MATHEMATICS

A number of the form x+iy, where x & y are integers and $i = \sqrt{-1}$ is called a complex number.

<u>Different Forms:</u>

1) Cartesian Form Z= x + iy

2) Polar Form $Z = r\left(\cos\theta + i\sin\theta\right)$ or $Z = r\ cis\ \theta$

3) Exponential Form $Z = r\ e^{i\theta}$

Demoivier's Theorem: If n is any integer then

$$\left(\cos\theta + i\sin\theta\right)^n = \cos n\theta + i\sin n\theta$$

If n is fraction then one of the value of

$$\left(\cos\theta + i\sin\theta\right)^n\ is\ \cos n\theta + i\sin n\theta$$

MATHEMATICS

1) The value of $\left(\dfrac{1+i}{1-i}\right)^{4n} =$

1) -1 2) –i 3) i 4) 1

$$\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)^2}{1^2 - i^2}$$

$$= \frac{1^2 + i^2 + 2i}{1^2 - i^2} = \frac{1-1+2i}{1+1} = \frac{2i}{2} = i \qquad (i^2 = -1)$$

$$\therefore \left(\frac{1+i}{1-i}\right)^{4n} = i^{4n} = 1 \qquad (\because \text{For any integer } n, i^{4n} = 1)$$

Ans (4)

MATHEMATICS

2) Exponential form of 1 – i is

1) $\sqrt{2}e^{i\frac{\pi}{4}}$ 2) $\sqrt{2}e^{-i\frac{\pi}{4}}$ 3) $e^{i\frac{\pi}{2}}$ 4) $e^{-i\frac{\pi}{2}}$

$$Sol : \mathrm{Z} = \mathrm{x} + \mathrm{iy} = 1 - i$$

$$\therefore r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$\theta = -\tan^{-1}\left|\frac{y}{x}\right| = -\tan^{-1}\left|\frac{-1}{1}\right| = -\tan^{-1} 1$$

$$\theta = \frac{-\pi}{4} \quad \therefore 1 - i = r\,e^{i\theta} = \sqrt{2}\,e^{-i\frac{\pi}{4}}$$

Ans(2)

MATHEMATICS

3) Find the smallest $+$ve integer for which $(1+i)^{2n} = (1-i)^{2n}$

1) 1 2) 2 3) 3 4) None of these

Sol : using $(1+i)^2 = 2i$ $(1-i)^2 = -2i$

$$we\ get\ (2i)^n = (-2i)^n \Rightarrow (-1)^n = 1$$

$$\therefore n = 2$$

Ans (2)

MATHEMATICS

4) *Find the* modulus *and Amplitude of* $\dfrac{1+2i}{1-(1-i)^2}$

1) (0, 1) 2) (1 ,0) 3) $\left(1, \dfrac{\pi}{2}\right)$ 4) None of these.

$$Ans = \frac{1+2i}{1-1^2-i^2+2i}$$

$$= \frac{1+2i}{1+2i} = 1 = 1 + i(0) \therefore \text{modulus} = 1$$

$$amp(\theta) = 0$$

Ans 2)

KEA

MATHEMATICS

$5)\ Amplitude\ of\ \ i\left(\sqrt{3}-i\right)=$

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{6}$ 4) *None of these*

$Sol\ :\ i\left(\sqrt{3-1}\right)= i+i\sqrt{3}\ \ \therefore\ \theta=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$

or

$W.K.T\ \ Amp\ \left[Z_1\ \ Z_2\right]=\ Amp\ \ Z_1\ +\ Amp\ \ Z_2$

$\therefore\ Amp\ \left[i\left(\sqrt{3}\ -i\right)\right]=\ Amp\ \ i\ +\ Amp\left(\sqrt{3}\ -\ i\right)$

$=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$

Ans(2)

MATHEMATICS

6) $Amplitude\ of\ \left(\dfrac{1+\sqrt{3}\ i}{\sqrt{3}+i}\right)$

1) $\dfrac{\pi}{3}$ 2) $\dfrac{\pi}{6}$ 3) $\dfrac{\pi}{4}$ 4) $\dfrac{\pi}{12}$

$$Sol: Amp\left(\frac{Z_1}{Z_2}\right) = Amp\ Z_1 - Amp\ Z_2$$

$$= Amp\left(1+\sqrt{3}\ i\right) - Amp\left(\sqrt{3}+i\right)$$

$$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \qquad Ans(2)$$

7) Amplitude of 'O' is

1) 0   2) $\frac{\pi}{2}$ 3) $\frac{-\pi}{6}$ 4) *None of these*

$$Sol: Z=0=0+i0$$

$$x=0,\ y=0 \quad Amp(\theta)=\tan^{-1}\left|\frac{y}{x}\right|=\tan^{-1}\left|\frac{0}{0}\right|$$

$$r=0 \quad \therefore \theta\, can\ have\ any\ value$$

$$Amp\ of\ '0'\ not\ defined \qquad Ans\ \ (4)$$

MATHEMATICS

8) The Amplitude of $\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$ is

1) $\frac{2\pi}{5}$ 2) $\frac{\pi}{5}$ 3) $\frac{\pi}{15}$ 4) $\frac{\pi}{10}$

$$Sol := 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i2\sin^2\frac{\pi}{10} \quad \because\left[\sin\theta = 2\sin\theta/2\cos\theta/2\right]$$

$$= 2\sin\frac{\pi}{10}\left[\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right] \quad \left[1 - \cos\theta = 2\sin^2\theta/2\right]$$

MATHEMATICS

$$= 2\sin\frac{\pi}{10}\left[\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right]$$

$$it\,is\,of\;the\;form\quad r\left[\cos\theta + \sin\theta\right]$$

$$\therefore \theta = \frac{\pi}{10}$$

Ans(4)

