y^2 = 4x$ ------ (2)

$x\frac{dy}{dx} + y = 0$ $2y\frac{dy}{dx} = 4$

solving (1) and (2) we get x= 1 and y = 2

therefore $m_1 = -2$ $m_2 = 1$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-2-1}{1+(-2)(1)}\right| = 3$ The angle is $\tan^{-1}3$

10. (3) Number of cut vertices is p=2 and number of cut edges is q=1.

11. (1)

The given equation is (2x – 3y + 2)+k(5x – 6y – 1) =0

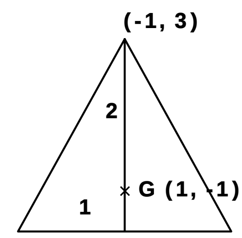
Solve equations 2x – 3y + 2 = 0 & 5x – 6y – 1 = 0

12. (1)

$m_1 + m_2 = m_1 m_2$

$\Rightarrow \frac{-2h}{b} = \frac{a}{b} \Rightarrow \frac{-2h}{-1} = \frac{4}{-1} \Rightarrow h = -2$

13. (2)



Let (x, y) be the mid point of BC

Then (1, -1) = $\left(\frac{2x-1}{2+1}, \frac{2y+3}{2+1}\right) \Rightarrow \frac{2x-1}{3} = 1, \frac{2y+3}{3} = -1$

x = 2, y = -3 ∴ (2, -3) is the required point

14. (3)

Put x = 0 we get $d = \begin{vmatrix} 0 & 1 & -1 \\ -3 & 0 & 2 \\ 3 & -4 & 0 \end{vmatrix}$ = -1(0 – 6) – 1(12 – 0) = 6 – 12 = -6

15. (4)

The given equation is (sinA – sinB)(sinB – sinC)(sinC – sinA) = 0

Therefore either A = B or B = C or C = A ⇒ a=b or b=c or c=a.

16. (1)

Apply Cayley Hamilton therem.

17. (4)

$A^2 + B^2$ = AA + BB = A(BA) + B(AB) = (AB)A + (BA)B = BA + AB = A + B

18. (4)

It is advised to cross check with simple integer say n = 1

we get the value of $3^{3^4} = 3^{81} = (9^2)^{20}.3 \equiv 3(\text{mod}10)$

therefore the last digit is 3 + 2 = 5 which is (d).

19. (3) $S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{1}{1-(1/3)} + \frac{2(1/3)}{4/9} = \frac{9}{4}$

20. (3) Substitute n = 1 $2^3 - 1 = 7$ is divisible by 7 only.

21. (1) If $\frac{2x}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$, then

a) A = B = C b) A ≠ B ≠ C c) A ≠ B = C d) A = B ≠ C

22. (4)

If in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, n ∈ N, sum of the coefficient of x^5 and x^{10} is 0, then value of n is d) 15

23. Ans: (2) $y = \sqrt{\log(\sin x) + \sqrt{\log(\sin x) + \sqrt{\log(\sin x) + \text{........}to\ \infty}}}$ squaring on both sides

$y^2 = \log(\sin x) + \sqrt{\log(\sin x) + \sqrt{\log(\sin x) + \sqrt{\log(\sin x) + \text{........}to\ \infty}}} = \log(\sin x) + y$

differentiating w. r. t. x we get $2y\frac{dy}{dx}$ = cotx + $\frac{dy}{dx} \Rightarrow (2y-1)\frac{dy}{dx} = \cot x$

24. (2)

Sol: Given y = $\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$

= $\cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$ [using $\sec^{-1} = \cos^{-1}1/x$]

$= \frac{\pi}{2} \quad \Rightarrow \frac{dy}{dx} = 0.$

25. (1)

Sol: $y = ae^x + be^{2x} \Rightarrow \frac{dy}{dx} = ae^x + 2be^{2x} \quad \Rightarrow \quad \frac{d^2y}{dx^2} = ae^x + 4be^{2x}$

