Areas bounded by the Curves

By

Umesh H

Lecturer

B L R P U College Sirigere

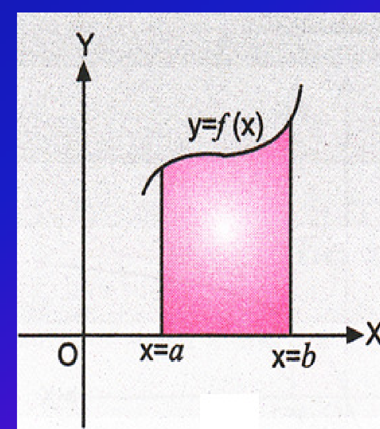
Vikasana - CET 2012

In this chapter, while calculating the definite integral as the '***limit of the sum***'. We have learnt the process of finding the area bounded by the curve $y=f(x)$, the x-axis and the ordinates $x=a$ and $x=b$.

In this chapter we shall discuss the use of definite integrals. In computing areas bounded by simple curves such as straight lines, circles, parabolas and other conics.

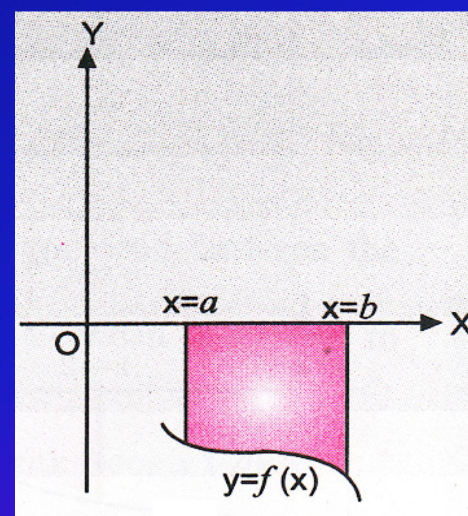
Let $y=f(x)$ be a finite and continuous curve in the interval **[a,b].** Then the area between the curve $y=f(x)$, x-axis and two ordinates at the points $x = a$ and $x = b$ is given by,

$$A = \int_a^b y\,dx = \int_a^b f(x)\,dx$$



Let ***y=f(x)*** be a continuous curve below the ***x***-axis. Then the area between the curve ***y=f(x)***, ***x***-axis and the ordinates ***x=a*** and ***x=b*** is given by

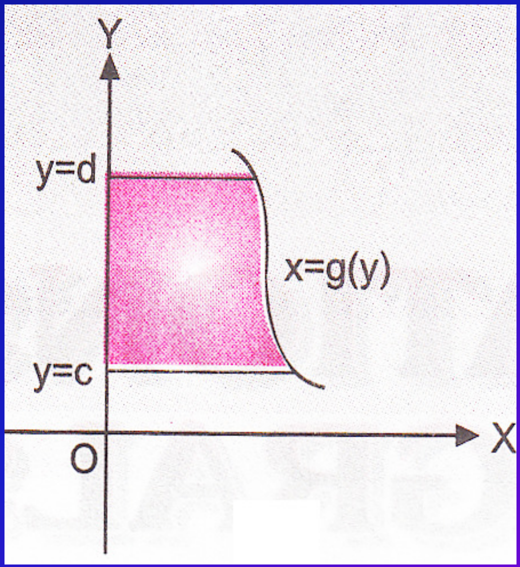
$$A = \int_a^b - y\,dx = -\int_a^b f(x)\,dx$$



$$A = \left| \int_a^b f(x)\,dx \right|$$

The area bounded by the curve ***x=f(y),*** *y*-axis and the lines ***y=c*** and ***y=d (c < d)*** is given by

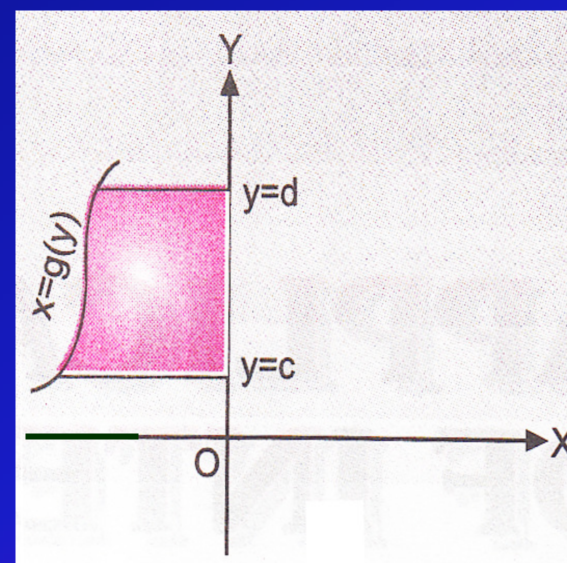
$$A = \int_c^d x\,dy = \int_c^d f(y)\,dy$$



If the curve $x=f(y)$ lies to the left of y-axis then the area bounded by the curve $y=f(x)$ and the lines $y=c$ and $y=d$ is given by

