MOCK TEST – 02

SUBJECT: MATHEMATICS

ANSWERS: COMMON ENTRANCE TEST 2012

1. $\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right]$

$=\sin^{-1}\left[\cot\left(\sin^{-1}\sqrt{\frac{4-2\sqrt{3}}{8}}+\cos^{-1}\frac{2\sqrt{3}}{4}+\sec^{-1}\sqrt{2}\right)\right]$

$=\sin^{-1}\left[\cot\left(\sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}}+\cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}\sqrt{2}\right)\right]$

$=\sin^{-1}[\cot(15^0+30^0+45^0)]=\sin^{-1}(\cot 90^0)$

$=\sin^{-1}(0)=0$

Answer: a

2. Put $x^2=\sin 2\propto$

Then $\sqrt{1+x^2}=\sqrt{1+\sin 2\alpha}$

$=\sqrt{cos^2\alpha+sin^2\alpha+2\sin\alpha\cos\alpha}=\cos\alpha+\sin\alpha$

$\sqrt{1-x^2}=\cos\alpha-\sin\alpha$

$\therefore\ LHS=\tan^{-1}\left[\frac{cos\alpha+\sin\alpha-\cos\alpha+\sin\alpha}{cos\alpha+\sin\alpha+\cos\alpha-\sin\alpha}\right]=\tan^{-1}\left(\frac{2\sin\alpha}{2\cos\alpha}\right)$

$=\tan^{-1}(\tan\alpha)=\alpha$

Answer: c

3. $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

$\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{\pi}{2}+\frac{\pi}{2}+\frac{\pi}{2}$

$\sin^{-1}=\frac{\pi}{2}\ \ \therefore x=\sin\frac{\pi}{2}\ \ \therefore x=1$

Similarly, y =1 & z = 1

$\therefore(x^{2012}+y^{2012}+z^{2012})-\frac{9}{(x^{2013}+y^{2013}+z^{2013})}=3-\frac{9}{3}$

$=3-3=0$

Answer: b

4. $2^{1+\cos^2x+cos^4x+\,.....\infty}=4$

$2^{1+\cos^2x+cos^4x+\,.....\infty}=2^2$

$\therefore 1+cos^2x+cos^4x+\cdots\infty=2$

The LHS is in G.P a=1. r=cos^2x

$\therefore S\infty=2$

$\frac{1}{1-cos^2x}=2$

$\frac{1}{sin^2x}=2\quad\therefore\sin x=\pm\frac{1}{\sqrt{2}}\quad\therefore x=\frac{\pi}{4}$ and $\frac{-\pi}{4}$

Answer: b

5. $9^{\sin x}-2.3^{\sin x}+1=0$

$(3^{\sin x})^2-2.3^{\sin x}+1=0$

$(3^{\sin x}-1)^2=0$

$3^{\sin x}=1=3^0$

$\therefore$ sin x = 0 $\therefore x=0^\circ,\propto=0^\circ$

The G.S is $x=n\pi+(-1)^n0^\circ$ $\qquad\therefore\ x=n\pi$

Answer: a

6. $\sum_{k=1}^{2012}\left(\sin\frac{2k\pi}{2013}+i\cos\frac{2k\pi}{2013}\right)$

$=\sum_{k=1}^{2012}\mathrm{i}\left(\cos\frac{2k\pi}{2013}-i\sin\frac{2k\pi}{2013}\right)=\mathrm{i}\sum_{k=1}^{2012}\ \mathrm{e}^{-\left(\mathrm{i}\frac{2\pi}{2013}\right)k}$

$= i \cdot \sum_{k=1}^{2012} z^k$, Where $z = e^{-i\frac{2\pi}{2013}}$ $\therefore z^{2013} = e^{-i\,2\pi}$

$= \cos 2\pi - i\sin 2\pi = 1$

$= i(z + z^2 + \cdots + z^{2012})$ is in G.P. a = z . r = z. n = 2012

$= \frac{i(1-z^{2012})z}{(1-z)} = i\frac{(z-z^{2013})}{-(z-1)} = -\frac{i(z-1)}{(z-1)} = -i.$

