

I PUC ALGEBRA

- 1. SET THEORY
- 2. LOGARITHMS
- 3. SUMMATION OF SERIES
- 4. THEORY OF EQUATIONS
- 5. MATHEMATICAL LOGIC
- 6. PARTIAL FRACTIONS
- 7. BINOMIAL THEOREM



1. Which of the following is not a singleton?

a) { x : $|x| < 1, x \in Z$ } b) { x : $|x| = 5, x \in N$ } c) { x : $x^3 + 27 = 0, x \in R$ } d) { x : $x^2 + 3x + 4 = 0, x \in R$ }

Ans: (d).



Solution :

[A singleton set is a set having only one element.]

- a) : {0}
- b) : { 5 }
- c) : {-3}
- d): φ
- ∴ Answer is option (d)



2. If $g(x) = 1 + \sqrt{x}$ and $f(g(x)) = 3 + 2\sqrt{x} + x$, then f(4) =a) 3 b) 18 c) 11 d) 4 Ans b.



Solution:

f(g(x)) = 3 + 2
$$\sqrt{x}$$
 + x
= [1+2 \sqrt{x} + $\sqrt{x^2}$] +2
= $(1 + \sqrt{x})^2$ +2
= [g(x)]^2 +2
 \therefore f(g(x))=[g(x)]^2 +2 put g(x)=4
 \therefore f(4) = (4)^2 +2 = 18
 \therefore Answer is option (b)



- 3. Let A={ -1, 0, 1} and B= { 0, 2} and a function f: A \rightarrow B defined by y = 2 x⁴, then f is
 - a) one one onto
 - b) one one into
 - c) many one onto
 - d) many one into.

Ans c.

K. MATHEMATICS Solution: f: $A \rightarrow B$ defined by $y = 2 x^4$. when x = -1, y = 2; x = 0, y = 0 and x = 1, y = 2∴ -1 and 1 have 2 as their image. \therefore f is not one-one \therefore f is many one. Answer is either option c or option d Since both 0 and 2 are images, f is onto. ∴ f is many one and onto

∴ Answer is option (c)

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- 4. Let R be a relation on the set of real numbers defined by a R b
 if | a b | ≤ 1, then R is
 - a) Reflexive and symmetric
 - b) Reflexive and transitive
 - c) symmetric and transitive
 - d) only reflexive.

Solution:

K

 $|a - a| \le 1 \Longrightarrow 0 \le 1$ is true. $\therefore a R a$

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- \therefore R is reflexive.
- We have $|\mathbf{x}| = |-\mathbf{x}| \implies |\mathbf{a} \mathbf{b}| \le 1$
- $\Rightarrow | \mathbf{b} \mathbf{a} | \leq 1 \qquad \qquad \therefore \mathbf{R} \text{ is symmetric}$
- $|3-2| \le 1$ and $|2-1| \le 1$ but $|3-1| \le 1$
- . R is not transitive.
- \therefore R is only reflexive and symmetric .
 - : Answer is option (a)



5. If a function f: N \rightarrow N such that f(1) = 1, f(n+1) = 2 f(n) + 1, then f(n) =

a)
$$2^{n-1}$$
 b) $2^n - 1$
c) $2^{n-1} - 1$ d) $2^{n+1} + 1$
Ans b.

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Solution:

Given f(1) = 1, Now f(n+1) = 2 f(n) + 1.

 \therefore put n=1, f(2) = 2 f(1) + 1 = 2 + 1 = 3 (:: f(1) = 1) ∴ f(2) = 3. When n= 2, (a) $\rightarrow 2^{n-1} = 2$; (b) $\rightarrow 2^n - 1 = 4 - 1 = 3$; (c) $\rightarrow 2^{n-1}$ - 1=1; (d) $\rightarrow 2^{n+1}$ + 1=9 \therefore f(2) = 3 matches with option (b) \therefore Answer is option (b)



6. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is a) { 1, 2, 3, 4 } b) { 1, 2, 3, 4, 5, 6 } c) { 1, 2, 3 } d) { 3, 4, 5 } Ans c.



Solution: $f(x) = {}^{7-x}P_{x-3}$. clearly 7- $x \ge x - 3 \implies 10 \ge 2x \implies 5 \ge x$ $\implies x \le 5$

But $x \ge 3$: x = 3,4,5.