MATHEMATICS

9) Imaginary part of $\dfrac{1}{1+\cos\theta - i\sin\theta}$ *is*

1) $2\tan\theta/2$ 2) $\dfrac{-1}{2}\tan\theta/2$ 3) $-\cos ec\theta$ 4) $\dfrac{1}{2}\tan\theta/2$

$$Sol: \quad \frac{1}{1+\cos\theta - i\ \sin\theta} = \frac{1}{2\cos^2\theta/2 - i2\sin\theta/2\cos\theta/2}$$

$$= \frac{1}{2\cos\theta/2\ \left(\cos\theta/2 - \ i\ \sin\theta/2\right)}$$

$$= \frac{1}{2\cos\theta/2}\left(\cos\theta/2 + i\,\sin\theta/2\ \right)$$

$$= \frac{\cos\theta/2}{2\cos\theta/2} + \frac{i\,\sin\theta/2}{2\,\cos\theta/2} = \frac{1}{2} + i\frac{1}{2}\tan\theta/2$$

Ans (4)

10) Which of the following are correct for any two complex numbers Z_1 and Z_2?

1) $|Z_1 \; Z_2| = |Z_1| \; |Z_2|$

2) $\arg(Z_1 \; Z_2) = (\arg \; Z_1)(\arg \; Z_2)$

3) $|Z_1 + Z_2| = |Z_1| + |Z_2|$

4) $|Z_1 + Z_2| \geq |Z_1| - |Z_2|$

Sol: By the properties of complex numbers

2), 3) & 4) are not correct.

Ans (1)

11) The modulus of $\dfrac{\sqrt{3}+i}{(1+i)\left(1+\sqrt{3}\,i\right)}$ is

1) 2 2) $\dfrac{1}{2}$ 3) $\sqrt{2}$ 4) $\dfrac{1}{\sqrt{2}}$

$$Sol:\frac{\left|\sqrt{3}+i\right|}{\left|1+i\right|\left|1+\sqrt{3}\,i\right|}=\frac{\cancel{2}}{\sqrt{2}\;\cancel{2}}=\frac{1}{\sqrt{2}}$$

Ans (4)

MATHEMATICS

12) If $\omega = \dfrac{-1 + i\sqrt{3}}{2}$ find $\left(3 + \omega + 3\omega^2\right)^4$

1) 16 2) 16ω 3) -16 4) -16ω

$$Sol: \quad 3 + \omega + 3\omega^2 = 3\left(1 + \omega^2\right) + \omega \quad \left(\because \omega^3 = 1\right)$$

$$= 3(-\omega) + \omega = -3\omega + \omega = -2\omega$$

$$\therefore \left(3 + \omega + 3\omega^2\right)^4 = (-2\omega)^4$$

$$= 16\omega^4 = 16\,\omega$$

Ans (2)

MATHEMATICS

13) The value of $\left(\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}\right)^3$ is

1) 1 2)-1 3) i 4) -i

Sol:

$$\left[i\left(\cos\frac{\pi}{3}-i\ \sin\frac{\pi}{3}\right)\right]^3 = i^3(\cos\pi - i\sin\pi)$$

$$= (-i)(-1) = i$$

Ans (3)

14) If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3 + 8 = 0$ *are*

1) $-1, 1+2\omega, 12\omega^2$
2) $-1, 1-2\omega, 1-2\omega^2$
3) $1, 1, -1$
4) None of these

Sol : $(x-1)^3 + 8 = 0 \quad \Rightarrow (x-1)^3 = -8$

$\Rightarrow (x-1)^3 = (-2)^3$ using $x^3 = r^3 \Rightarrow x = r, r\omega, r\omega^2$

$x - 1 = -2\,(1, \ \omega, \ \omega^2)$

$x = -2+1, \ -2\omega+1, \ -2\omega^2+1$

$x = -1, \quad 1-2\omega, \ 1-2\omega^2$

Ans (2)

MATHEMATICS

15) If the complex number Z= x + iy which satisfy $\left|\frac{Z - 5i}{Z + 5i}\right| = 1$

lies on the

1) x- axis 2) y-axis 3) both the axes 4) None

$$Sol: \quad Consider \quad |Z - 5i| = |x + iy - 5i|$$

$$= |x + i(y - 5)| = \sqrt{x^2 + (y - 5)^2}$$

$$\therefore |Z - 5i| = \sqrt{x^2 + (y - 5)^2}$$

$$\& |z + 5i| = |x + iy + 5i| = |x + i(y + 5)|$$

MATHEMATICS

$$\therefore |Z+5i| = \sqrt{x^2+(y+5)^2}$$

$$\therefore \left|\frac{Z-5i}{Z+5i}\right| = 1 \Rightarrow |Z-5i|^2 = |Z+5i|^2$$

$$\Rightarrow x^2+(y-5)^2 = x^2+(y+5)^2$$

Simplifying we get, $20y = 0$

$$\therefore \quad y = 0$$

Ans(1)

16) The modulus and amplitude of $\left(1+\sqrt{3}\right)^8$ are respectively

1) $256 \, and \, \frac{2\pi}{3}$ 2) $256 \, and \, \frac{\pi}{3}$ 3) $256 \, and \, \frac{8\pi}{3}$ 4) $2 \, and \, \frac{2\pi}{3}$

Sol:

$$1+i\sqrt{3} = 2 \, cis \frac{\pi}{3}$$

$$\left(1+i\sqrt{3}\right)^8 = 2^8 \, cis \frac{8\pi}{3} \qquad = 2^8 \, cis\left(2\pi + \frac{2\pi}{3}\right) = 2^8 \, cis \frac{2\pi}{3}$$