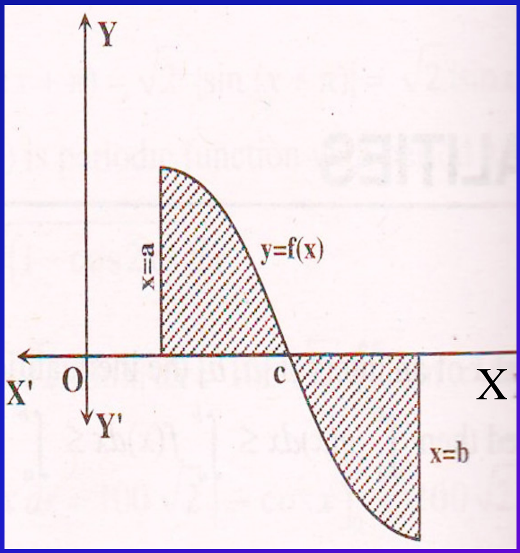
$$A=\int_{c}^{d}(-x)dy=-\int_{c}^{d}x\,dy$$

$$A=\left|\int_{c}^{d}f(y)dy\right|$$



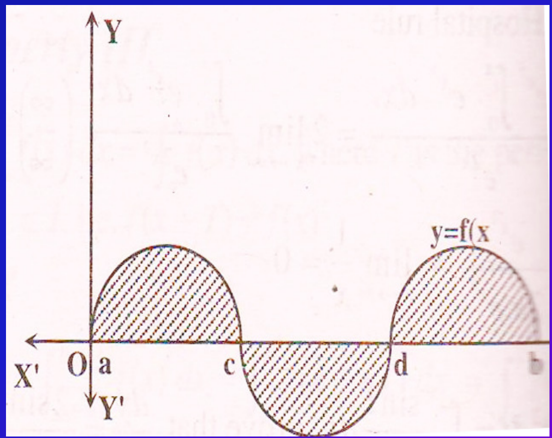
If the curve crosses x-axis at one point 'C' then the area bounded by the curve is given by.

$$A = \left| \int_a^c f(x)dx \right| + \left| \int_c^b f(x)dx \right|$$



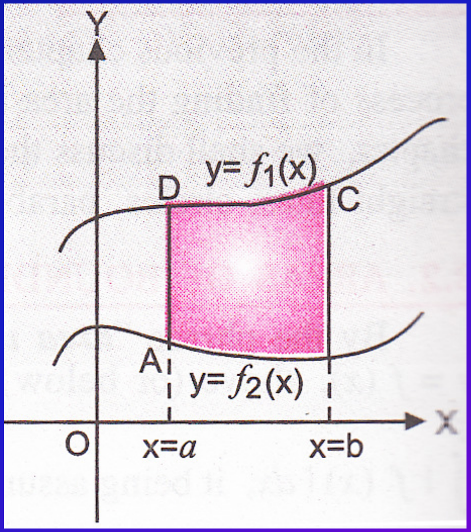
If the curve crosses x-axis in two points **c&d**, then the area between the curve $y=f(x)$, the x-***axis*** and the ordinates $x=a$ & $x=b$ is

$$A=\left|\int_a^c f(x)dx\right|+\left|\int_c^d f(x)dx\right|+\left|\int_d^b f(x)dx\right|$$



The area enclosed between the curves ***$y=f_1(x)$*** and ***$y=f_2(x)$*** between the ordinates ***$x=a$*** **&** ***$x=b$*** is given by

$$\int_a^b | f_1(x) - f_2(x) | \, dx$$



If the two curves do not cross each other between lines $x=a$ & $x=b$, then the area is

$$\left| \int_a^b f_1(x)dx - \int_a^b f_2(x)dx \right|$$

Curve Sketching for Area

For the evaluation of area of bounded regions, it is very essential to draw the rough sketch of the curves. The following points are very useful to draw a rough sketch of the curve.

- For all 'x' for which $y=f(x)=0$ $(a \le x \le b)$
- Mark these points on x - *axis*.
- In case of two curves, find the point of intersection of two curves.
- Use symmetry of the curve in finding area.

Symmetry about x-*axis* –

If the equation of the curve does not change when 'y' is changed to '$-y$', then the curve is symmetrical about x - axis.

(i.e. If only even power of 'y' occur, then the curve is symmetrical about x-axis).

Ex: $y^2 = 4ax$ is symmetrical about x-axis.

Symmetry about y-axis :

If the equation of the curve does not change, when x is changed to $-x$, then the curve is symmetrical about y-axis. (If only even power of x occur in the equation then then curve is symmetrical about y-*axis*)

Ex: $x^2=4ay$ is symmetrical about y-axis.

Symmetry in opposite quadrants :

If on replacing x by $-x$ and y by $-y$. Then the equation of the curve does not change (remains same). Then the curve is symmetric in opposite quadrants.

Ex: $y = Sin\ x$ is symmetrical in opposite quadrants.

Symmetric about the line $y = x$:

If the equation of the curve remains same on interchanging x and y, then the curve is symmetrical about the line $y=x$.

Ex: $x^3+y^3=3axy$ *is symmetrical about the line* $y=x$.

Some standard results on area :

- The area of the region bounded by $y^2=4ax$ and $x^2=4by$ is $\frac{16ab}{3}$ sq units.
- Area of the region bounded by $y^2=4ax$ and $y=mx$ is $\frac{8a^2}{3m^3}$ sq units.
- Area of the region bounded by $y^2=4ax$ and its latus return is $\frac{8a^2}{3}$ sq units.

- Area bounded by $y=sinx$, x–axis is 2 sq units. Infact, area of one loop of $y=sinx$ and $y=cosx$ is 2sq. units
- Area bounded by, $y=log_e x$, $y=0$ and $x=0$ is 1sq units

- Area of region bounded by the curve $y = sin ax$ and x-axis in $[0, np\ ]$ is $\frac{2n}{a}$
- Area of region bounded by the curve $y = cos ax$ and x-axis in $[0, np\ ]$ is $\frac{2n}{a}$
- Area of region bounded by one arch of $sin ax$ or $cos ax$ and x-axis is $\frac{2}{a}$ sq units.
- Area of circle $x^2 + y^2 = a^2$ is πa^2 sq. uints

πab

- The area of region bounded by parabola $y=ax^2+bx+c$ or $x=ay^2+by+c$ & x-axis is $\dfrac{(b^2-4ac)^{\frac{3}{2}}}{6a^2}$

- The area ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is πab sq units.