∴ Domain is

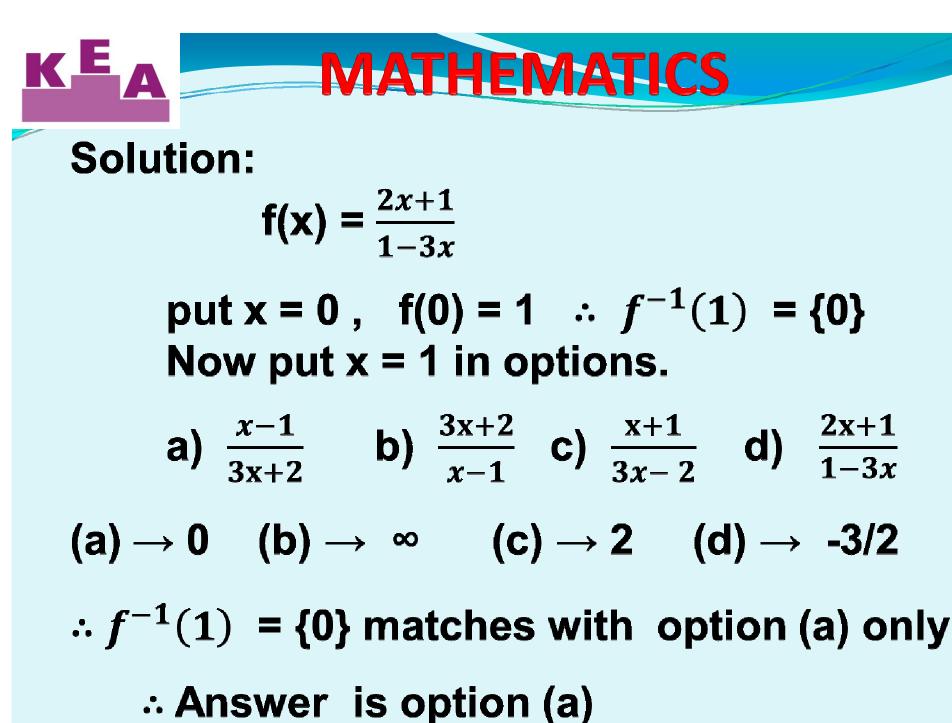
 ${7^{-x}P_{x-3} \setminus x = 3,4,5} = {4P_0, 3P_1, 2P_2} = {1,3,2}$

∴ Answer is option (c)



7. If $f(x) = \frac{2x+1}{1-3x}$ then $f^{-1}(x) =$ _____ a) $\frac{x-1}{3x+2}$ b) $\frac{3x+2}{x-1}$ c) $\frac{x+1}{3x-2}$ d) $\frac{2x+1}{1-3x}$

Ans a.





8. If A = { 1, 2, 3, 4} Then which of the following is a function from A to itself

a)
$$f_1 = \{ (x, y) \setminus y = x + 1 \}$$

b) $f_2 = \{ (x, y) \setminus (x + y) > 4 \}$
c) $f_3 = \{ (x, y) \setminus (y < x) \}$
d) $f_4 = \{ (x, y) \setminus (x + y = 5) \}$

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Solution :

a) when x= 4, y = x+1=5 \notin A \therefore 4 has no image b) $f_2 = \{ (x, y) \setminus (x + y) > 4 \}$ 2+3>4 and 2+4>4 \therefore 2 has two images # c) $f_3 = \{ (x, y) \setminus (y < x) \}$ when x = 1, $y < 1 \therefore y \notin A$ \therefore options a, b, c are rejected. Hence only possibility is option (d). \therefore Answer is option (d) 17



9. If log_e^2 , $log_e^{(2x-1)}$ and $log_e^{(2x+3)}$ are in A P then the value of x is _____

a)
$$-\frac{1}{2}$$
 b) $\frac{5}{2}$

c) 1 d)
$$\frac{1}{2}$$

Ans b.



Solution : when (a) $\rightarrow x = -\frac{1}{2}$, $log_e^{(2x-1)} = log_e^{(-2)}$, meaningless When (c) \rightarrow x = 1; log_{e}^{2} , $log_{e}^{(1)} = 0$ and $log_e^{(5)}$, which are not in AP when (d) $\rightarrow x = \frac{1}{2}$, $log_e^{(2x-1)} = log_e^{(0)}$, meaningless Hence only possibility is option (b) ∴ Answer is option (b) 19



10. If
$$x = log_4^2$$
, $y = log_6^4$ and
 $z = log_8^6$,
then yz(2 - x) =

Ans c.



