

1. $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x(1 + \cos x)} \right)$ is equal to

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) -1

2. $\lim_{x \rightarrow 0} \frac{|x|}{x}$ is equal to :

- (A) 1
- (B) -1
- (C) 0
- (D) Does not exist

3. $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}}$ is :

- (A) 2
- (B) 0
- (C) 1
- (D) -1.

4. Let $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$, the quadratic equation whose roots are $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$ is :

- (A) $x^2 - 6x + 9 = 0$
- (B) $x^2 - 7x + 8 = 0$
- (C) $x^2 - 14x + 49 = 0$
- (D) $x^2 - 10x + 21 = 0$

5. $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x}$ is :

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) $\frac{1}{4}$

6. $\lim_{x \rightarrow \infty} \frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$ equals :

(A) 0

(B) $\frac{1}{10}$

(C) $\frac{1}{5}$

(D) $\frac{3}{10}$

7. $\lim_{x \rightarrow \infty} \left[\frac{x+6}{x+1} \right]^{x+4}$ is :

(A) 0

(B) 1

(C) e^4

(D) e^5 .

8. If f be a function such that $f(9) = 9$ and $f'(9) = 3$, then :

$\lim_{x \rightarrow 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is :

(A) 1

(B) 9

(C) 3

(D) None of these

9. $\lim_{x \rightarrow \infty} \left[\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)} \right]$ is :

(A) 1

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) 0.

10. If a, b, c, d are + ve, then :

$\lim_{x \rightarrow \infty} \left[1 + \frac{1}{a+bx} \right]^{c+dx}$ is equal to :

(A) e

(B) $e^{(c+d)(a+b)}$

(C) $e^{d/b}$

(D) $e^{c/a}$

11. The value of constants a and b , so that :

$\lim_{x \rightarrow \infty} \left[\frac{x^2+1}{x+1} - ax - b \right] = 0$, is

(A) $a = 0, b = 0$

(B) $a = 1, b = -1$

(C) $a = -1, b = -1$

(D) $a = 2, b = -1$.

12. $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+1} - \sqrt[3]{3^3+1}}{\sqrt[4]{x^4+1} - \sqrt{x^4+1}}$ equals :

(A) -1

(B) 0

(C) -1

(D) None of these

13. $f(x) = \frac{\sqrt{1+px} - \sqrt{1-px}}{x}$, $-1 \leq x \leq 0$
 $= \frac{2x+1}{x-x}$, $0 \leq x \leq 1$ is continuous in the interval $[-1,1]$, then p is :

(A) -1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1.

14. If $f(x) = \frac{2 - (256 - 7x)^{\frac{1}{8}}}{(5x + 32)^{\frac{1}{5}} - 2}$, $x = 0$, then for f to be continuous everywhere, $f(0)$ is equal to :

(A) 2^4

(B) 1

(C) -1

(D) None of these

15. Let $f(x) = \begin{cases} \frac{\sqrt{1+ax} - \sqrt{1-ax}}{x} & : -1 \leq x < 0 \\ \frac{2x+1}{x-2} & : 0 \leq x \leq 1 \end{cases}$

is continuous in $[-1,1]$. Then a equals :

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

16. $\lim_{x \rightarrow \infty} \frac{(2+x)^{40} (4+x)^5}{(2+x)^{45}}$ is :

- (A) -1
- (B) 1
- (C) 16
- (D) 32.

17. $\lim_{n \rightarrow \infty} \frac{\log 1+x+x^2 + \log(1-x-x^2)}{\sec x - \cos x}$ is equal to :

- (A) 2
- (B) 1
- (C) 0
- (D) does not exist

18. $\lim_{x \rightarrow 0} \frac{\log_{\sec x/2} \cos x}{\log_{\sec x} \cos x/2}$ is equal to :

- (A) 2
- (B) 4
- (C) 8
- (D) 16.

19. $\lim_{x \rightarrow 0} \left[\int_y^a e^{\sin^2 t} dt - \int_{x+y}^a e^{\sin^2 t} dt \right] + x$ is equal to :

- (A) $e^{\sin^2 y}$
- (B) $\sin 2y e^{\sin^2 y}$
- (C) 0
- (D) None of these.

20. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \cos t^2 dt}{x \sin x}$ is :

- (A) $\frac{3}{2}$
- (B) 1
- (C) -1
- (D) None of these

21. $\lim_{x \rightarrow \infty} \frac{\int_0^{2x} se^2 dx}{e^{4x^2}}$ equals :

- (A) 0
- (B) ∞
- (C) 2
- (D) $\frac{1}{2}$

22. $\lim_{x \rightarrow 2} \left(\frac{[x]^2}{3} - \left[\frac{x}{3} \right]^2 \right)$ equals :

- a) 0
- b) $\frac{1}{3}$
- c) $\frac{64}{27}$
- d) None of these

23. Let $f(x) = \begin{cases} \frac{g(x)-g(a)}{x-a} & x \neq a \\ g'(a) & x = a \end{cases}$ where g is a function derivable at $x = a$, then at $x = a$:

- (A) f is continuous only if $g'(a) = 0$
- (B) f is continuous
- (C) f is continuous only if $\lambda = 0$
- (D) None of these.

24. Let $f(x) = \begin{cases} 1, & x \leq -1 \\ |x| - 1 & -1 < x < 1 \\ 0 & x \geq 1 \end{cases}$ then :

- A) f is continuous at $x = -1$
- B) f is differentiable at $x = -1$
- C) f is continuous everywhere
- D) f is differentiable for all x .

25. The function of $f(x) = [x]$ is :

- (A) Discontinuous only for integral x
- (B) A constant function
- (C) Continuous for all x
- (D) Derivable for all x .

26. Let $f(x) = \frac{1 - \sin x}{(\pi - 2x)^2}$, when $x \neq \pi/2$ and $f(\pi/2) = k$,

The value of k which makes f continuous at $\pi/2$ is :

- (A) $1/8$
- (B) $1/4$
- (C) $1/2$
- (D) None of these

27. Let $f(x) = \begin{cases} \frac{x^2}{a}, & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2} & \sqrt{2} \leq x \end{cases}$

be continuous in $[0, \infty)$, then the most suitable values of a and b are :

- A) $a = -1, b = -1$
- B) $a = 1, b = -1$
- C) $a = -1, b = 1 + \sqrt{2}$
- D) None of these.

28. The value of $f(0)$ so that the function :

$f(x) = \frac{\sqrt{1+x} - (1+x)^2}{x}$ becomes continuous, is equal to :

- (A) $\frac{1}{3}$
- (B) 2

(C) $\frac{1}{6}$

(D) $\frac{1}{4}$

29. The value x , where the function :

$$f(x) = \frac{\tan x \log(x-2)}{x^2 - 4x + 3}$$

(A) $-\infty$

(B) $(-\infty, 2) \cup (3, \infty)$

(C) $(-\infty, 2) \cup \left\{3, \pi n + \frac{\pi}{2}, n \geq 1\right\}$

(D) None of these.

30. The value of $f(0)$ for which the function :

$$\frac{\log_e(1-ax) - \log_3(1-bx)}{x} \text{ is}$$

(A) $b - a$

(B) $a + b$

(C) $-(a + b)$

(D) $a - b$

31. If $f(x) = [x] + [-x], x \neq 2$
 $= \lambda, x = 2.$

then f is continuous at $x = 2$, provided λ is equal to :

(A) 2

(B) -1

(C) 1

(D) 0.

33. Let $f(x)$ be defined by $f(x) = \begin{cases} \frac{|x^2-x|}{x^2-x}, & x \neq 0,1 \\ 1, & x = 0 \\ -1, & x = 1. \end{cases}$

Then $f(x)$ is continuous for all :

- (A) x
- (B) x except at $x = 0$
- (C) x except at $x = 1$
- (D) x except at $x = 0$ and $x = 1$.

34. $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = k$, the value of k is :

(A) $-\frac{3}{2}$

(B) $\frac{2}{3}$

(C) $-\frac{2}{3}$

(D) 0.