Answer: c

7. let $Z = re^{i\theta}$ & Z *is rotated by angle* $\frac{\pi}{2}$

then the new position is $z = r \cdot e^{i(\theta+\frac{\pi}{2})}$

$= r \cdot e^{i\theta} \cdot e^{i\frac{\pi}{2}}$

$= z\,(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

= z . i

= i . z

Answer: b

8. The continued product of the cube root of $\sqrt{3} + i$ is it self

Answer: a

9. $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$

i^{100} = a + i b

$(i^4)^{25}$ = a + i b

1^{25} = a + i b,

1 + i . 0 = a + i b $\therefore (a, b) = (1, 0)$

Answer: b

10. $(49^2 - 4)\,(49^3 - 49) = (49^2 - 2^2)49.(49^2 - 1)$

= (49 – 2) (49 + 2) 49. (49 – 1) (49 + 1)

= 47.51.49.48.50

= 47. 48. 49. 50 .51

It is the product of 5 consecutive integers and hence it is divisible by 5!

Answer: a

11. Put $x = 1,\ y = 1,\ z = 1$

Then $\frac{(x^2+x+1)(y^2+y+1)(z^2+z+1)}{xyz} = \frac{3x3x3}{1x1x1} = 27$

Answer: c

12. $(2009)! \equiv 0(mod 10)$&

$3^2 \equiv -1(mod 10)$

$(3^2)^{3943} \equiv (-1)^{3943}(mod\ 10)$

$\therefore\ 3^{7886} \equiv -1(mod\ 10)$

$3^{7886} \equiv 9(mod\ 10)$

$\therefore (2009)! + 3^{7886} \equiv (0 + 9)(mod\ 10)$

$\therefore (2009)! + 3^{7886} \equiv 9(mod\ 10)$

Answer: d

13. $\sin ce\ A\ is\ 4 \times 4\ matrix$

$then\ A^n = 4^{n-1}A \quad \therefore A^4 = 4^{4-1}A \quad = 4^3A$ = 64 A

Answer: b

14. A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $adj\ A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$|adj\ A| = cos^2\theta + \sin^2\theta = 1$

$adj(adj\ A) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\therefore (adj\ A)^{-1} = \frac{adj(adj\ A)}{|adj(A)|} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = A$

Answer: a

15. The characteristic roots of Triangular matrix are the diagonal elements, 1 ,3 ,6

Answer: c

16. $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$

$1(c^2 - ab) - a(c - a) + b(b - c) = 0$

$c^2 - ab - ac + a^2 + b^2 - bc = 0\ \times by\ 2$

$2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

$a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2\,ca = 0$

$(a-b)^2+(b-c)^2+(c-a)^2=0$
$\therefore a-b=0 \quad b-c=0 \quad c-a=0$
$\therefore a=b \quad b=c \quad c-a=0$
$\therefore a=b=c \quad \therefore A=B=C=60^0$
$sin^2A+sin^2B+sin^2C=\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{9}{4}$

Answer: d

17. $A_x=\begin{bmatrix} x & x \\ x & x \end{bmatrix}$ & $A_e=\begin{bmatrix} e & e \\ e & e \end{bmatrix}$

$A_x\ A_e=A_x$

$\begin{bmatrix} 2xe & 2xe \\ 2xe & 2xe \end{bmatrix}=\begin{bmatrix} x & x \\ x & x \end{bmatrix} \therefore 2xe=x \therefore e=½$

$\therefore A_e=\begin{bmatrix} ½ & ½ \\ ½ & ½ \end{bmatrix}=½\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Answer: c

18. **Answer: b**
19. **Answer: c**
20. $\vec{u}\times\vec{v}=(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$ & $|\vec{a}|=2, |\vec{b}|=2$