1. The area region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is

1. $\frac{16a^2}{3}$

2. $\frac{32a^2}{3}$

3. $\frac{9a^2}{2}$

4. *none*

Sol^n : WKT The area of the region bounded by $y^2 = 4ax$ and $x^2=4ay$ is $\dfrac{16ab}{3}$ sq. units.

Here replace **'b'** by **'a'** we get $\dfrac{16a^2}{3}$ sq. units.

2. The area enclosed between the parabolas $y^2=4x$ and $x^2=4y$ is

1. $\frac{3}{4}$ *sq units*

2. 16 *sq units*

3. $\frac{16}{3}$ *sq units*

4. $\frac{32a^2}{3}$ *sq units*

Here **a=1** & **b=1**

$\therefore$ Required area is

$$A=\frac{16}{3}\,squn$$

3. The area enclosed between the parabolas $y^2=6x$ and $x^2=6y$ is

1. **12** sq. uints
2. $\frac{16}{3}$ sq. uints
3. **36** sq. uints
4. none of these

$$y^2 = 6x$$

$$y^2 = 4ax$$

$$4a = 6$$

$$\boxed{a = \frac{3}{2}}$$

$$x^2 = 6y$$

$$x^2 = 4by$$

$$4b = 6$$

$$\boxed{b = \frac{3}{2}}$$

[illegible]

4. The area inside the parabola $y^2=4ax$ between the lines $x=a$ and $x=4a$ is

1. $4a^2$
2. $28a^2$
3. $\frac{28a^2}{3}$
4. $\frac{56a^2}{3}$

Since $y^2=4ax$ and is symmetrical about x-**axis**

Area of the region = 2(area of the region in the 1^{st} *quadrant)*

$$= 2\int_a^{4a} y\,dx = 2\int_a^{4a} \sqrt{4ax}\,dx = 2 \quad 2\sqrt{a}\int_a^{4a}\sqrt{x}\;dx$$

$$= 4\sqrt{a}\left.\frac{x^{3/2}}{\frac{3}{2}}\right]_a^{4a} = \frac{8}{3}\sqrt{a}\left[(4a)^{\frac{3}{2}} - a^{\frac{3}{2}}\right] = \frac{8}{3}\sqrt{a}\left[8a^{3/2} - a^{3/2}\right]$$

$$= \frac{8}{3}a^{\frac{1}{2}}\left[7a^{\frac{3}{2}}\right] = \frac{56}{3}a^{2}$$

Vikasana - CET 2013

5. The area bounded by the parabola $y^2=4ax$ and the line $x=a$ and $x=4a$ and x-*axis* is

1. $\dfrac{35a^2}{3}$

2. $\dfrac{4a^2}{3}$

3. $\dfrac{7a^2}{3}$

4. $\dfrac{28a^2}{3}$

Since the area bounded by the parabola $y^2 = 4ax$ & x–*axis* and lines $x = a$ and $x = 4a$ is

$$A = \int_a^{4a} y\,dx = \int_a^{4a} 2\sqrt{a}\sqrt{x}\,dx$$

$$= 2\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a}$$

$$= \frac{4\sqrt{a}}{3}\left[(4a)^{3/2} - a^{3/2}\right]$$

$$= \frac{4\sqrt{a}}{3}\left[8a^{3/2} - a^{3/2}\right]$$

$$= \frac{4\sqrt{a}}{3}\left[7a^{3/2}\right]$$

$$A = \frac{28a^2}{3}\, squn$$

6. The area of the figure bounded by $y = Cos\ x$ and $y = Sin\ x$ and the ordinates $x = 0$ and $x = \frac{\pi}{4}$ is

1. $\frac{1}{2}\left(\sqrt{2}-1\right)$
2. $\frac{1}{\sqrt{2}}$
3. $\sqrt{2}-1$
4. $\sqrt{2}+1$

$$\text{Required } Area = \int_0^{\frac{\pi}{4}} (Cos\, x - \sin\, x)\, dx$$

$$= [\sin\, x + \cos x]_0^{\frac{\pi}{4}}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4} \right) - (Sin 0 + Cos 0)$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (0+1) = \frac{2}{\sqrt{2}} - 1 = \sqrt{2} - 1$$

7. The area bounded by $y = log_e x$, the x-axis and the line $x = e$ is

1. 1
2. $1 - \frac{1}{e}$
3. $1 + \frac{1}{e}$
4. e

$$Area = \int_{1}^{e} \log x dx$$

$$= [x \log x - x]_{1}^{e}$$

$$= (e \log e - e) - (1 \log 1 - 1)$$

$$= (e - e) - (0 - 1) = 1$$

$y = \log x$

at $x = e$

$y = \log_e e = 1$

8. The area of the region bounded by the parabola $y=x^2+1$ and the straight line $x+y=3$ is given by,

1. $\frac{45}{7}$

2. $\frac{25}{4}$

3. $\frac{\pi}{18}$

4. $\frac{9}{2}$

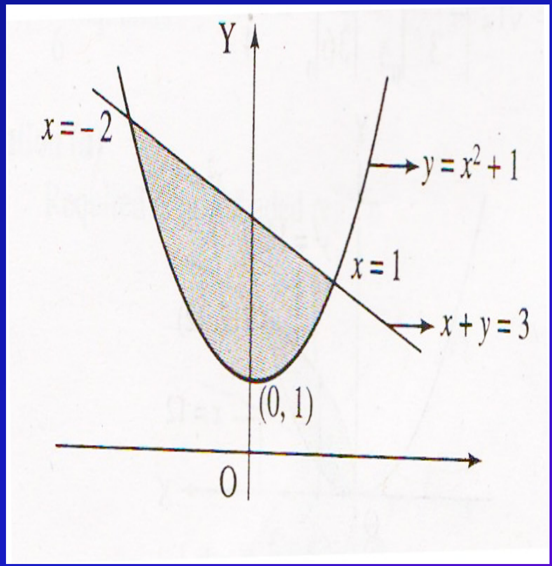
$$Given \quad y = x^2 + 1 \ \& \ x + y = 3 \Rightarrow \quad y = 3 - x$$