35. The value of $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t \, dt}{x \sin x}$ is :

(A) 2

(B) 1

(C) 0

(D) 3.

36. The value of p and q for which the function :

$$f(x) = \begin{cases} \sin(p+1)x + \sin x, & x < 0 \\ \frac{\sin(p+1)x + \sin x}{x}, & x = 0 \\ q, & x > 0 \end{cases}$$

is :

$$\begin{cases} \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} \end{cases}$$

continuous for all x in \mathbf{R} , are :

(A) $p = \frac{1}{2}, q = -\frac{3}{2}$

(B) $p = \frac{5}{2}, q = \frac{1}{2}$

(C) $p = -\frac{3}{2}, q = \frac{1}{2}$

(D) $p = \frac{1}{2}, q = \frac{3}{2}$

37. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to “

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) $-\frac{1}{4}$

38. If $f(x) = \begin{cases} \frac{3 \sin \pi x}{5y}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$ is continuous at $x = 0$.

then the value of k is equal to :

(A) $\frac{3\pi}{10}$

(B) $\frac{3\pi}{5}$

(C) $\frac{\pi}{10}$

(D) $\frac{3\pi}{2}$

39. If the function $f(x)$ satisfies $\lim_{x \rightarrow 1} \frac{f(x)-2}{x^2-1} = \pi$

then $\lim_{x \rightarrow 1} f(x) =$

(A) 2

(B) 3

(C) 1

(D) 0

(E)

40. If $\lim_{x \rightarrow \infty} \frac{(1^2+2^2+\dots+n^2) (n)^{1/n}}{(n+1)(n+10)(n+100)} =$

(A) 3

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) ∞

41. $\lim_{x \rightarrow 0} \frac{1-\cos x}{x^2}$ is:

(A) 2

(B) 3

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

42. The function of $f(x) = [x]$, where $[x]$ denotes greatest integer function is not continuous at:

(A) 4

(B) -2

(C) 1

(D) 1.5

43. $f(x) = \begin{cases} 3x - 8 & \text{if } x \leq 5 \\ 2k, & \text{if } x < 5 \end{cases}$ is continuous, find k .

(A) $\frac{2}{7}$

(B) $\frac{3}{7}$

(C) $\frac{4}{7}$

(D) $\frac{7}{2}$

44. For a real number x let $[x]$ denote the greatest integer less than or equal to x . Then $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$ is:

(A) continuous at some x

(B) continuous at all x but $f(x)$ does not exist

(C) $f(x)$ exists for all x , but $f'(x)$ does not exist

(D) $f(x)$ exists for all x .

45. Every continuous function is :

(A) differentiable

(B) increasing

(C) decreasing

(D) not differentiable

46. Let $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$, then :

(A) $f(x)$ is not derivable at $x = 0$

(B) $f(x)$ is derivable at $x = 0$

(C) $f(x)$ is not continuous at $x = 0$

(D) $f(x)$ is continuous but not derivable at $x = 0$.

47. The function $f(x) = 1 + |\sin x|$ is :

(A) differentiable everywhere

(B) differentiable nowhere

(C) continuous everywhere

(D) continuous nowhere.

48. At the point, $x = 1$, the function :

$$f(x) = \begin{cases} x^3 - 1, & 1 < x < \infty \\ x - 1, & -\infty < x \leq 1 \end{cases}$$

(A) discontinuous and not differentiable

(B) continuous and differentiable

- (C) continuous and not differentiable
 (D) discontinuous and differentiable.

49. $g(x) = xf(x)$, where $f(x) = x\sin\frac{1}{x}$, $x \neq 0$
 $= 0$, $x = 0$.
 At $x=0$:

- (A) g is differentiable but g' is continuous
 (B) g is differentiable but g' is not continuous
 (C) g is differentiable while f is not continuous
 (D) both f and g are differentiable

50. If $y = \frac{\sin(x+9)}{\cos x}$, then $\frac{dy}{dx}$ at $x = 0$ is:

- (A) $\cos 9$
 (B) $\sin 9$
 (C) 0
 (D) 1

51. If $f(x) = 1 + x + \frac{x^2}{2} + \dots + \frac{x^{100}}{100}$, then $f'(1)$ is equal to :

- (A) $\frac{1}{100}$
 (B) 100
 (C) Does not exist
 (D) 0.