$\therefore \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = (ae^x + 4be^{2x}) - 3(ae^x + 2be^{2x}) + 2(ae^x + be^{2x}) = 0.$

26. (2)

Sol: $y = \frac{(1-x)^2}{x^2} = \frac{1+x^2-2x}{x^2} = \frac{1}{x^2} + 1 - \frac{2}{x}$

$\therefore \quad \frac{dy}{dx} = -\frac{2}{x^3} + \frac{2}{x^2}.$

27. (4) $\left(1+\cos\frac{\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right) = \sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{3\pi}{8}\right)$

$= \frac{\left(1-\cos\frac{\pi}{4}\right)}{2} \cdot \frac{\left(1-\cos\frac{3\pi}{4}\right)}{2} = \frac{1}{8}$

28. (2) In a triangle ABC, a = 2b and $\angle A = 3\angle B$, then angle A is 90^o

29. (3) If sin A + sin B + sin C = 3, then A = B= C = 90^o $\sin\frac{A}{3} + \sin\frac{B}{3} + \sin\frac{C}{3} = \frac{3}{2}$

30. (3) If Secθ = m and Tanθ = n, then $\frac{1}{m}\left[(m+n) + \frac{1}{(m+n)}\right] = 2$

31. (4)

$\int \frac{5\cot x - 2}{2\cot x + 3}dx = \int \frac{5\cos x - 2\sin x}{2\cos x + 3\sin x}dx = \int \frac{l(Dr) + m\left[\frac{dDr}{dx}\right]}{Dr}dx$

$l = \frac{ac+bd}{c^2+d^2} = \frac{4}{13} \quad m = \frac{ad-bc}{c^2+d^2} = \frac{19}{13}$

solution is $= lx + m\log(Dr.) = \frac{19}{13}\log(2\cos x + 3\sin x) + \frac{4}{13}x + C$

32. (4)

$3\sinh x + 3\cosh x = 3\left(\frac{e^x - e^{-x}}{2}\right) + 3\left(\frac{e^x + e^{-x}}{2}\right) = 3e^x$

$\int \frac{e^x}{3\sinh x + 3\cosh x}dx = \int \frac{e^x}{3e^x}dx = \int \frac{1}{3}dx = \frac{x}{3}$

33. (2)

Integration by parts take u = $\sec^{-1}x$ and dv = dx

$\int \sec^{-1}x\,dx = \quad x\sec^{-1}x - \cosh^{-1}x$

34. (3)

$\int \sqrt{\frac{(1-\cos x)(1-\cos x)}{1-\cos^2 x}}dx = \int \frac{1-\cos x}{\sin x}dx$

$= \int \frac{2\sin^2 x/2}{2\sin x/2\cos x/2}dx = \int \tan x/2\,dx = 2\log\left(\sec\left(\frac{x}{2}\right)\right)$

35. (1) For every a, b $\in$R, 3a -2b $\in$R (b) (c) are also b.o.

$\therefore$ a * b = $\frac{a}{b+2}$ not a b.o. because b can be -2. is a binary operation

36. (1) $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -3 \\ 1 & -2 & 1 \end{vmatrix}$ = -5i -5j -5k $\quad |\vec{a} \times \vec{b}| = \sqrt{25+25+25} = 5\sqrt{3}$

$\therefore$ Vector of magnitude 5 perpendicular to both $\vec{a}$ and $\vec{b}$ is

$\pm 5\hat{n} = \pm 5\left(\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}\right) = \pm\frac{5}{\sqrt{3}}$ (-i –j –k)

37. (2) Vector in the direction of $\vec{a}$ having magnitude equal to the magnitude of $\vec{b}$ =

$\frac{\vec{a}}{|\vec{a}|}|\vec{b}|$

$= \frac{(i + 2j + 2k)}{\sqrt{9}} \sqrt{9 + 36 + 4} = \frac{7}{3}(i + 2j + 2k)$

38. (2) $\vec{OA} = i + 2j + 4k$; $\vec{OB} = -2i + 2j + k$, $\vec{OC} = 2i + 4j - 3k$