$$ie. \ 3 - x = x^2 + 1$$

$$(x+2)(x-1) = 0 \Rightarrow x = 1, -2$$

$$\text{Required area} = \int_{-2}^{1} (3-x) - (x^2+1)\,dx$$

$$= \int_{-2}^{1} (2 - x - x^2)\,dx = 2x - \frac{x^2}{2} - \frac{x^3}{3} \Big]_{-2}^{1}$$



$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - \frac{\cancel{4}^{\,2}}{\cancel{2}} + \frac{8}{3}\right)$$

$$= \left(\frac{12 - 3 - 2}{6}\right) - \left(\frac{-18 + 8}{3}\right)$$

$$= \frac{7 + 20}{6} = \frac{\cancel{27}^{\,9}}{\cancel{6}_{\,2}}$$

$$= \frac{7 + 20}{6} = \frac{\cancel{27}^{\,9}}{\cancel{6}_{\,2}} = \frac{9}{2}$$

9. The area of portion of the circle $x^2+y^2=64$ which is exterior to the parabola $y^2=12x$

1. $\frac{16}{3}\left(4\pi-\sqrt{3}\right) sq\ units$
2. $\frac{16}{3}\left(8\pi-\sqrt{3}\right) squnits$
3. $\frac{16}{3}\left(8+\sqrt{3}\right) squnits$
4. *None of these*

In the first quadrant the point of intersection of the circle $x^2+y^2=64$ and the parabola $y^2=12x$ is $(4, \pm 4\sqrt{3})$

$$x^2+y^2=64 \quad ie., \; x^2+12x-64=0$$

$$\Rightarrow x^2+16x-4x-64=0 \;\; \Rightarrow (x-4)(x+16)=0$$

$$\Rightarrow x=4 \;\&\; x=-16 \;(neglet\;it)$$

$$ie., \quad y^2=48 \quad \therefore y=\pm 4\sqrt{3}$$

Required area =

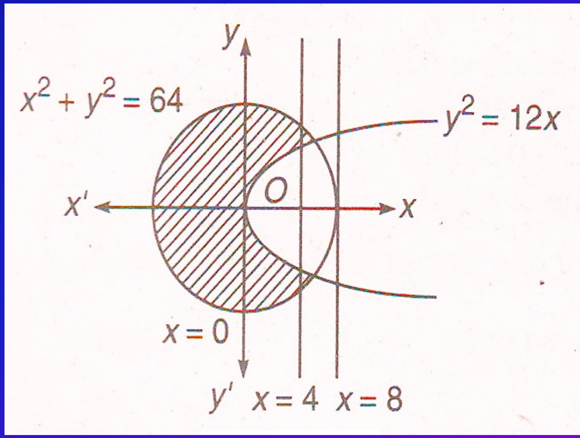
Area of the circle – Area of circle exterior to the parabola.

$$= 64\pi - 2\int_0^4 y\,dx - 2\int_4^8 y\,dx$$

$$= 64\pi - 2\int_0^4 2\sqrt{3}\sqrt{x}\,dx - 2\int_4^8 \sqrt{64 - x^2}\,dx$$

$$= 64\pi - 4\sqrt{3}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 - 2\left[\frac{x}{2}\sqrt{64 - x^2} + \frac{64}{2} Sin^{-1}\left(\frac{x}{8}\right)\right]_4^8$$

$$= 64\pi - \frac{8\sqrt{3}}{3}\left[4^{\frac{3}{2}} - 0\right] - \left[8(0) + 64 Sin^{-1}(1)\right] - \left(4\sqrt{64-16} + 64\, Sin^{-1}\left(\frac{1}{2}\right)\right)$$



$$= 64\pi - \frac{8\sqrt{3}}{3}(8) - \left[64\left(\frac{\pi}{2}\right) - 4\sqrt{48} - 64\,\frac{\pi}{6}\right]$$

$$= 64\pi - \frac{64\sqrt{3}}{3} - 32\pi + 16\sqrt{3} + \frac{32\pi}{3}$$

$$= \frac{192\pi - 64\sqrt{3} - 96\pi + 48\sqrt{3} + 32\pi}{3}$$

$$= \frac{128\pi - 16\sqrt{3}}{3} = \frac{16}{3}\left[8\pi - \sqrt{3}\right] sq.\ units.$$

10. The area enclosed between the concentric circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$ is

1. 5π sq un
2. 4π sq un
3. 9π sq un
4. 36π sq un

Given $x^2 + y^2 = 9 \longrightarrow (1)$ $x^2 + y^2 = 4 \longrightarrow (2)$

Let A_1 be the area of circle (1) is

$A_1 = 9\pi$ sq. units.

Let A_2 be the area of circle (2) is

$A_2 = 4\pi$ sq. units.