Ans (1)

MATHEMATICS

17) The points representing the complex numbers

$7+9i, 3-7i$ and $-3+3i$ form a

1) Right angled triangle
2) Isosceles triangle
3) Equilateral triangle
4) None of these

$$Sol: A \equiv (7,9) \qquad B \equiv (3,-7) \qquad C \equiv (-3,3)$$

$$AB^2 = (3-7)^2 + (-7-9)^2$$

$$= (-4)^2 + (-16)^2 = 16 + 256 = 272$$

$$BC^2 = (-3-3)^2 + (3+7)^2$$

$$=(-6)^2+(10)^2=36+100=136$$

$$AC^2=(-3-7)^2+(3-9)^2=100+36=136$$

$$\therefore AB^2=BC^2+AC^2$$

$$\therefore Ans\ (1)$$

18) If Z *and* ω *are* two non - zero complex number such that $|Z\omega| = 1$ and $\arg Z - \arg\omega = \pi/2$ then $(\bar{z}\omega)$

1) i 2) -i 3) 1 4) -1

Sol:

$$Note: |\bar{Z}\omega| = |\bar{Z}|\ |\omega| = |Z|\ |\omega| = 1$$

$$\arg(\bar{Z}\omega) = \arg\bar{Z} + \arg\ \omega \qquad \because \arg\bar{z} = -\arg Z$$

$$= \arg\ \omega - \arg\ Z = -\pi/2$$

$$\therefore \bar{z}\omega = -i$$

$$\therefore Ans\ (2)$$

MATHEMATICS

19) If $x + iy = \sqrt{\dfrac{a+ib}{c+id}}$ *then* $\left(x^2 + y^2\right)^2 =$

1) $\dfrac{a^2+d^2}{a^2+b^2}$ 2) $\dfrac{a^2-b^2}{a^2-d^2}$ 3) $\dfrac{\sqrt{a^2+b^2}}{c^2+d^2}$ 4) $\dfrac{a^2+b^2}{c^2+d^2}$

Sol : $x + iy = \sqrt{\dfrac{a+ib}{c+id}}$ $x - iy = \sqrt{\dfrac{a-ib}{c-id}}$

$$\therefore \ (x+iy)(x-iy) = \sqrt{\frac{a+ib}{c+id} \times \frac{a-ib}{c-id}}$$

$$\left(x^2 + y^2\right) = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$$ squaring

$$\therefore \ \left(x^2 + y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$$

$\therefore$ *Ans* (4)

20) The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\frac{2\pi k}{7}\right)$ *is*

1) -1 2) 0 3) –i 4) i

$$sol:-i\left(\cos\frac{2k\pi}{7}+i\sin\frac{2k\pi}{7}\right)$$

$$=-i\sum_{k=1}^{6}cis\frac{2k\pi}{7}=-i\left(\text{sum of }7^{\text{th}}\text{ root of unity except }1\right)$$

$$=-i\sum_{k=0}^{6}cis\frac{2k\pi}{7}-1=-i(0-1)=i\quad\left(\because\sum_{k=0}^{6}cis\frac{2k\pi}{7}=0\right)$$

Ans (4)

MATHEMATICS

$$21)\ If\ \ x_n = cis\frac{\pi}{2^n}\ \ then\ \ x_1.x_2.x_3......\ to\ \infty\ is$$

1) -i 2) -1 3) i 4) 1

$$Sol: x_1 = cis\frac{\pi}{2},\ x_2 = cis\frac{\pi}{2^2},\ x_3 = cis\frac{\pi}{2^3}.......$$

$$x_1.\ x_2.x_3.........\ \infty \quad = cis\frac{\pi}{2}\ cis\frac{\pi}{4}\ \ cis\frac{\pi}{8}.............to\ \infty$$

$$= cis\left(\frac{\pi}{2}+\frac{\pi}{4}+\frac{\pi}{8}\right) = cis\pi\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\infty\right)$$

MATHEMATICS

$$s_\infty \text{ of } GP \text{ with } a = 1/2,\ r = 1/2$$

$$s_\infty = \frac{a}{1-r}$$

$$= cis\,\pi\left(\frac{1/2}{1-1/2}\right) = cis\,\pi(1) = -1$$

Ans (2)

MATHEMATICS

22) *The* real and imaginary part of $\log_e\left(1+\sqrt{3}\,i\right)$ are

1) $\log_e 2 \ \ and \ \ \frac{\pi}{3}$ 2) $\log_e \sqrt{2} \, and \ \ \frac{\pi}{3}$ 3) $\log_e 2 \ \ and \ \ \frac{\pi}{6}$

4) $\log_e \sqrt{2} \ \ and \ \ \frac{\pi}{6}$

$$Sol: 1+i\sqrt{3} = 2e^{i\frac{\pi}{3}}$$

$$\log_e\left(1+i\sqrt{3}\right) = \log 2e^{i\frac{\pi}{3}}$$

$$= \log 2 + \log e^{i\frac{\pi}{3}} \quad = \log 2 + i\frac{\pi}{3}\log_e e \quad \left(\because \ \log_e e = 1\right)$$

$$= \log 2 + i\frac{\pi}{3}$$

Ans (1)

23) If Z is a complex number such that $Z = -\overline{Z}$ then

1) Real Part of Z is the same as its imaginary part.

2) Z is any complex number.