$= 2(\vec{a}\ \times\vec{b})$

$|\vec{u}\times\vec{v}|=\sqrt{4|\vec{a}\times\vec{b}|^2}$

$=\sqrt{4|\vec{a}|^2|b|^2\sin^2\theta}$

$=2\sqrt{|\vec{a}|^2|b|^2(1-\cos^2\theta)}$

$=2\sqrt{|\vec{a}|^2|b|^2-(|\vec{a}||\vec{b}|\cos\theta)^2}$

$=2\sqrt{4(4)-(\vec{a}\cdot\vec{b})^2}$

$=2\sqrt{16-(\vec{a}\cdot\vec{b})^2}$

Answer: a

21. Given $[\ \vec{a}\ \vec{b}\ \vec{c}\ ]=\ 40$

Then Required volume $=[\vec{b}+\vec{c}, \vec{c}+\vec{a}, \vec{a}+\vec{b}]$

$=2[\vec{a}\ \vec{b}\ \vec{c}]$

$=2(40)$

= 80 cubic units

Answer: c

22. $|\vec{a}|=1 \quad |\vec{b}|=1 \quad |\vec{c}|=1$ & $\vec{c}=\vec{a}+\vec{b}$

$|\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+2(\vec{a}.\vec{b})$

$\therefore 2(\vec{a}\cdot\vec{b})=1-1-1=-1$

$\therefore |\vec{a}-\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2-2(\vec{a}.\vec{b})$

= 1 + 1 - (-1) = 1 + 1 + 1 = 3

$|\vec{a}-\vec{b}|=\sqrt{3}$

Answer: b

23. Given $|\vec{u}|=1. |\vec{v}|=2$ & $|\vec{w}|=3$ & $\vec{v}.\vec{w}=0$ & $\frac{\vec{v}.\vec{u}}{|\vec{u}|}=\frac{\vec{w}.\vec{u}}{|\vec{u}|} \therefore \ \vec{v}.\vec{u}=\ \vec{w}.\vec{u}$

$|\vec{u}-\vec{v}+\vec{w}|^2=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2-2(\vec{u}.\vec{v})-2(\vec{v}.\vec{w})+2(\vec{w}.\vec{u})$

$=(1)^2+(2)^2+(3)^2-2(\vec{w}.\vec{u})-2(0)+2(\vec{w}.\vec{u})$

$=1+4+9$

$=14$

$|\vec{u}-\vec{v}+\vec{w}|=\sqrt{14}$

Answer: b

24. $f(x)=1+nx+\frac{n(n-1)}{2}x^2+\ \frac{n\,(n-1)(\,n-2)}{6}\ x^3+\cdots.+x^n$

$f(x)=(1+x)^n$

$f^{|}(x)=n(1+x)^{n-1}$

$f^{||}(x)=n(n-1)(1+x)^{n-2}$

$f^{||}(1)=n(n-1)2^{n-2}$

Answer: d

25. If y $=cos^2\left(\frac{3x}{2}\right)$ - $sin^2\left(\frac{3x}{2}\right)$

$y = \cos 2\left(\frac{3x}{2}\right) = \cos 3x$

$\frac{dy}{dx} = -\sin 3x.3$

$\frac{d^2y}{dx^2} = -3\cos 3x.3 \qquad = -9y$

Answer: c

26. $\sin(x+y) + \cos(x+y) = \log(x+y)$.

$\cos(x+y)(1+y_1) - \sin(x+y)(1+y_1) - \frac{1}{x+y}(1+y_1) = 0$

$\therefore 1 + y_1 = 0 \quad \therefore y_1 = -1 \quad \therefore y_2 = 0 \quad \therefore \frac{d^2y}{dx^2} = 0$

Answer: a

27. $y = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \cot hx$

$\frac{dy}{dx} = -\cos ech^2 x$

Answer: c

28. $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$

$\frac{dy}{dx} = 0 + 1 + \frac{2x}{2\times 1!} + \frac{3x^2}{3\times 2!} + \cdots + \frac{x^{n-1}}{(n-1)!}$

$\frac{dy}{dx} + \frac{x^n}{n!} = 1 + \frac{2x}{2\times 1!} + \frac{3x^2}{3\times 2!} + \cdots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} \quad = y$

Answer: c

29. $y = \sin^{-1}\left[\sqrt{x - ax} - \sqrt{a - ax}\right]$

$y = \sin^{-1}\left[\sqrt{x(1-a)} - \sqrt{a(1-x)}\right]$

$= \sin^{-1}\left[\sqrt{x}\sqrt{1-\left(\sqrt{a}\right)^2} - \sqrt{a}\sqrt{1-\left(\sqrt{x}\right)^2}\right]$ $Put\ \sqrt{x} = \sin\theta, \sqrt{a} = \sin\alpha$

