QUESTION BANK
PART E

4 Marks Questions

1)

a) if $\vec{a}+\vec{b}+\vec{c} = 0$, $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, find the angle between $\vec{a}$ and $\vec{b}$

b) if $\vec{a}+\vec{b}+\vec{c} = 0$, $|\vec{a}|=5, |\vec{b}|=7, |\vec{c}|=9$, find the angle between $\vec{b}$ and $\vec{c}$

c) if $\vec{a}+\vec{b}+\vec{c} = 0$, $|\vec{a}|=6, |\vec{b}|=8, |\vec{c}|=10$, find the angle between $\vec{a}$ and $\vec{b}$

d) if $\vec{a}+\vec{b}+\vec{c} = 0$, $|\vec{a}|=4, |\vec{b}|=5, |\vec{c}|=6$, find the value of
$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

e) Find $|\vec{a}+\vec{b}|$ Given that

I) $|\vec{a}|=13, |\vec{b}|=19, |\vec{a}+\vec{b}|=24$
II) $|\vec{a}|=8, |\vec{b}|=11, |\vec{a}+\vec{b}|=15$

f) Find $|\vec{a}+\vec{b}|$ Given that $|\vec{a}|=4, |\vec{b}|=6, |\vec{a}-\vec{b}|=\sqrt{40}$

2) Find the cuberoots of the following and represent them in Argand diagram

a) $\sqrt{3}-i$ b) $-1+i$ c) $\frac{1}{2}-i\sqrt{\frac{3}{2}}$ d) $-2+2i$ e) $-4\sqrt{3}-4i$

3) Find the fourth roots of the following and represent them in argand diagram.

a) $\sqrt{3}-i$ b) $-1+i$ c) $1+i\sqrt{3}$ d) $-4i$

4) Find the Cube roots of the following and find their Continued product

a) $-\sqrt{3}+i$ b) $1-i$ c) $-2-2\sqrt{3}i$

5) Find the fourth roots of the following and find their Continued product.

a) $-1-i$ b) $2-2\sqrt{3}i$ c) $-\sqrt{3}+i$ d) $-\frac{1}{2}+i\sqrt{\frac{3}{2}}$

6) a) Find the length of the common chord of the two intersecting circles

$x^2+y^2-6x-2y+5=0$ and $x^2+y^2-5x-3y+6=0$

b) Find the length of the common chord of the two intersecting circles

$x^2+y^2-4x-4y+3=0$ and $x^2+y^2-6x-2y+5=0$

c) Find the length of the common chord of the two intersecting circles

$x^2+y^2+2x+3y-7=0$ and $x^2+y^2+3x-2y-1=0$

7) Integrate the following wrt x .

a) $\sin^3 ax$ b) $\cos^3 ax$ c) $\text{Tan}^3 ax$ d) $\text{Cot}^3 ax$ e) $\sec^3 ax$ f) $\text{Cosec}^3 ax$ g) $\text{Sin}^4 ax$

h) $\text{Cos}^4 ax$ i) $\text{Tan}^4 ax$ j) $\text{Cot}^4 ax$ k) $\sec^4 ax$ l) $\text{cosec}^4 ax$ m) $\sin^3 4x$ n) $\sec^4 5x$.

8) a) The sum of two numbers is given, then prove that their product is maximum if they are Equal.

b) The sum of the four sides of a rectangle is constant. Show that the area of the rectangle is maximum when it a square.

c) Prove that among all rectangles of a given hypotenuse, the isosceles triangle has maximum area.

d) Prove that among all rectangles of a given area, the square has minimum perimeter.

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2 Marks Questions

1) Find the least positive remainder obtained when

a) 2^{99} is divided by 5

b) 2^{150} is divided by 11

c) 2^{126} is divided by 13

d) 3^{151} is divided by 7

e) 657x 219x 78 is divided by 11

2) Find the digit in the unit place of the following.

a) 3^{63} b) 7^{203} c) 9^{201}

3) Find the least positive integer x in the following

a) $3^{101} \equiv x(\text{mod}16)$ b) $4^{65} \equiv x\ (\text{mod}7)$

4) If 1, w and w^2 are the cube roots of unity, then show that.

a) $(1+w+4w^2)(1+4w+w^2)(4+w+w^2)=27$

b) $(3-w)(3-w^2)(3-w^4)(3-w^5) = 169$

c) $(a-b)(a-bw)(a-bw^2) = a^3-b^3$

d) $(a+b)(a+bw)(a+bw^2) = a^3+b^3$

e) $(1-w+w^2)^{10} + (1+w-w^2)^{10} = -2^{10}$

f) $\frac{1}{1+2w} - \frac{1}{1+w} + \frac{1}{2+w} = 0$

5) Solve the following linear congruence equations

a) $2x \equiv 3 \pmod 5$ b) $4x \equiv 3 \pmod 9$

c) $6x \equiv 3 \pmod 9$ d) $2x \equiv 4 \pmod 8$

6) Differentiate the following wrt x

a) $\log_5(\sin x)$ b) $\log_{10}\sqrt{\sin x}$ c) $\log_6\sqrt{\cos x}$ d) $\log_4(\tan x)$ e) $\log_{10}\sqrt{\cos x}$

f) $\log_5\sqrt{x^2+2x+1}$ g) $\log_6(\cosh x)$

7) Differentiate wrt x

a) $\sin(3x)^0$ b) $\cos(2x)^0$ c) $\tan(5x)^0$ d) $\sin^2(2x)^0$ e) $\cos^2(ax)^0$ f) $\cot^2(x^0)$

8) Evaluate the following.

a) $\int_0^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{2-x}}\,dx$ b) $\int_0^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\,dx$ c) $\int_0^6 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{6-x}}\,dx$

d) $\int_0^3 \frac{\sqrt{x+2}}{\sqrt{x+2}+\sqrt{5-x}}\,dx$ e) $\int_0^2 \frac{\sqrt{x+3}}{\sqrt{x+3}+\sqrt{5-x}}\,dx$ f) $\int_0^4 \frac{\sqrt{x+5}}{\sqrt{x+5}+\sqrt{9-x}}\,dx$

9) Find the degree and the order of the following differential equations :-

a) $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2} = \frac{d^2y}{dx^2}$

b) $\left(1-\left(\frac{dy}{dx}\right)^4\right)^{2/3} = 2\frac{d^2y}{dx^2}$

c) $\left(x-\frac{d^3y}{dx^3}\right)^{1/3} = 3\left(\frac{d^2y}{dx^2}\right)^2$