Let 'A' be the area enclosed between the two circles

$A = A_1 - A_2 \quad = 9\pi - 4\pi \quad \therefore A = 5\pi \; sq\; units$

11. Area bounded by the curves

$y = \log x, \; y = \log|x|, \; y = |\log x| \; \& \; y = |\log|x|| \text{ is}$

1. 4 sq un
2. 6 sq un
3. 10 sq un
4. none of these

W.K.T. $log\ x$ is defined for $x>0$ and $log\ |x|$ is defined for all $x \in \mathbb{R} - \{0\}$

Also $|log\ x| \geq 0$ *and* $|log\ |x|| \geq 0$

Required area is symmetrical in all the four quadrants

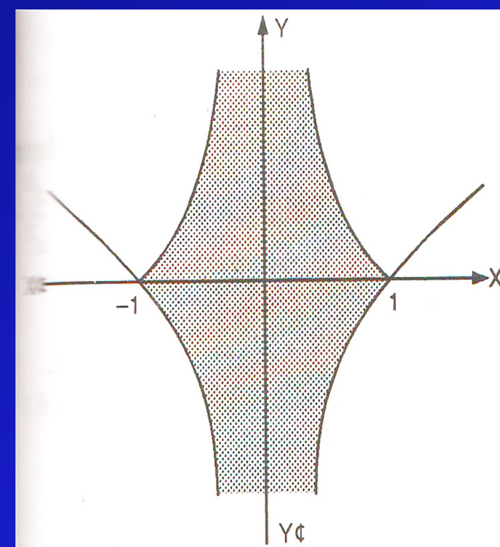
$$So\ the\ area = 4\int_0^1 |\log x|\,dx$$

$$= -4\int_0^1 \log x \, dx \quad [\ldots]$$

$$= -4\left[x \log x - x\right]_0^1$$

$$= -4\left[(1 \log 1 - 1) - (0 - 0)\right]_0^1$$

$$= -4(-1) = 4 \text{ sq. units}$$



12. The area bounded by the curves $y = x$ & $y = x^3$ *is*

1. 1 sq unit 2. $\frac{1}{2}$ sq unit

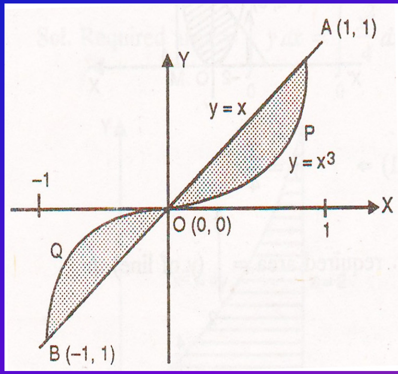
3. 2 sq unit 4. none of these

When $x=0,\ y=0 \quad x=\pm 1 \Rightarrow y=\pm 1$

$i.e.\ x = x^3 \Rightarrow x(x^2-1)=0 \quad \therefore x=0, \quad x=\pm 1$

$\therefore$ The line $y=x$ intersect the curve $y=x^3$ at three points (-1,-1),(0,0) & (1,1) Hence it is symmetric in opposite quadrant.

Required



$$= \int_0^1 (x - x^3)\,dx + \int_{-1}^0 (x^3 - x)\,dx$$

$$= \left.\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 + \left.\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^0$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \;\; sq\ units$$

13. **The area bounded by the curves** $|x| + |y| \geq 1$ **and** $x^2 + y^2 \leq 1$ **is**

1. 2 squn

2. π squn

3. $(\pi - 2)$ squ

4. none of

Area of square ABCD = **2** sq.

Area of circle = π sq. units.

Required area = $(\pi - 2)$ sq.un

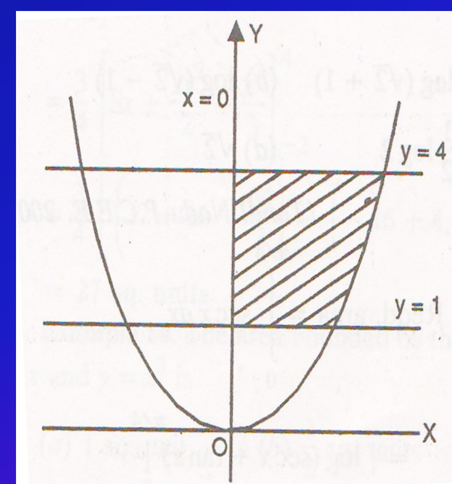
14) **The area of region bounded by** $x^2=16y$ **&** $x = 0$ **and** $y = 1, y = 4$ ***and*** y **- axis in the** 1^{st} **quadrant is**

1. $\dfrac{64}{3} sq\,un$
2. $\dfrac{56}{3} sq\,un$
3. $\dfrac{16}{3} sq\,un$
4. $\dfrac{4}{3} sq\,un$

$$\text{Required Area} = \int_1^4 x\,dy = \int_1^4 4\sqrt{y}\,dy$$

$$= 4\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\Bigg]_1^4 = \frac{8}{3}y^{\frac{3}{2}}\Bigg]_1^4 = \frac{8}{3}\left[4^{\frac{3}{2}} - 1\right]$$

$$= \frac{8}{3}(8-1) = \frac{56}{3}\text{ sq units}$$



15) The area of the region bounded by $y = x^2 - 5x + 4$ and x-*axis* is

1. $\frac{3}{2}$

2. $\frac{5}{2}$

3. $\frac{7}{2}$

4. $\frac{9}{2}$

Since the curve $y = x^2 - 5x + 4$ crosses *x-axis* $y=0$

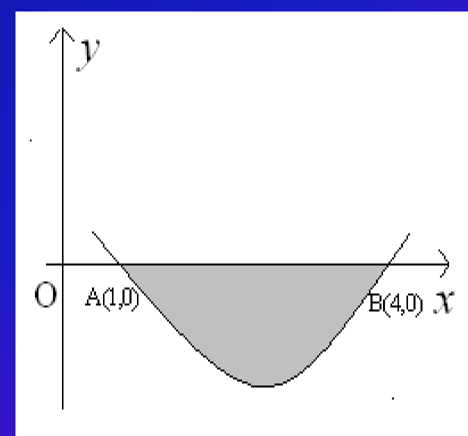
$$i.e., x^2 - 5x + 4 = 0 \qquad x^2 - 4x - 1x + 4 = 0$$