$= \sin^{-1}(\sin\theta\cos\alpha - \sin\alpha\ \cos\theta)$

$= \sin^{-1}(\sin(\theta - \alpha)) = \theta - \alpha$

$y = \sin^{-1}\sqrt{x} - \sin^{-1} a$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}} - 0 = \frac{1}{2\sqrt{x}\sqrt{1-x}}$

Answer: a

30. $x = 48t - 16t^2 \qquad \frac{dx}{dt} = 48 - 32t \qquad \frac{d^2x}{dt^2} = -32(-ve)$

$\therefore$ The stone reaches maximum height $\therefore \frac{dx}{dt} = 0$ 48 - 32t = 0 32t = 48 $\therefore t = \frac{3}{2} sec$

when t = $^3/_2\, secs.$ $x = 48\left(\frac{3}{2}\right) - 16\left(\frac{3}{2}\right)^2 = 36mts.$

total maximum height from the ground = 64 + 36 =100mts.

Answer: a

31. $y = x^2 - x + 4, \qquad \frac{dy}{dx} = 2x - 1$

$\left(\frac{dy}{dx}\right)(1.4) = m = 2 - 1 = 1$

$The\ tangent\ is\ y - y_1 = m(x - x_1)$

$y - 4 = 1(x - 1)$

$x - y - 1 + 4 = 0$

$x - y + 3 = 0$ -------------(1)

The $normal\ is\ y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 4 = -1(x - 1)$

$x + y - 4 - 1 = 0$

$x + y - 5 = 0$ ------------(2)

The tgt cuts the x – axis at A $\therefore y = 0$ from eqn (1) x = -3 $\therefore A = (-3.0)$

The normal cuts the x – axis at B $\therefore y = 0$

From eqn (2) x = 5 B = (5,0)

Area of the $\Delta\ APB = \frac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ -3 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix} = 16\ sq, unit$

Answer: c

32. Surface area of the sphere is $s = 4\pi r^2$ & $\frac{dr}{dt} = 2$

$\frac{ds}{dt} = 8\pi r\ \frac{dr}{dt}$

$= 8\pi r(2)$

$= 16\pi r$

$\therefore \frac{ds}{dt} \propto r$

Answer: d

33. $\int e^x x^5\, dx = x^5 e^x - e^x.5x^4 + e^x.20x^3 - e^x,60x^2 + e^x.120x - e^x.120 + c$.(by Bernoullis rule)

$= e^x [x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120] + c$

Answer: c

34. $\sin ce\ ax^3 + bx\ is\ an\ odd\ \ function\ \therefore \int_{-2}^{2}(ax^3 + bx)dx = 0$

$\int_{-2}^{2}(ax^3 + bx + c)dx = \int_{-2}^{2}(ax^3 + bx)dx + \int_{-2}^{2} cdx$

$= 0 + c[x]_{-2}^{2}$

$= c[2-(-2)] = 4c$

$\therefore it\ depends\ on\ the\ value\ of\ c$

Answer: b

35. $I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x^{2012}} = \int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2012}}{(\cos x)^{2012}+(\sin x)^{2012}} dx = \frac{\pi}{4}$

Answer: c

36. The $area\ of\ the\ ellipse = \pi ab \qquad here\ \ a = 8\ \ \& b = 6$

$= 48\pi\ squnits.$

Answer: c

37. $\int \frac{cosx\ dx}{\cos(x-\alpha)} = \int \frac{\cos(x-\alpha+\alpha)}{\cos(x-\alpha)} dx$

$= \int \frac{\cos(x-\propto)\cos\alpha - \sin(x-\alpha)\sin\alpha}{\cos(x-\alpha)}$ dx

$= \int \frac{\cos(x-\alpha)\cos\alpha}{\cos(x-\alpha)} dx - \int \frac{\sin(x-\alpha)sin\alpha}{\cos(x-\alpha)}$ dx