3) Z is purely imaginary

4) Z is purely real.

sol : let $z = x + iy \quad -\bar{z} = -x + iy$

$given \quad z = -\bar{z}$

$\Rightarrow x + iy = -x + iy \quad \therefore x = 0 \therefore z \text{ is purely imaginary}$

Ans (3)

MATHEMATICS

24) If we express $\dfrac{(\cos2\theta - i\sin2\theta)^4\ (\cos4\theta + i\sin4\theta)^{-5}}{(\cos3\theta + i\sin3\theta)^{-2}\ (\cos3\theta - i\sin3\theta)^{-9}}$

in the form $x + iy$, we get

1) $\cos21\theta - i\sin21\theta$

2) $\cos49\theta + i\sin49\theta$

3) $\cos23\theta + i\sin23\theta$

4) $\cos49\theta - i\sin49\theta$

$$Sol: \frac{(\text{cis}(-2\theta))^4\,(\text{cis}4\theta)^{-5}}{(\text{cis}3\theta)^{-2}(\text{cis}(-3\theta))^{-9}} = \frac{\text{cis}(-8\theta)\,cis(-20\theta)}{\text{cis}(-6\theta)\,cis27\theta}$$

$$= \frac{\text{cis}(-8\theta - 20\theta)}{\text{cis}(-6\theta + 27\theta)} = \frac{cis(-28\theta)}{cis21\theta} = cis(-28\theta - 21\theta)$$

$$= cis(-49\theta) \qquad \therefore Ans(4)$$

MATHEMATICS

25) If $\omega \neq 1$ is a cube root of unity find the least + ve value of n for which $(1+\omega^2)^n = (1+\omega^4)^n$

1) 1 2) 2 3) 3 4) 4

$$Sol: (1+\omega^2)^n = (1+\omega)^n$$

$$\Rightarrow (-\omega)^n = (-\omega^2)^n$$

$$\Rightarrow (-1)^n(\omega)^n = (-1)^n(\omega)^{2n}$$

$$\Rightarrow \omega^n = 1 \qquad \Rightarrow n = 3$$

Ans(3)

MATHEMATICS

26) $If\ \omega = cis\dfrac{2\pi}{3}$, then the value of $\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$ is

1) 0　　2) 2　　3) 1　　4) -1

$$Sol: \quad \frac{1}{\omega}\ \frac{a\omega + b\omega^2 + c\omega^3}{c + a\omega + b\omega^2} + \frac{1}{\omega^2}\ \frac{a\omega^2 + b\omega^3 + c\omega^4}{b + c\omega + a\omega^2}$$

$$\frac{1}{\omega}\ \frac{a\omega + b\omega^2 + c}{c + a\omega + b\omega^2} + \frac{1}{\omega^2}\ \frac{a\omega^2 + b + c\omega}{b + c\omega + a\omega^2}$$

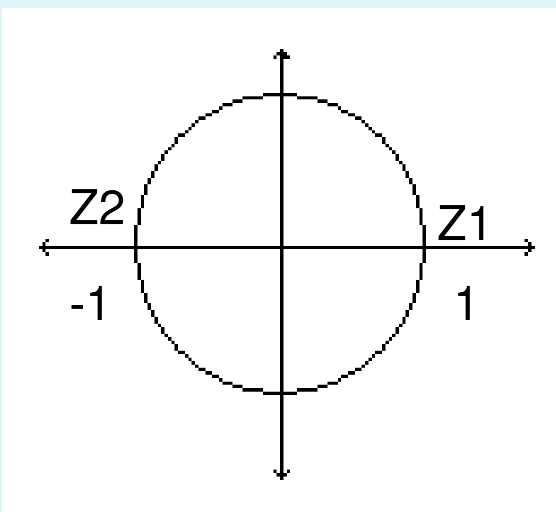
$$\frac{1}{\omega} + \frac{1}{\omega^2} = \frac{\omega^2 + \omega}{\omega\ \omega^2} = \frac{-1}{\omega^3} = -1 \quad \because \omega^3 = 1$$ **Ans(4)**

MATHEMATICS

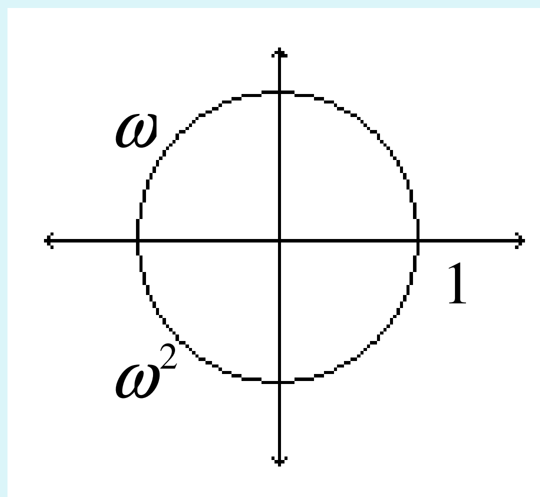
27) Let Z_1 & Z_2 be n^{th} roots of unity which subtend a right angle at the origin. Then n must be of the form

1) 4k+3 2)4k 3) 4k+1 5) 4k+2

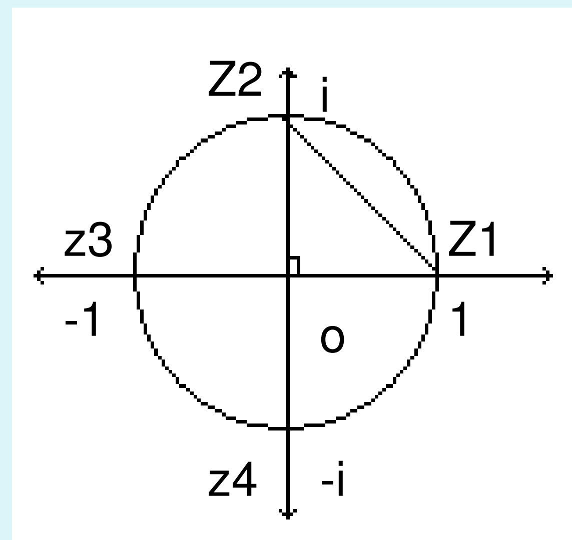
Sol : Recall Argand diagram of square roots, cube roots, fouth roots of a complex number