Solution:

yz(2-x) = 2yz - xyz $= 2log_{6}^{4}log_{8}^{6} - log_{8}^{2}$ $= 2log_8^4 - log_8^2$ $= log_8^{16} - log_8^2$ $= log_8^8 = 1$ Hence answer is option (c)



11. If $\frac{1}{\log_3^{\pi}} + \frac{1}{\log_4^{\pi}} > k$ then the greatest integral value of k =

a) 3
b) 2
c) 1
d) 4

Ans b.

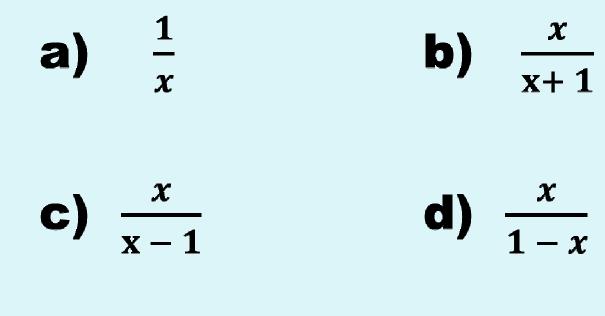


Solution:

Given $log_{\pi}^{3} + log_{\pi}^{4} > k$ $\Rightarrow log_{\pi}^{12} > k$ $\Rightarrow 12 > \pi^{k}$. (k greatest integer) Now 12 > π^{2} and 12 $\Rightarrow \pi^{3}$ \therefore k= 2 \therefore Answer is option (b)



12. If $log_a^{ab} = \mathbf{x}$ then $log_b^{ab} =$



Ans c.

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Solution:

We have $\log_a^{ab} = x$ $\therefore \log_{ab}^{a} = \frac{1}{x}$ Now $\log_{ab}^{a} + \log_{ab}^{b} = \log_{ab}^{ab} = 1$ $\Rightarrow \log_{ab}^{b} = 1 - \log_{ab}^{a} = 1 - \frac{1}{x} = \frac{x-1}{x}$ $\therefore \log_b^{ab} = \frac{1}{\log_{ab}^{b}} = \frac{x}{x-1}$

∴ Answer is option (c).



13. If x = 27, y = log_3^4 then $x^y =$ a) 64 b) 16 **b)** $\frac{1}{16}$ **c)** $\frac{3}{7}$



Solution :

$$x^{y} = 27^{\log_{3}^{4}}$$

$$= 3^{3 \log_{3}^{4}}$$

$$= 3^{\log_{3}^{64}}$$

$$= 64 \quad (\because a^{\log_{a}^{x}} = x)$$

$$\therefore \text{Answer is option (a)}$$



14. If $2 \log_{10}^{a} - 3 \log_{10}^{b} = 2$ then 100 $b^{3} =$ _____

a) a^2 b) a

- c) a^3 d) 3a
 - Ans a.

KEA MATHEMATICSSolution :

Given $2 \log_{10}^{a} - 3 \log_{10}^{b} = 2$ $\Rightarrow log_{10}^{a^2} - log_{10}^{b^3} = 2$ $\Rightarrow \log_{10}^{\frac{a^2}{b^3}} = 2 \Rightarrow \frac{a^2}{b^3} = 100$ \Rightarrow **100** $b^3 = a^2$ \therefore Answer is option (a)



15. If x , y and z are any three odd consecutive odd positive integers then log _e (xz +4) =

a)
$$\log_{e}^{2y}$$
 b) \log_{e}^{y}
c) $2\log_{e}^{y}$ d) $4\log_{e}^{y}$

Ans c.



Solution :

Take 3 consecutive odd positive integers as x = 1 y = 3 z = 5then xz + 4 = 9 $\therefore \log_{e} (xz + 4) = \log_{e} 9$ $= 2 \log_{e} 3$ = $2log_e^{y}$ (:: y = 3) ∴ Answer is option (c)



16. If
$$S_n = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots$$
 to n
terms, then $S_n =$

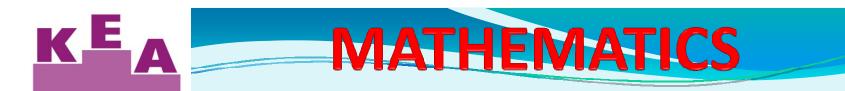
a)
$$\frac{2n}{2n+1}$$
 b) $\frac{n}{2n+1}$
c) $\frac{n}{n+2}$ d) $\frac{2n}{n+5}$

Ans b.