$= \cos\alpha.x - \sin\alpha.\log\sec(x-\alpha) + c$

$= x\cos\alpha + \sin\alpha.\log\cos(x-\alpha) + c$

Answer: a

38. $\left[1 + \left(\frac{dy}{dx}\right)^5\right]^{\frac{1}{3}} = \frac{d^2y}{dx^2}$ cubing on both sides

$1 + \left(\frac{dy}{dx}\right)^5 = \left(\frac{d^2y}{dx^2}\right)^3$

$\therefore order\ is\ 2\ \ and\ degree\ is\ 3.$

Answer: d

39. $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$

$\frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$

$\frac{dy}{\sqrt{1-y^2}} = \sqrt{(1-x^2)}dx$

$\sin^{-1} y = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x + c$

$2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + c$

Answer: c

40. $3x + y + k = 0\ \&\ x^2 + y^2 = 10$

$y = -3x - k \qquad a^2 = 10$

$m = -3\ \&\ c = -k$

$\therefore c^2 = a^2(m^2 + 1)$

$k^2 = 10(9 + 1) = 100$

$\therefore k = \pm 10$

Answer: c

41. A = $(x.3)$ $B = (3.5)\&\ c = (2.y)$

C = mid point of AB

$(2.y) = \left(\frac{x+3}{2}, \frac{3+5}{2}\right)$

$\frac{x+3}{2}$ = 2 4 = y

$x = 4 - 3 = 1$

$x = 1\ \&\ y = 4$

Answer: a

42. $x = 2 + 3\ \cos\theta\ y = 3\ \sin\theta - 1$

$x - 2 = 3\cos\theta\ \ y + 1 = 3\sin\theta$

$\therefore (x-2)^2 + (y+1)^2 = 3^2$

$r = 3$

Area of the circle = πr^2

$= 9\pi$ Sq.units

Answer: a

43. $y = 4x + c, m = 4, c = c$

$\& \ \frac{x^2}{4} + \frac{y^2}{1} = 1 \quad a = 2 \quad b = 1$

The condition is $c^2 = a^2m^2 + b^2$

$= 4\,(16) + 1 = 65 \qquad \therefore \ C = \pm\sqrt{65}$

Number of values of c is 2

Answer: d

44. $-\frac{x^2}{2-\lambda} - \frac{y^2}{\lambda-5} - 1 = 0$

$\frac{-x^2}{(2-\gamma)} - \frac{y^2}{(\lambda-5)} = 1$

$\frac{x^2}{\lambda-2} + \frac{y^2}{5-\lambda} = 1$

$if\ represents\ an\ ellipse\ \ if\ \lambda - 2 > 0 \quad \& \quad 5 - \lambda > 0$

$\lambda > 2 \ \ \& 5 > \lambda$

$\lambda < 2 \,\&\, \lambda < 5$

$\therefore 2 < \lambda < 5$

Answer: c

45. $\frac{Distance\ between\ Foci}{Distance\ between\ Directrix} = \frac{3}{2}$

$\frac{2ae}{\frac{2a}{e}} = \frac{3}{2} \quad e^2 = {}^{3}/_{2}$

$1 + \frac{b^2}{a^2} = \frac{3}{2}$

$\frac{b^2}{a^2} = \frac{3}{2} - 1$

$\frac{b^2}{a^2} = \frac{1}{2} \quad \frac{b}{a} = \frac{1}{\sqrt{2}}$

$\frac{a}{b} = \frac{\sqrt{2}}{1} \qquad \therefore a{:}b = \sqrt{2}{:}1$

Answer: b

46. $y^2 = -16x \quad \therefore 4a = -16 \quad a = -4$

Focus = (a . 0) = (- 4 . 0)

Answer: d

47. $x^2 + 2xy - y^2 = 0$

a = 1 b = -1 $\therefore a + b = 0 \qquad \therefore \theta = 90^0 = \frac{\pi}{2}$

Answer: d

48. Area = $\frac{1}{2}\begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$4 = \frac{1}{2}\,[x\,(1-2) - \ 0 + 1\,(\,2-0\,)]$

8 = - x + 2 , ∴ x = 2 – 8 , x = - 6

Answer: c

49. The equation is $\frac{x}{a} + \frac{y}{b} = 1$ ---------------1

The equation , if p is the length of the perpendicular from the origin is

x cosα + y sin α = p.