$$(x-4)(x-1) = 0 \qquad \therefore x = 1, 4 \qquad \therefore A(1,0),\ B(4,0)$$

$$\therefore \text{Required area} = \int_1^4 y\, dx$$

$$= \int_1^4 (x^2 - 5x + 4)\, dx$$

$$= \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x\right]_1^4$$



$$= \left(\frac{64}{3} - \frac{80}{2} + 16\right) - \left(\frac{1}{3} - \frac{5}{2} + 4\right)$$

$$= \left(\frac{64}{3} - 24\right) - \left(\frac{2 - 15 + 24}{6}\right) \quad = \left(\frac{64 - 72}{3}\right) - \left(\frac{11}{6}\right)$$

$$= \left|\frac{-16 - 11}{6}\right| = \frac{27}{6} = \frac{9}{2} \quad \textbf{\textit{OR}} \quad a = 1,\ b = -5,\ c = 4$$

$$W.K.T \ \frac{(b^2 - 4ac)^{3/2}}{6a^2} \quad = \frac{(25 - 16)^{3/2}}{6(1)^2} = \frac{9^{3/2}}{6} = \frac{9}{2}$$

16. The area enclosed by the parabola $y^2=16x$ and its latus rectum

1. $\frac{108}{3}$ sq un

2. $\frac{16}{3}$ sq un

3. $\frac{128}{3}$ sq un

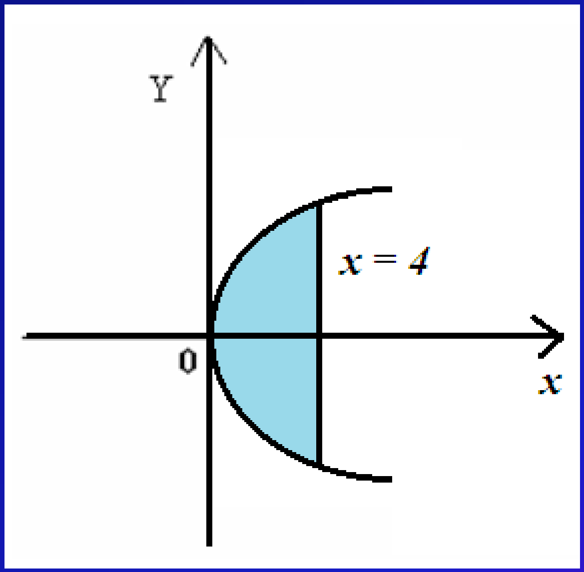
4. $\frac{4}{3}$ sq un

$$\text{Required Area} = 2\int_0^4 y\,dx$$

$$= 2\int_0^4 4\sqrt{x}\,dx = 8\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$= \frac{16}{3}\left[4^{\frac{3}{2}} - 0\right] = \frac{16}{3}(8)$$

$$A = \frac{128}{3}\ \text{Sq units}$$



17. The area of smaller segment cut off from the circle $x^2+y^2=9$ by $x=1$ is

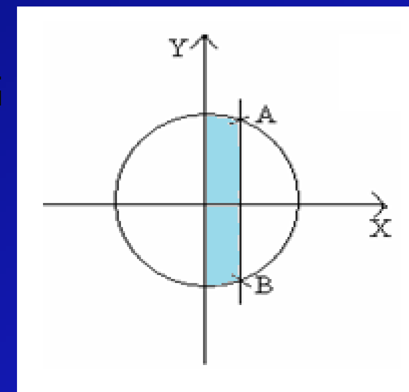
1. $\frac{1}{2}\left[9\sec^{-1}3-\sqrt{8}\right]$ sq units

2. $\left(9\sec^{-1}3-\sqrt{8}\right)sq\,units$

3. $\left(\sqrt{8}-9\sec^{-1}3\right)sq\,units$

4. None of these

$$\text{Required Area} = 2\int_1^3 y\,dx = 2\int_1^3 \sqrt{9-x^2}\,dx$$



$$= 2\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_1^3$$

$$= \left[0 + 9\sin^{-1}(1)\right] - \left[\sqrt{8} + 9\sin^{-1}\left(\frac{1}{3}\right)\right]$$

$$= 9\left(\frac{\pi}{2}\right) - \sqrt{8} - 9\sin^{-1}\frac{1}{3} \quad = 9\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)\right) - \sqrt{8}$$

$$= 9\cos^{-1}\left(\frac{1}{3}\right) - \sqrt{8} \quad = 9\,Sec^{-1}3 - \sqrt{8}$$

18. The ratio of which the area bounded by the curves $y^2 = 12x$ and $x^2 = 12y$ is divided by the line $x = 3$ is

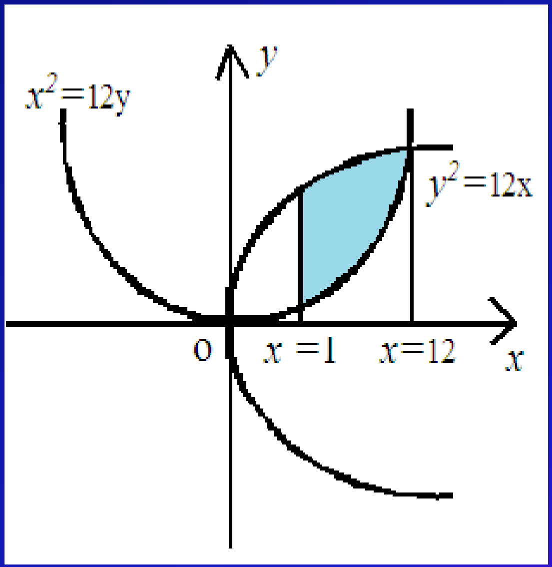
1. 154

2. 134

3. 123

4. Not

$$L.A = \int_0^3 \ldots \, dx - \int_0^3 \frac{x^2}{12} \ldots$$



$$= \left[2\sqrt{3}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{36} \right]_0^3$$