Solution: By inspection method
a)
$$\frac{2n}{2n+1}$$
 b) $\frac{n}{2n+1}$ c) $\frac{n}{n+2}$ d) $\frac{2n}{n+5}$
Put n= 2, Then LHS= $S_2 = \frac{1}{3} + \frac{1}{15} = \frac{6}{15} = \frac{2}{5}$
 $\Rightarrow S_2 = \frac{2}{5}$
In options, $a \rightarrow \frac{4}{5}$ $b \rightarrow \frac{2}{5}$ $c \rightarrow \frac{1}{2}$ $d \rightarrow \frac{4}{7}$

- \therefore The value of S_2 matches with option (b).
- \therefore Answer is option (b).

Or
$$S_n = \frac{n}{a(a+nd)} = \frac{n}{1(1+2n)} = \frac{n}{2n+1}$$



17. The sum 1.3 + 3.5 + 5. 7 +.... up to n terms is

a)
$$\frac{n}{5}$$
 [3 n^2 + 7n + 5)
b) $\frac{n}{2}$ [2 n^2 + 3n + 1)
c) $\frac{n}{3}$ [4 n^2 + 6n - 1)
d) $\frac{n}{3}$ [5 n^2 + 3n + 1)

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Solution: By inspection method:

instead of checking for n=1 or n=2, check for n=3 (fast) LHS = 1.3 + 3.5 + 5.7 = 53

When n= 3 the values of the options are

a) $\frac{n}{5}$ [3 n^2 + 7n + 5) b) $\frac{n}{2}$ [2 n^2 + 3n + 1)

c) $\frac{n}{3}$ [4 n^2 + 6n - 1) d) $\frac{n}{3}$ [5 n^2 + 3n + 1)

a) $\frac{3}{5}$. 53 b) $\frac{3}{2}$. 28 = 42 c) 53 d) 55 \therefore Answer is option (c).



18. The value of **1**+ $\frac{2}{5}$ + $\frac{3}{25}$ + ----- to ∞ is **b)** $\frac{16}{25}$ a) $\frac{1}{25}$ **c)** $\frac{25}{16}$ 5 4 **d)** Ans c.



Solution: Given is in AG series where a = 1, d = 1, r = $\frac{1}{5}$ $S_{\infty} = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{1}{1-\frac{1}{5}} + \frac{\frac{1}{5}}{(1-\frac{1}{5})^2}$ $=\frac{5}{4}+\frac{5}{16}=\frac{25}{16}$

∴ Answer is option (c)