$\frac{x \cos\alpha}{p} + \frac{y \sin\alpha}{p} = 1$-----------2

On comparing equations 1 & 2 we get $\frac{1}{a} = \frac{\cos\alpha}{p} \ \& \ \frac{1}{b} = \frac{\sin\alpha}{p}$

$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

Answer: c

50. $\lim_{n\to\infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n}$

$= \lim_{n\to\infty} \frac{5^n(\,3.\frac{2^n}{5^n}.2 - 4.5)}{5^n(5.\frac{2^n}{5^n} + \ 7\,)} = \frac{0-20}{0+7} = -\frac{20}{7}$

Answer: a

51. The function f(x) is not defined at x = 0

The function f(x) is to be continuous, then,

$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{\log(1+ax)-\log(1-bx)}{x}\right)$

$= \lim_{x\to 0}\left(\log\left(\frac{1+ax}{ax}\right).a - \frac{\log(1-bx)(-b)}{-bx}\right)$

$= \log_e e.a - \log_e e.(-b) = a + b$

Answer: d

52. $\sin^2 17.5^\circ + \sin^2 72.5^\circ$

$= \sin^2 17.5^\circ + \sin^2(90 - 17.5^\circ)$

$= \sin^2 17.5^\circ + \cos^2 17.5^\circ = 1 = \tan^2 45^\circ$

Answer: b

53. OA = a = 4, OB = b = 3 , c = 120°

$c^2 = a^2 + b^2 - 2ab\cos c.$

$= 16 + 9 - 2\times 4\times 3\times \cos 120^0$

$= 25 - 24\left(\frac{-1}{2}\right) = 25 + 12 = 37$

$C = \sqrt{37}\ Km/hr$

Answer: a

54. cos 2A + cos 2B +cos 2C = - 1 – 4 cos A . cos B. cos C

Answer: a

55. $\sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right) + \sin^2\left(\frac{5\pi}{8}\right) + \sin^2\left(\frac{7\pi}{8}\right)$

$= \sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right) + \sin^2\left(\pi - \frac{3\pi}{8}\right) + \sin^2\left(\pi - \frac{\pi}{8}\right)$

$= \sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right) + \sin^2\left(\frac{\pi}{8}\right)$

$= 2\left(\sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right)\right) = 2\left(\sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)$

$= 2\left(\sin^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{8}\right)\right) = 2\ (1) = 2$

Answer: b

56. A={1,2,3,4,5,6}

Since A has 6 elements.

∴ number of subsets of A = $2^6 = 64.$

Answer: a

57. $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \cdots + \frac{1}{\log_{2020} n}$

$= \log_n 2 + \log_n 3 + \log_n 4 + \cdots + \log_n 2020.$

$= \log_n(2\times 3\times 4\times \ldots 2020) = \log_n(2020)!.$

But n= (2020)! ∴ $\log_{(2020)!}(2020)! = 1$

Answer: d

58.

p	q	r	q V r	p→ (q v r)
T	F	F	F	F

Answer: a

59. Consider $\left(3x - \frac{1}{2x}\right)^8$,

Here x = 3x a= - $\frac{1}{2x} = \left(-\frac{1}{2}\right)x^{-1}$, n = 8,

$T_{r+1} = {}^nC_r\ x^{n-r}a^r$ & r = 8

$T_{8+1} = {}^8C_8\ (3x)^{8-8}\left(-\frac{1}{2}x^{-1}\right)^8$

$T_9 = {}^8C_8 . 1 . (-\frac{1}{2})^8 x^{-8} = 1 . \frac{1}{2^8 x^8} = \frac{1}{256x^8}$

Answer: d

60. $(1 + x - 3x^2)^{3148} = A_o + A_1 x + A_2 x^2 + \ldots$

Put x = 1 $(1 + 1 - 3)^{3148} = A_o + A_1 + A_2 + \ldots$

$(-1)^{3148} = A_0 + A_1 + A_2 + \ldots$

∴ $A_0 + A_1 + A_2 + \ldots = 1$

Answer: c