$\vec{AB} = -3i - 3k$; $\vec{BC} = 4i + 2j - 4k$; $\vec{AC} = i + 2j - 7k$

$|\vec{AB}| = \sqrt{18}$; $|\vec{BC}| = 6$; $|\vec{AC}| = \sqrt{54}$ $\therefore |\vec{AC}|^2 = |\vec{AB}|^2 + |\vec{BC}|^2$

Hence it is a right angled triangle

39. (3) In $\{1, 5, 7\}$, $5 \times_{12} 7 = 11 \notin \{1, 5, 7\}$

In $\{5, 7, 11\}$ and $\{7, 11\}$ identity 1 is not there

Hence $\{1, 5\}$ is a subgroup.

40. (1) If one root of $x^3 - 5x^2 + 9x - 5 = 0$ is 2 + I, then the other root will be 2 – i

($\because$ complexroots with real coefficients occur in conjugate pairs)

Let α be the third root

$\therefore$ Sum of the roots $= (2 + i) + (2 - i) + \alpha = 5$

$\Rightarrow \alpha = 1$

$\therefore$ The other roots of the given equation are 2 – i and 1

41. (4) Let α be the other root of $x^2 + kx - 24 = 0$

$\therefore 3 + \alpha = k; \quad 3\alpha = -24$

$\Rightarrow \alpha = -k - 3 \quad \alpha = -8 \Rightarrow \quad k = 8 - 3 = 5$

Alterte: by putting, k = 5 in the choices, we observe that $x^2 - 5x + 6 = 0$ has roots 2 and 3.

$\therefore$ 3 is also a root of $x^2 - kx + 6 + 0$, where k = 5.

42. (4) $\sqrt{10^{\left(2+\frac{1}{2}\right)\log_{10} 16}} = \sqrt{10^{\frac{5}{2}\log_{10} 2^4}} = \sqrt{10^{\log_{10} 2^{10}}} = \sqrt{2^{10}} = 2^5 = 32$

43. (3)

The negation of $\sim(x \rightarrow y) \equiv x \wedge \sim y$

The negation of the statement "If it rains then you get wet" is

"it rains and you don't get wet"

44.(1) $\int \sec^2 x \operatorname{cosec}^4 x dx = \int (1 + \tan^2 x)\frac{\sec^4 x}{\tan^4 x} dx = \int (1 + \tan^2 x)^2 \frac{\sec^2 x}{\tan^4 x} dx =$

$\int (1 + t^2)^2 \frac{dt}{t^4} = -\frac{1}{3}\cot^3 x + \tan x - 2\cot x$

45. (1)

Put a = 2, b = 3 and c = 0

Then $\int_0^\infty \frac{x^2 dx}{(x^2+4)(x^2+9)(x^2+0)} = \frac{\pi}{2(2+3)(3+0)(0+2)}$

i.e., $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} = \frac{\pi}{60}$

46. (4)

$4a = 6 \Rightarrow a = \frac{3}{2}$

Area $= \frac{16a^2}{3} = \frac{16}{3} \times \frac{9}{4} = 12$

47. (2)

$\sqrt{\frac{dy}{dx}} = 4\frac{dy}{dx} + 7x$

Squaring both sides: $\frac{dy}{dx} = \left(4\frac{dy}{dx} + 7x\right)^2$ $\therefore$ Order = 1, Degree = 2

48. (2)

$y(x^2 + 1)\, dy = x(y^2 + 1)\, dx$

$\frac{ydy}{y^3 + 1} = \frac{xdx}{x^2 + 1}$

$\int \frac{xdx}{x^2+1} = \int \frac{ydy}{y^2+1} \Rightarrow \frac{1}{2}\log(x^2 + 1) = \frac{1}{2}\log(y^2 + 1) + \frac{1}{2}\log k$