$$= \frac{4\sqrt{3}}{3}\left(3\sqrt{3}\right) - \frac{1}{36}(27) \quad = 12 - \frac{3}{4}$$

$$A = \frac{45}{4} \text{ Sq uni}$$

$$Let\ \ A_2 = \int_3^{12} \sqrt{12x}\,dx - \int_3^{12} \frac{x^2}{12}\,dx$$

$$= \frac{4\sqrt{3}}{3}\left[12^{\frac{3}{2}} - 3^{\frac{3}{2}}\right] - \frac{1}{36}\left(12^3 - 3^3\right)$$

$$= \frac{4\sqrt{3}}{3}\left[24\sqrt{3} - 3\sqrt{3}\right] - \frac{1}{36}\left(1728 - 27\right)$$

$$= \frac{4\sqrt{3}}{3}\left[21\sqrt{3}\right] - \frac{1}{36}\left(1701\right) = 84 - \frac{189}{4} = \frac{336-189}{4}$$

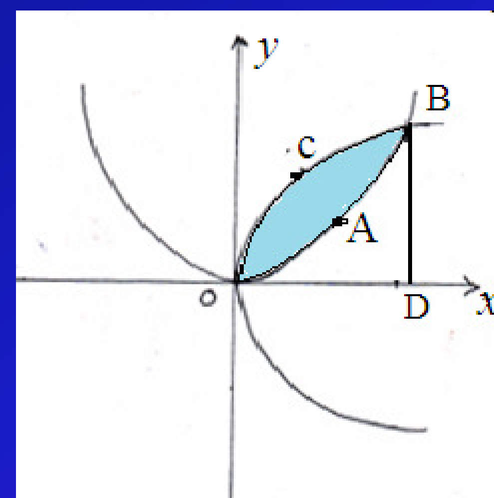
19. The area bounded by $y = ax^2$ and $x = ay^2$ $(a>0)$ is 1 then 'a' is

1. 1
2. $\dfrac{1}{\sqrt{3}}$
3. $\dfrac{1}{3}$
4. $\dfrac{-1}{\sqrt{3}}$

Solve the given equations, we get (0,0) & $\left(\frac{1}{a},\frac{1}{a}\right)$

Required Area $= Area\ of\ OCBDO - Area\ of\ OABDO$

$$1 = \int_0^{\frac{1}{a}}\left[\frac{\sqrt{x}}{\sqrt{a}} - a x^2\right]dx$$



$$\Rightarrow 1 = \left[\frac{1}{\sqrt{a}}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - a.\frac{x^3}{3}\right]_0^{1/a}$$

$$1 = \frac{2}{\sqrt{3}a}\left[\frac{1}{\sqrt{3}a} \; 0\right] \; \left[\frac{a}{3a} \; \ldots\right]$$

$$\Rightarrow 1 = \frac{2}{3a^2} \; \frac{1}{3a^2}$$

$$1 = \frac{1}{3a^2} \; 2a \; 1 \; \ldots \; \frac{1}{\sqrt{3}}$$

20. The area of the region

$\{ (x, y): x^2+y^2 \leq 1 \leq x + y \}$

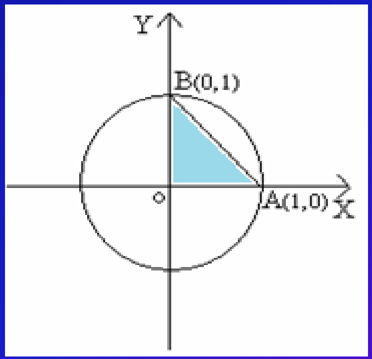
1. $\frac{\pi}{5}$ Squr   2. $\frac{\pi}{4}$ Squ

3. $\frac{2\pi}{3}$ Sq   4. $\left(\frac{\pi}{4} - \frac{1}{2}\right)$ Sq

Given equation of the circle and the line are $x^2 + y^2 = 1$ and $x + y = 1$

Solving these equations we get

$x = 0,\ x = 1$



$A(1,0)$ and $B(0,1)$

Required Area =

Area of OAB - Area of triangle OAB

$$=\left(\frac{\pi}{4}-\frac{1}{2}\right) \text{Sq}$$

21. Area of included between the curves $y = x^2 - 3x + 2$ and $y = -x^2 + 3x - 2$ is

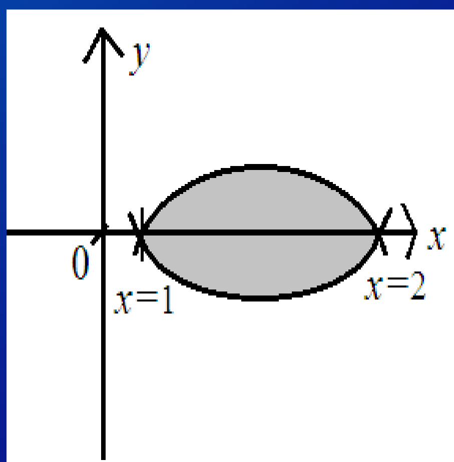
1 $\frac{1}{6}$ sq

2 $\frac{1}{2}$ sq

3 1 sq

4 $\frac{1}{3}$ sq

Required Area $= 2\int_1^2 \left(x^2 - 3x + 2\right) dx$ Both portions are same



$$= 2\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right)_1^2$$

$$= 2\left(\frac{8}{3} - 6 + 4\right) - \left(\frac{1}{3} - \frac{3}{2} + 2\right)$$