19. The 25^{th} term of the series 3 + 15 + 35 + 63 + is _

a) 2500 b) 2499

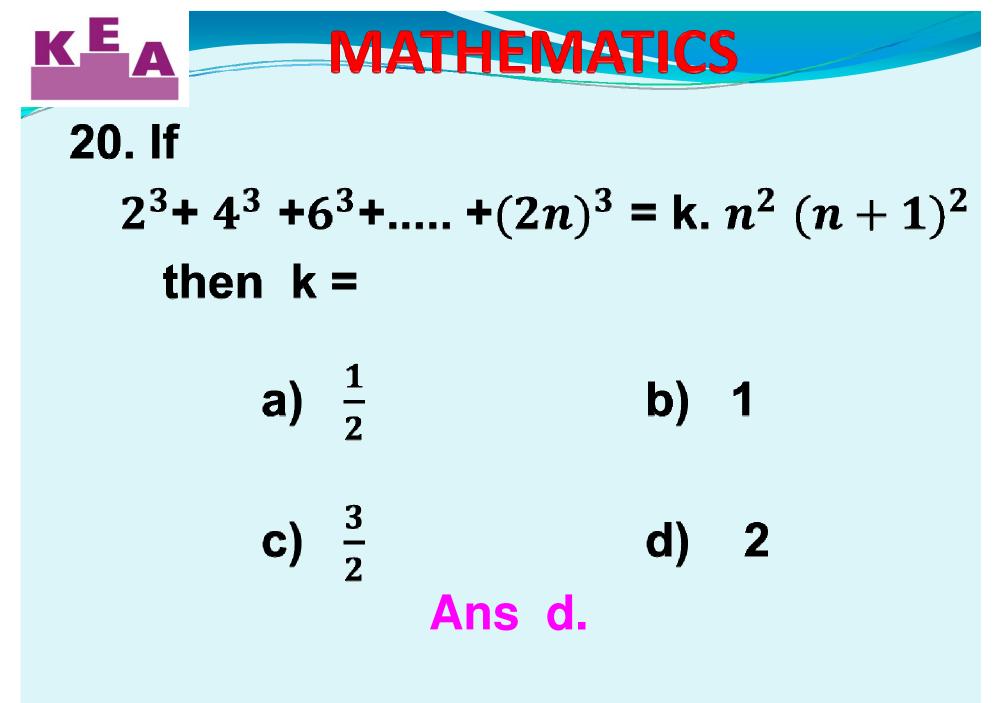
c) 2501 d) 1249

Ans b.

Solution:

summation by the method of differences. Here I differences : 12, 20, 28; II differences : 8,8,8,.... $\Delta = 12$ $\Delta^2 = 8$ $T_1 = 3$ $T_n = T_1 + (n-1) \Delta + \frac{1}{2} (n-1)(n-2) \Delta^2$ $T_{25} = 3 + 24.12 + \frac{1}{2} \cdot 24.23.8$ = 3 + 288 + 2208 = 2499 \therefore Answer is option (b) 39

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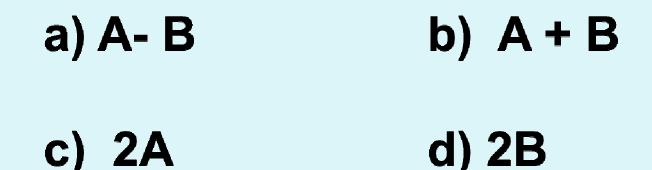


Solution:

Inspection Method : 2^{3} + 4^{3} + 6^{3} +....+ $(2n)^{3}$ = k. n^{2} $(n + 1)^{2}$ Put n = 1RHS = k.4LHS = 8 ∴ 4k = 8 \Rightarrow k = 2 ∴ Answer is option (d)



21. If the sum of n terms of an AP is $nA + n^2 B$ where A and B are constants. Then its common difference is



KEA MATHEMATICS Solution: Given $S_n = nA + n^2 B$. Put n=1, 2 $\therefore T_1 = S_1 = A + B$ And $S_2 = 2A + 4B$ But $S_2 = T_1 + T_2$ $:: T_2 = S_2 - T_1 = (2A + 4B) - (A + B)$ =A + 3B. Now $T_1 = A + B$ and $T_2 = A + 3B$: Common difference = T_2 - T_1 = 2B ∴ Answer is option (d)



22. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan30⁰ and tan15⁰, Then q=

c) p +1 d) $\sqrt{3}$ p

KEA MATHEMATICS Solution: Consider $x^2 + px + q = 0$ Let \propto = tan30⁰ and β = tan15⁰. The $\propto + \beta = -b/a = -p$ and $\propto \beta = c/a = q$. Now $\tan 45^0 = \tan (30^0 + 15^0)$ $\implies \mathbf{1} = \frac{tan30^0 + tan15^0}{1 - tan30^0 \cdot tan15^0} = \frac{\alpha + \beta}{1 - \alpha \beta} = \frac{-p}{1 - q}$ $\Rightarrow 1 = \frac{-p}{1-q} \Rightarrow 1-q = -p$ \Rightarrow q - p = 1 \Rightarrow q = 1 + p ∴ Answer is option (c)



23. The roots of the equation $x^3 - 12 x^2 + 39x - 28 = 0$ are in AP, then the roots are _____

a) 3, 4, 5 b) 2, 4, 6

c) 1, 4, 7 d) -1, -4, -7

Ans c.