Square roots of unity



Cube roots of unity



Fourth roots of unity

$\therefore$ It is obvious that n must be a multiple of 4 **Ans(2)**

MATHEMATICS

28) The smallest possible integral value of 'n' such that

$$\left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^n$$ to be purely imaginary then n =

1) 3 2) 2 3) 8 4) 4

MATHEMATICS

$$Sol\ :\ Put\ \sin\frac{\pi}{8}+i\cos\frac{\pi}{8}=Z$$

$$\left(\frac{1+Z}{1+\frac{1}{Z}}\right)^n=\left(\frac{1+Z}{\frac{Z+1}{Z}}\right)^n=\left((1+Z)\ \frac{Z}{Z+1}\right)^n=Z^n$$

$$=\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)^n=\left(\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right)^n$$

$$=\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)^n\ when\ \ n=4\ \ we\ get$$

$$=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}=0+i(-1)=-i\ is\ \text{purely imaginary}$$

Ans (2)

29) If Z_1, Z_2, Z_3 are three complex numbers in A.P. then they lie on

1) a Circle 2) a straight line 3) a parabola 4) an ellipse

$Sol: Since$ Z_1, Z_2, Z_3 are in A.P.

$$\therefore 2Z_2 = Z_1 + Z_3 \Rightarrow Z_2 = \frac{Z_1 + Z_3}{2}$$

$\Rightarrow Z_2$ is mid point of the line joining $Z_1 \& Z_3$

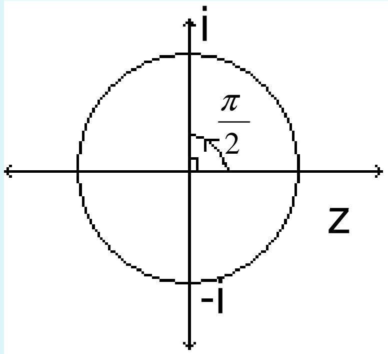
$\Rightarrow Z_1, Z_2, Z_3$ are lie on a straight line.

Ans (2)

30) Multiplication of a complex number Z by i corresponds to:

1) Clock wise rotation of the line joining Z to origin in Argand diagram through an angle of $\frac{\pi}{2}$

2)Anti clockwise rotation of the line joining Z to origin in Argand diagram through an angle of $\frac{\pi}{2}$



3)Rotation of the line joining Z to origin in Argand diagram through an angle π

4) No rotation

Ans (2)

MATHEMATICS

31) Which of the following is a fourth root of $\frac{1}{2}+i\frac{\sqrt{3}}{2}$?

1) $cis\,\frac{\pi}{12}$ 2) $cis\,\frac{\pi}{2}$ 3) $cis\,\frac{\pi}{3}$ 4) $cis\,\frac{\pi}{6}$

$$Sol\ :\ \frac{1}{2}+i\,\frac{\sqrt{3}}{2}=cis\,\frac{\pi}{3}\quad (Polar\quad form)$$

$$\therefore\ \text{Required}=\left(cis\,\frac{\pi}{3}\right)^{\frac{1}{4}}=cis\,\frac{\pi}{12}$$

Ans (1)

KEA MATHEMATICS

32) The principal amplitude of log2i is

1) $\frac{\pi}{2}$ 2) $\tan^{-1}\left(\frac{\pi}{\log 2}\right)$ 3) $\tan^{-1}\left(\frac{\pi}{\log 4}\right)$ 4) 0

$$Sol: \quad i = e^{\frac{i\pi}{2}}$$

$$\therefore \log 2i = \log 2e^{\frac{i\pi}{2}} = \log 2 + \log e^{\frac{i\pi}{2}}$$

$$= \log 2 + \frac{i\pi}{2}\log_e e \quad \left(\log m^n = n\log m\right) \qquad \because \log_e e = 1$$

$$= \log 2 + \frac{i\pi}{2} \qquad \theta = \tan^{-1}\left(\frac{\pi/2}{\log 2}\right) = \tan^{-1}\left(\frac{\pi}{\log 4}\right)$$

Ans(3)

MATHEMATICS

33) If $x + \frac{1}{x} = 2\cos\theta$, then $x^n + \frac{1}{x^n} =$

1) $2\cos n\theta$ 2) $2i\sin n\theta$ 3) $2\cos\theta$ 4) $2i\sin\theta$

Sol: If $x + \frac{1}{x} = 2\cos\theta$ then $x = \cos\theta \pm i\sin\theta$

take $x = \cos\theta + i\sin\theta$

$\Rightarrow x^n = \cos n\theta + i\sin n\theta$

$\Rightarrow \frac{1}{x^n} = \cos n\theta - i\sin n\theta \left[Note: x^n - \frac{1}{x^n} = 2i\sin n\theta \right]$

$\therefore x^n + \frac{1}{x^n} = 2\cos n\theta$ Ans (1)

MATHEMATICS

34) If $(1+i)(1+2i)(1+3i)........(1+ni) = x+iy$ *then* $2.5.10..............(1+n^2) = ?$