$$= 2\left(\frac{8}{3} - 2\right) - \left(\frac{2 - 9 + 12}{6}\right)$$

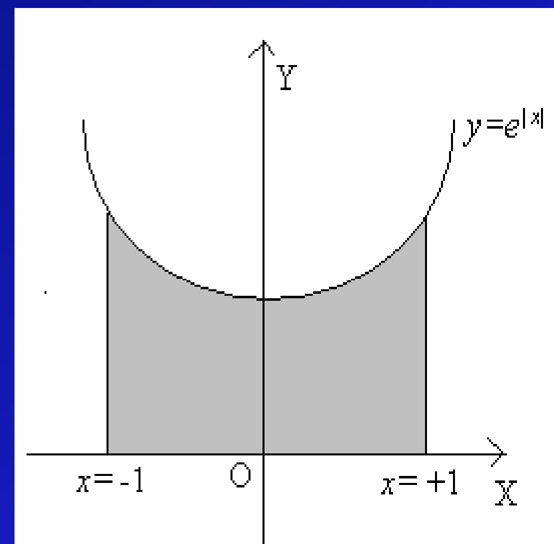
$$= 2\left[\frac{2}{3} - \frac{5}{6}\right] = 2\left[\frac{1}{6}\right] = \frac{1}{3}\ squnits$$

22. The area bounded by the curve $y = e^{|x|}$, ***x-axis*** and the lines ***x = -1*** and ***x = 1*** is

1) $(e-1)$ sq 2) $2(e-1)$ sq

3) $\left(e-\frac{1}{e}\right)$ sq 4) none of these

Required area $= \int_{-1}^{1} e^{|x|} dx$



$$= 2\int_0^1 e^x dx$$

$$= 2\left[e^x\right]_0^1$$

$$= 2\left[e^1 - e^0\right]$$

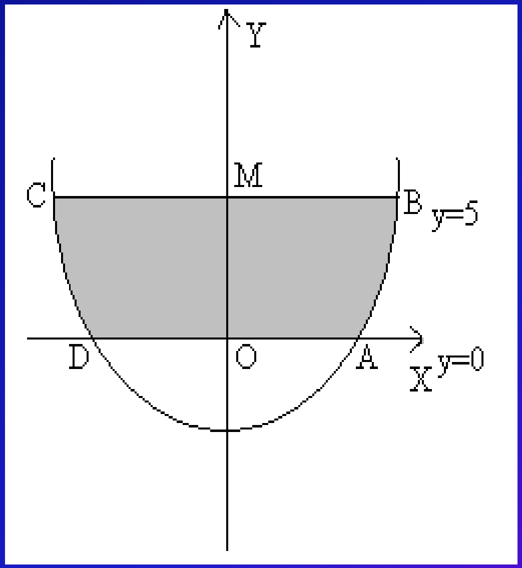
$$= 2\left[e - 1\right] sq units$$

23. The area bounded by the curve $x^2=y+4$ and the lines $y=0$ and $y=5$ *is*

1. $\frac{16}{3}$ *squr*   2 $\frac{76}{3}$ *squr*

3. $\frac{20}{3}$ *squr*   3 *None of the*

$$\text{Required Area} = Area\ of\ ABCDA = 2(Area\ of\ ABMOA)$$



$$= 2\int_0^5 x\,dy$$

$$= 2\int_0^5 \sqrt{y+4}\,dy$$

$$= 2\left[\frac{(y+4)^{3/2}}{3/2}\right]_0^5$$

$$= \frac{4}{3}\left[9^{3/2} - 4^{3/2}\right] = \frac{4}{3}[27 - 8] = \frac{4}{3}[19]$$

$$A = \frac{76}{3}\ squnits$$

24. The area region bounded by $x = a \cos\theta$ and $y = a \sin\theta$ *or* $x = a\left[\frac{1-t^2}{1+t^2}\right]$ & $y = a\left[\frac{2t}{1+t^2}\right]$ *is*

1. $2\pi a^2$

2. πa^2

3. $2\pi a$

4. πa

Since

$x = a\cos\theta \rightarrow 1$ $\quad y = a\sin\theta \rightarrow 2$

$x^2 + y^2 = a^2$

Required area = 4 area of OAB = $4\int_0^a y\,dx$

$$= 4\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right.$$

$$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= 4\left[0 + \frac{a^2}{2} \times \frac{\pi}{2}\right]$$

$$= \pi a^2 \text{ sq units}$$

25. The area of the region bounded by $x = a\cos\theta$ and $y = b\sin\theta$, i.e.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a>b)\ is$$

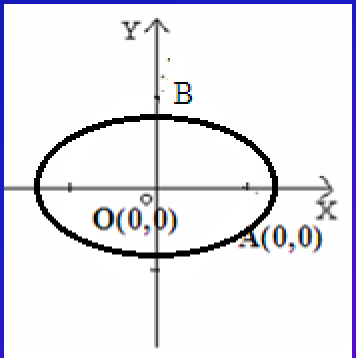
1. $2\pi a$ 2. πa

3. $4\pi a$ 4. a

Since $x = a\cos\theta$ and $y = b\sin\theta$

$$\Rightarrow \frac{x}{a} = \cos\theta \qquad \Rightarrow \frac{y}{a} = \sin\theta$$

$$\frac{x^2}{a^2} \qquad \frac{y^2}{b^2}$$



Required Area

$$= 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{4b}{a}\left[\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}(1)\right]$$

$$= \frac{4b}{2}\left[0+\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right]$$

$A = \pi ab$ squni

End