$(x^2 + 1) = k(y^2 + 1)$

49.(1)

$$\lim_{n \to \infty} (3^n + 4^n)^{\frac{1}{n}} = \max\{3, 4\} = 4$$

50. (1)

The function $f(x) = |x| + \frac{|x|}{x}$ is

discontinuous at the origin $\frac{|x|}{x}$ is discontinuous there

51. (4)

f (x) is defined if

$1 - x \neq 1$ and $1 - x > 0$ and $x + 2 \geq 0$ i. e. $x \neq 0$ and $1 > x$ and $x \geq -2$

i. e. $-2 \leq x < 1$ and $x \neq 0$

i. e. [-2, 1) excluding 0

52. (2)

Number of proper subsets = $2^6 - 1 = 63$

53. (3)

Only (3, - 1) satisfies the equation $x^2 + y^2 - 8x + 6y + 20 = 0$

54. (3)

Equation of the circle passing through the intersection is $S_1 + kS_2 = 0$.

$\Rightarrow x^2 + y^2 - 6 + k(x^2 + y^2 - 6x + 8) = 0$ -----------------------(1)

Since the required circle passes through (1, 1)

$1 + 1 - 6 + k(1 + 1 - 6 + 8) = 0 \Rightarrow k = 1$.

$\therefore$ From (1) equation is $x^2 + y^2 - 3x + 1 = 0$.

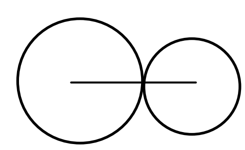
55. (4) The centres are A (1, - 1) and B (-1, 1)

Since the circles cut orthogonally, $AB^2 = r_1^2 + r_2^2$

i.e $(1 + 1)^2 + (-1 - 1)^2 = r^2 + r^2 \Rightarrow 8 = 2r^2 \Rightarrow r = 2$

56. (4)

The locus is a circle concentric with the given circle. But only last option represents a circle concentric with given circle



57. (2)

$3x^2 - 4(x - 1)^2 = 12 \Rightarrow 3x^2 - 4x^2 + 8x - 4 = 12 \Rightarrow -x^2 + 8x - 16 = 0$

$\Rightarrow x^2 - 8x + 16 = 0$

$(x - 4)^2 = 0 \Rightarrow x = 4, y = 3$

$\therefore$ Point of contact is (4, 3)

58. (3)

The tangent to parabola $y^2 = 8x$ is $y = mx + \frac{2}{m}$. If this makes an angle **45^o** with

$y = 3x + 5$

$\therefore \tan 45^o = \left| \frac{m - 3}{1 + 3m} \right|$

$\therefore \pm 1 = \frac{m - 3}{1 + 3m} \Rightarrow 1 + 3m = m - 3$ or $3 - m \Rightarrow m = -2$

$\therefore$ Equation of tangent is $y = -2x - 1$ i. e. $2x + y + 1 = 0$ or $y = \frac{1}{2}x + 4$

i. e., $x - 2y + 8 = 0$

59. (1)

$x^2 - 4y^2 - 2x + 8y - 2 = 0 \Rightarrow x^2 - 2x + 1 - 4(y^2 - 2y + 1) = -1$

$\Rightarrow (x - 1)^2 - 4(y - 1)^2 = -1$ which is a hyperbola with a vertical axis

$\therefore$ Transverse axis = $2b = 2\left(\frac{1}{2}\right) = 1$

60. (1)

Given $2a = 2.2b$ ------- (1) and $\frac{2b^2}{a} = 3$ --------- (2)

From (1) $a = 2b$ and hence from (2)

$$\frac{2b^2}{2b} = 3 \Rightarrow b = 3 \therefore a = 6$$

$\therefore$ Distance between foci $= 2\sqrt{a^2 - b^2} = 2\sqrt{36 - 9} = 6\sqrt{3}$