52. If $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then $\frac{du}{dv}$ is :

- (A) $\frac{1}{2}$
 (B) x
 (C) $\frac{1-x^2}{1+x^2}$

(D) 1.

53. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \dots \dots \text{to } \infty}}$, then the value of $\frac{dy}{dx}$ is equal to :

- (A) $\sqrt{\frac{\sin x}{y+1}}$
- (B) $\frac{\sin x}{y+1}$
- (C) $\frac{\cos x}{2y+1}$
- (D) $\frac{\cos x}{2y-1}$

54. If $y = x^{x^{x^{\dots \infty}}}$, then $x \frac{dy}{dx}$ equals :

- (A) $\frac{x(1-y \log x)}{y^2}$
- (B) $\frac{y^2}{x(1-y \log x)}$
- (C) $\frac{y^2}{1-y \log x}$

(D) None of these

55. If $y = \sin(\sin x)$ and $\frac{d^2y}{dx^2} + \frac{dy}{dx} \tan x + f(x) = 0$, then

$f(x)$ equals to :

- (A) $\sin^2 x \sin(\cos x)$
- (B) $\sin^2 x \cos(\sin x)$
- (C) $\cos^2 x \sin(\cos x)$
- (D) $\cos^2 x \sin(\sin x)$

56. If $y = a e^{ms} + b e^{-ms}$ then y_2 is:

- (A) $-m^2 y$
- (B) my_1
- (C) $m^2 y$
- (D) None of these

57. If $\sqrt{x+y} + \sqrt{y-x} = c$ then $\frac{d^2y}{dx^2}$ is equal to :

(A) $\frac{2}{c^2}$

(B) $\frac{2}{c}$

(C) $-\frac{2}{c^2}$

(D) None of these.

58. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is :

(A) $\frac{1}{(1+e^2)^2}$

(B) $\frac{e^x}{(1+e^x)^2}$

(C) $-\frac{e^x}{(1+e^x)^3}$

(D) e^x .

59. Let $f(2) = 4$ and $f'(2) = 4$.

Then $\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x-2}$ is given by :

(A) 2

(B) -2

(C) -4

(D) 3.

60. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx}$ is :

(A) a^2y

(B) $-n^2y$

(C) $-y$

(D) $2x^2y$.

61. If $f(x) = \int_{-1}^x |t| dt, x \geq -1$ then :

- (A) f and f' are continuous for $x + 1 > 0$
- (B) f is continuous but f' is not so for $x + 1 > 0$
- (C) f and f' are continuous at $x = 0$
- (D) f is continuous at $x = 0$ but f' is not so.

62. If $x \sin y + y \cos x = \pi$, then the value of $y''(10)$ is :

- (A) π
- (B) $-\pi$
- (C) 1
- (D) 0.

63. Let $\mathbf{R} \rightarrow \mathbf{R}$ be a function defined by :

$$f(x) = \text{Min. } \{x + 1, |x| + 1\},$$

Then which of the following is true?

- (A) $f(x)$ is not differentiable at $x = 1$
- (B) $f(x)$ is differentiable everywhere
- (C) $f(x)$ is not differentiable at $x = 1$
- (D) $f(x) \geq 1$ for all $x \in \mathbf{R}$.

64. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^2}$.