1) $x^2 - y^2$ 2) $x^2 + y^2$ 3) $\sqrt{x^2+y^2}$ 4) None of these

$$Sol: |1+i|\,|1+2i|\,|1+3i|............|1+ni| = |x+iy|$$

$$\Rightarrow \sqrt{1^2+1^2}\,\sqrt{1^2+2^2}\,\sqrt{1^2+3^2}\,..............\sqrt{1^2+n^2} = \sqrt{x^2+y^2}$$

$$\Rightarrow \sqrt{2}\,\sqrt{5}\,\sqrt{10}\,...............\sqrt{1+n^2} = \sqrt{x^2+y^2}$$

squaring B.S. weget,

$$2.5.10........(1+n^2) = x^2+y^2 \qquad \therefore Ans(2)$$

MATHEMATICS

35) The argument of the complex number $\frac{1+2i}{1-i}$ lies in

1) Third quadrant
2) Fourth quadrant
3) First quadrant
4) Second quadrant

$$Sol: \frac{1+2i}{1-i} \times \frac{1+i}{1+i} = \frac{1+i+2i+2i^2}{1^2-i^2}$$

$$= \frac{1+3i-2}{1+1} = \frac{-1+3i}{2} = \left(\frac{-1}{2}, \frac{3}{2}\right)$$

Ans (4)

MATHEMATICS

36) Two of the three cube roots of a complex number are $\cos 10^0 + i \sin 10^0$ and $\cos 250^0 + i \sin 250^0$ the third root is

1) $\frac{1}{2} - i\frac{\sqrt{3}}{2}$ 2) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ 3) $cis 130^0$ 4) $None$

Sol :

$$third \quad root \quad is \quad cis\ 130^{0}$$

Ans (3)

37) The real part of log(3 – 4i) is

1) log 3   2) ½ log 5   3) log 5   4) none of these

$$Sol: 3-4i = 5e^{-i\tan^{-1}\left(\frac{4}{3}\right)}$$

$$\log(3-4i) = \log\left(5e^{-i\tan^{-1}\left(\frac{4}{3}\right)}\right) = \log 5 + \log e^{-i\tan^{-1}\left(\frac{4}{3}\right)}$$

$$= \log 5 - i\tan^{-1}\left(\frac{4}{3}\right)\log_e e \therefore real\ \ part = \log 5$$

Ans(3)

38) The Real part of $e^{e^{ix}}$ is

$1)\, e^{e^{x}} \quad 2)\, e^{\cos x} \quad 3)\, e^{\cos x}\cos \quad 4)\, none\, of\, the\, above$

$Sol: e^{ix} = \cos x + i\sin x \qquad real\, part = e^{\cos x}\cos(\sin x)$

$\therefore e^{e^{ix}} = e^{\cos x + i\sin x} = e^{\cos x} e^{i\sin x} = e^{\cos x}\left[\cos(\sin x) + i\sin(\sin x)\right]$

$Ans\,(3)$

39) If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)......(\cos n\theta + i\sin n\theta) = 1$ then the value of θ is :

1) $4m\pi$ 2) $\frac{2m\pi}{n(n+1)}$ 3) $\frac{4m\pi}{n(n+1)}$ 4) $\frac{m\pi}{n(n+1)}$

$$Sol:\ cis\theta\ cis\,2\theta\ cis\ 3\theta............\ cis\ n\theta = 1$$

$$\Rightarrow cis(\theta + 2\theta + 3\theta + ... + n\theta) = 1$$

$$\Rightarrow cis(1 + 2 + 3 + ... + n) = 1$$

MATHEMATICS

$$\Rightarrow cis\left(\frac{n(n+1)\,\theta}{2}\right)=1$$

$$\Rightarrow \cos\left(\frac{n(n+1)\,\theta}{2}\right)=1 \;\;\&\; \sin\left(\frac{n(n+1)\,\theta}{2}\right)=0$$

$$\Rightarrow \frac{n(n+1)\,\theta}{2}=2m\pi \Rightarrow \theta=4m\pi$$

$$\Rightarrow \theta=\frac{4m\pi}{n(n+1)},\quad m\in I$$

Ans (3)

MATHEMATICS

40) Modulus and Amplitude of $1+\cos\theta+i\sin\theta$

1) $2\cos\theta/2, \theta/2$ 2) $2\sin\theta/2, \theta/2$

3) $2\cos\theta, \theta/2$ 4) $2\sin\theta, \theta/2$

$Sol: 1+\cos\theta+i\sin\theta$

$2\cos^2\theta/2 + i\,2\cos\theta/2\cos\theta/2$

$= 2\cos\theta/2(\cos\theta/2 + i\sin\theta/2)$

modulus $= 2\cos\theta/2$ $\quad$ $Amp.\ of\ (Z) = \theta/2$

$Ans(1)$

MATHEMATICS

41) If n= 4m + 3, m is an integer then i^n =

1) i 2) –i 3) -1 4) 1

$$Sol: i^n = i^{4m+3}$$

$$= i^{4m} \; i^3 = 1(-i) = -i$$

Ans (2)

$$Note : \text{ For any integer } 'm' \; i^{4m} = 1$$

$$i^{4m+1} = i$$

$$i^{4m+2} = i^2 = -1$$

$$i^{4m+3} = -i$$

MATHEMATICS

42) The value of $\dfrac{i^{1000} + i^{100} + 1}{i^{100} + i^{10} - 1}$

1) -1 2) -2 3) -3 4) 1

$$Sol: i^{1000} = 1,\ i^{100} = 1,\ i^{10} = -1$$

$$\therefore \frac{1+1+1}{1-1-1} = \frac{3}{-1} = -3$$

Ans (3)

43) $If\ 1, \omega, \omega^2$ are the cube roots of unity, then $(1+\omega)(1+\omega)^2\ (1+\omega)^8\ is\ equal\quad to$

1) 1 2) -1 3) $-\omega$ 4) ω^2

$Sol:$ $(1+\omega)(1+\omega)^2\ (1+\omega)^8 = (1+\omega)^{11}$

$= \left(-\omega^2\right)^{11} = (-1)^{11}\ \omega^{22} = -1\left(\omega^3\right)^7\ \omega$