- (A) $5x^4$
- (B) $\frac{1}{1+\{g(x)\}^5}$
- (C) $1 + \{g(x)\}^5$
- (D) $1 + x^5$

65. $\frac{d}{dx} \left[\cos^2 \left(\cot^{-1} \sqrt{\frac{2+x}{2-x}} \right) \right]$ is :

(A) $\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) $-\frac{3}{4}$

66. If $f(x) = \frac{\sin^2 x}{1+\cot x} + \frac{\cos^2 x}{1+\tan x}$, then $f\left(\frac{\pi}{4}\right)$ is :

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) $-\frac{1}{2}$

(D) $\frac{-3}{4}$

67. If $\cos^{-1}\left(\frac{y}{b}\right) = n \log\left(\frac{x}{n}\right)$, then:

(A) $y_1 = x\sqrt{b^2 - y^2}$

(B) $xy_1 - \sqrt{b^2 - y^2} = 0$

(C) $xy_1 - n\sqrt{b^2 - y^2}$

(D) $xy_1 + n\sqrt{b^2 - y^2} = 0$.

68. If $f(x) = \begin{cases} 2a - x & \text{when } -a \leq x < a \\ 3x - 2a & \text{when } a \leq x. \end{cases}$

- (A) $f(x)$ is not differentiable at $x = a$
- (B) $f(x)$ is continuous at $x = a$
- (C) $f(x)$ is continuous for all $x < a$
- (D) $f(x)$ is differentiable for all $x \geq a$.

69. If $y = (1 + x)(1 + x^2)(1 + x^4)$, then $\frac{dy}{dx}$ at $x = 1$ is :

- (A) 28
- (B) 0
- (C) 20
- (D) 1.

70. If $y = (\tan^{-1}x)^2$, then $(x^2 + 1)y_2 + 2x(x^2 + 1)y_1$ is equal to :

- (A) 0
- (B) 1
- (C) 4
- (D) 2.

71. If $f(x)$ is defined by:

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + 1,$$

then $f'(0) =$

- (A) 100
- (B) -1
- (C) $100 f'(0)$
- (D) 1.

72. Which of the following functions is differentiable at $x = 0$?

- (A) $\cos(|x|) + |x|$
- (B) $\cos(|x|) - |x|$
- (C) $\sin(|x|) + |x|$
- (D) $\sin(|x|) - |x|$

73. If $x = a(\cos t + \log \tan t/2)$, $y = a \sin t$, then $\frac{dy}{dx} =$

- (A) $\tan t$
- (B) $\cot t$
- (C) $-\cot t$
- (D) $-\tan t$

74. If $f(x) = \tan^{-1}x$, then $\lim_{x \rightarrow 1} \frac{f(x)-f(1)}{x-1} =$

- (A) $\frac{\pi+3}{4}$
- (B) $\frac{x}{4}$
- (C) $\frac{\pi+1}{4}$
- (D) $\frac{\pi+2}{4}$

75. If $f(y) = f(x^2 + 2)$ and $f'(y) = 5$, then $\frac{dy}{dx}$ at $x=1$ is

- (A) 5
- (B) 25
- (C) 15
- (D) 10.

76. If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, then $1 + \left(\frac{dy}{dx}\right)^2$ is :

- (A) $\tan \theta$

(B) $\tan^2\theta$

(C) $\sec^2\theta$

(D) 1

77. If the function $g(x)$ is defined by:

$$g(x) = \frac{x^{200}}{200} + \frac{x^{199}}{199} + \frac{x^{198}}{198} + \cdots \cdots \cdots + \frac{x^2}{2} + x + 5,$$

then $g'(0) =$

(A) 1

(B) 200

(C) 100

(D) 5.

78. If $f(x) = 2x^2$, find $\frac{f(3.8) - f(4)}{3.8 - 4}$:

(A) 1.56

(B) 156

(C) 15.6

(D) 0.156.

79. If $x = ct$ and $y = \frac{c}{t}$, find $\frac{dy}{dx}$ at $t = 2$:

(A) $\frac{1}{2}$

(B) 4

(C) $\frac{1}{4}$

(D) 0.

80. If $y = \log \left(\frac{1-x^2}{1+x^2} \right)$, then $\frac{dy}{dx}$ is equal to:

- (A) $\frac{-4x}{1-x^4}$
- (B) $\frac{4x^2}{1-x^4}$
- (C) $\frac{1}{4-x^4}$
- (D) $\frac{4x^3}{4-x^4}$