Ans (3)

MATHEMATICS

44) The complex number z which satisfies the equation $\left|\dfrac{i+z}{i-z}\right| = 1$ **lies on**

a) a circle $x^2+y^2=1$ b) The x-axis

c) The y-axis d) The line $x+y=1$

MATHEMATICS

$$Sol: \quad \left|\frac{z+i}{i-z}\right| = 1$$

$$\Rightarrow |z+i| = |i-z|$$

$$\Rightarrow |i+z|^2 = |i-z|^2$$

$$\Rightarrow x^2 + (y+1)^2 = x^2 + (y-1)^2$$

simplifying we get, $4y = 0 \qquad \therefore\ y = 0$

$\Rightarrow z$ lies on the x axis

Ans (2)

45) Find modulus and Amplitude of $1-\sin\theta+i\cos\theta$

1) $2\sin\left(\frac{\pi}{4}-\theta/2\right),\frac{\pi}{4}+\theta/2$ 2) $2\cos\left(\frac{\pi}{4}-\theta/2\right),\frac{\pi}{4}-\theta/2$

3) $2\sin\left(\frac{\pi}{4}+\theta/2\right),\frac{\pi}{4}+\theta/2$ 4) None of this

MATHEMATICS

$$Sol: 1-\cos(\pi/2-\theta)+i\ \sin(\pi/2-\theta)$$

$$\text{let} \qquad \pi/2-\theta=\alpha$$

$$\therefore 1-\cos\alpha+i\sin\alpha$$

$$=2\sin^2(\alpha/2)+i\ 2\sin(\alpha/2)\cos(\alpha/2)$$

$$=2\sin\alpha/2[\sin(\alpha/2)+\cos(\alpha/2)]$$

$$=2\sin\alpha/2\{\cos(\pi/2-\alpha/2\ )+\sin(\pi/2-\alpha/2\ )\}$$

$$compairing\ \text{with}\ Polar\ form: \quad r\ (\cos\theta+i\sin\theta)$$

$$\text{Modulus}=2\sin\alpha/2 \qquad\qquad Amplitude=(\pi/2-\alpha/2\ )$$

$$\text{Substuting}\ \alpha=\pi/2-\theta \quad \text{we get}$$

$$Ans(1)$$

MATHEMATICS

46) The value of $(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}}$ *is equal to*

1) $(a^2+b^2)^{\frac{m}{n}}\left(\cos\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$

2) $2(a^2+b^2)^{\frac{m}{2n}}\left(\cos\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$

3) $(a^2+b^2)^{\frac{m}{2n}}\left(\cos\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$

4) $2(a^2+b^2)^{\frac{m}{2n}}\left(\cos\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$

Remember

Sol: Standard result

Ans(2)

MATHEMATICS

47) Find the conjugate of $Z = \frac{(2+i)^2}{3+i}$

1) $\frac{13}{10} + \frac{9}{10}i$ 2) $\frac{13}{10} - \frac{9}{10}i$ 3) $\frac{-13}{10} - \frac{9}{10}i$ 4) *None of this*

$$Sol : Z = \frac{(2+i)^2}{3+i} = \frac{3+4i}{3+i}$$

simplifing we get ,

$$z = \frac{13}{10} + \frac{9}{10}i$$

$$\therefore \overline{Z} = \frac{13}{10} - \frac{9}{10}i$$

Ans(2)

MATHEMATICS

48) $Amplitude \quad of \left(\dfrac{1-\sqrt{3}i}{(-1-i)i}\right)$

1) $\dfrac{\pi}{12}$ 2) $\dfrac{\pi}{8}$ 3) $\dfrac{\pi}{6}$ 4) $\dfrac{-\pi}{12}$

$$Sol: Amp\left(\frac{1-\sqrt{3i}}{(-1-i)i}\right)=\frac{-\pi}{3}-\left(\frac{-3\pi}{4}-\frac{\pi}{2}\right)=\frac{-\pi}{12}$$

$Ans(4)$

49) If $\omega = \dfrac{Z}{Z - \frac{1}{3}i}$ *and* $|\omega| = 1$ name the locus of Z.

1) Circle 2)straight line 3)ellipse 4)parabola

$Sol : |\omega| = 1 \Rightarrow |Z| = \left|Z - \frac{1}{3}i\right|$

$Note : |Z - Z_1| = |Z - Z_2|$

Distance of Z from Z_1 = Distance *of Z from* Z_2.Hence Z lies on the $\perp$ bisector of the line joining the points $Z_1 and Z_2$.

Required locus is a straight line. which is the perpendicular bisector of the line joining (0,0) and (0,-1/3)

$\therefore Ans(2)$

50) *The* complex numbers $1, -1, i\sqrt{3}$ form a triangle which is

1) *Right angled* 2) Isosceles

3) *Equilateral* 3) Isosceles Right angled

$Sol: Given\ A = (1, 0), \quad B = (-1, 0), \ C = (0, \sqrt{3})$

Using distance formula

$|AB| = 2$

$|BC| = 2$

$|CA| = 2$

$\therefore \Delta ABC$ is equilateral

Ans (3)

MATHEMATICS

51) The locus of the point satisfying the condition

$$\text{amp}\left(\frac{Z-1}{Z+1}\right)=\frac{\pi}{3}\ is$$

1) Straight line passing through origin.

2) Circle

3) Parabola

4) Straight line not passing through origin

Sol :

$$\frac{Z-1}{Z+1}=\frac{x+iy-1}{x+iy+1}$$

$$=\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}=\frac{(x^2+y^2-1)+2iy}{(x+1)^2+y^2}$$

$$=Amp\left(\frac{Z-1}{Z+1}\right)=\frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right)=\tan^{-1}\left(\sqrt{3}\right)$$

$$\Rightarrow \frac{2y}{x^2+y^2-1}=\sqrt{3}\Rightarrow\frac{2y}{\sqrt{3}}=x^2+y^2-1$$

$$\Rightarrow x^2+y^2-\frac{2}{\sqrt{3}}y-1=0 \quad \text{which is a circle}$$

Ans(2)

52) If Z (2 – i) = 3 + i then value of Z^{20}.

1) -1024 2) 1 – i 3) 1 + i 4)1024

$$Sol: Z = \frac{3+i}{2-i}$$

$$= \frac{3+i}{2-i} \times \frac{2+i}{2+i} = 1+i$$

$$\Rightarrow Z^2 = (1+i)^2 = 2i$$

$$\therefore z^{20} = (z^2)^{10} = (2i)^{10} = 2^{10}\ i^{10} = -1024 \left(\because i^{10} = -1\right)$$

Ans 1)

53) If Z_1 & Z_2 are two n^{th} roots of unity then $\arg.\left(\frac{Z_1}{Z_2}\right)$ *is* a multiple of

1) $n\pi$ 2) $\frac{3\pi}{n}$ 3) $\frac{2\pi}{n}$ 4) *None of these*

$Sol: Polar\ form\ of\ 1 = cis0$

$$1 = cis(2r\pi)$$

$$\therefore 1^{\frac{1}{n}} = cis(2r\pi)^{\frac{1}{n}} = cis\left(\frac{2r\pi}{n}\right) \text{ (using Demoivres theorem)}$$

MATHEMATICS

$$Z_1 = cis\left(\frac{2r\pi}{n}\right)\ Z_2 = \ cis\left(\frac{2s\pi}{n}\right)\left(\text{wherer \& s integersbetn.0 \& n}\right)$$

$$\therefore \arg.\frac{Z_1}{Z_2} = \frac{2r\pi}{n} - \frac{2s\pi}{n} = \frac{(r-s)2\pi}{n} = a\ multipleof\ \frac{2\pi}{n}$$

$Ans(3)$

54) If $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\cos\theta$, *then* $x^6 + x^{-6} =$

1) $2\cos 6\theta$ 2) $2\cos 12\theta$ 3) $2\cos 3\theta$ 4) *None*

Sol : $\sqrt{x} + \frac{1}{\sqrt{x}} = 2\cos\theta$

$$\Rightarrow \sqrt{x} = cis\,\theta \quad or \quad cis(\theta)$$

$$x = cis\,2\theta$$

$$\therefore x^6 + x^{-6} = (cis\,2\theta)^6 + (cis\,2\theta)^{-6}$$

$$= cis\,12\theta + cis(-12\theta)$$

$$= \cos 12\theta + i\sin 12\theta + \cos 12\theta - i\sin 12\theta$$

$$= 2\cos 12\,\theta$$

Ans(2)

55) If $|Z| = 1$ *and* $\omega = \dfrac{Z-1}{Z+1}$ $(where\ Z \neq -1)$, *then* $\text{Re}(\omega)$ *is*

1) 0 2) $\dfrac{-1}{|Z+1|^2}$ 3) $\left|\dfrac{Z}{Z+1}\right|\dfrac{1}{|Z+1|^2}$ 4) $\dfrac{\sqrt{2}}{|Z+1|^2}$

$$\text{Sol}: \ \omega = \frac{Z-1}{Z+1} = \frac{Z^2-1}{(Z+1)^2} = 0 \quad \left(\because |Z| = 1 \Rightarrow Z^2 = 1\right)$$

Ans (1)

MATHEMATICS

56) The least +ve integer ‘n’ for which $\left(\dfrac{1+i}{1-i}\right)^n = 1$

1) 2 2) 1 3) 3 4) 4

$$Sol: \frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = i$$

$$\Rightarrow \left(\frac{1+i}{1-i}\right)^n = 1 \qquad \Rightarrow i^n = 1$$

$$\therefore\ By\ inspection\ \ n = 4\ (n\ \ must\ \ be\ \ a\ \ multiple\ \ of\ \ 4)$$

Ans (4)

KEA

MATHEMATICS

57) If $i = \sqrt{-1}$ and n is a + ve integer, then $i^n + i^{n+1} + i^{n+2} + i^{n+3} =$

1) 1 2) i 3) i^n 4) 0

Ans (4)

MATHEMATICS

58) If n is an integer, then i^n is

1) $1, -1, i, -i$ 2) $i, -i$ 3) $1, -1$ 3) i

$Sol: i^1 = i, \quad i^2 = -1, \; i^3 = -i, \; i^4 = 1 \; and \;\; so \; on.$

$\therefore i^n = 1, -1, \; i, -i$

$Ans\,(1)$

MATHEMATICS

59) If $\omega = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$ then $\left(1 - 2\omega + \omega^2\right)^6 =$

1) 729
2) -729
3) 2^6
4) -2^6

$$Sol: \left(1 - 2\omega + \omega^2\right)^6 = \left(1 + \omega^2 - 2\omega\right)^6$$

$$= \left(\omega - 2\omega\right)^6 \quad \left(\because 1 + \omega + \omega^2 = 0\right)$$

$$= \left(-3\omega^3\right) = 3^6\ \omega^6 = 729\left(\omega^3\right)^2 \left(\because \omega^3 = 1\right)$$

$$= 729$$

Ans (1)

MATHEMATICS

60)The continued product of the cube roots of $3+\sqrt{3}i$ is

1) 3 2) -3 3) $3+\sqrt{3}i$ 4)$3-\sqrt{3}i$

continued product of cube roots of Z is Z.

Ans (3)