KEA MATHEMATICS Solution: $x^3 - 12 x^2 + 39x - 28 = 0$ The sum of the roots = -b/a = 12. \rightarrow (m) Now product of the roots=-d/a = $28 \rightarrow (n)$ (a) 3, 4, 5 (b) 2, 4, 6 (c) 1, 4, 7 (d) -1, -4, -7 (m) and (n) matches with option (c) only. ∴ Answer is option (c) 47

MATHEMATICS KEA 24. If two roots of $x^3 + p x^2 + qx + r = 0$ are connected by the relation $\propto \beta$ + 1 =0, then the condition is ____ a) r^2 - pr + q + 1 = 0 b) r^2 + pr + q + 1 = 0 c) p^2 + pr + q + 1 = 0 d) q^2 + pr + q + 1 = 0 Ans c.

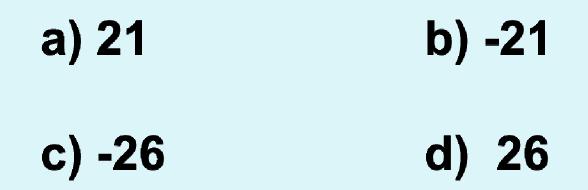
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Solution: Consider $x^3 + p x^2 + qx + r = 0$ Let the roots be \propto,β and γ . Now by data $\propto \beta = -1$ Then the sum = $\propto + \beta + \gamma$ = -b/a = -p; product = $\propto \beta \gamma$ = -d/a = -r Now $\propto \beta \gamma = -r \implies (-1) \gamma = -r \implies \gamma = r$ Now $\gamma = r$ satisfies $x^3 + p x^2 + qx + r = 0$ $\Rightarrow r^3 + p r^2 + qr + r = 0$ \Rightarrow r[r^2 + p r + q + 1]=0 \Rightarrow r^2 + p r + q + 1 =0 .: Answer is option (c)



25. If the roots of the equation $3x^3 - kx^2 + 52x - 24 = 0$ are in GP, then k=





Solution: Since the roots are in GP ,

$$x = \sqrt[3]{\frac{-d}{a}} = \sqrt[3]{\frac{24}{3}} = \sqrt[3]{8} = 2$$
 is a root.

Put x = 2 in $3x^3$ - k x^2 + 52x - 24 = 0

we have 24 - 4k + 104 - 24 = 0

$$\Rightarrow$$
 4k = 104 \Rightarrow k = 26

∴ Answer is option (d)



26. Two roots of the equation $x^3 - 7 x^2 + kx + m = 0$ are related by $\beta = 2 \propto$ and the third root being -2, then k and m are respectively,

a) 1 and 36 b) -1 and-36

c) 0 and 36 d) 36 and 0

Ans c.

K MATHEMATICS Solution: $x^3 - 7x^2 + kx + m = 0 \rightarrow (*)$ Let the roots be \propto , β and γ . Then by data $\gamma = -2$ and $\beta = 2 \propto$ Sum of the roots = $\propto + \beta + \gamma = - b/a = 7$ $\Rightarrow \propto + 2 \propto + (-2) = 7 \Rightarrow 3 \propto = 9 \Rightarrow \propto = 3$ Now $\propto = 3$ satisfies (*) $\therefore 27 - 63 + 3k + m = 0$ \Rightarrow 3k + m = 36 which is satisfied by option (c) only, i.e. (c) 0 and 36 where k = 0 and m = 36, by inspection.

Answer is option (c)

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27. If the equation $x^3 + ax + 1 = 0$ and $x^4 + a x^2 + 1 = 0$ have a root in common, then a =

a) 2 b) -2 c) 1 d) -1

Ans b.

KEA MATHEMATICS Solution: Given $x^3 + ax + 1 = 0 \Rightarrow x^3 + ax = -1$ multiply by x, $x^4 + a x^2 = -x$ Now given eq2 ($x^4 + a x^2$) + 1 =0 \Rightarrow -x + 1 = 0 \Rightarrow x = 1 Put x = 1 in x^3 + ax + 1 = 0 \Rightarrow |a = -2 | Given options (a) 2 (b) -2 (c) 1 (d) -1 ∴ Answer is option (b) 55



28. If α , β , γ and δ are the roots of the equation $x^4 - 3x^2 + 7 = 0$, then $\sum \frac{1}{\alpha\beta} =$ ______ $a) \frac{3}{7}$ $b) \frac{7}{3}$

c) $-\frac{7}{3}$ d) $-\frac{3}{7}$

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Solution:

Consider $x^4 - 3x^2 + 7 = 0$ here a = 1, b = 0, c = -3, d = 0, e = 7 $\sum \alpha = -b/a = 0$, $\sum \alpha \beta = c/a = -3$ product = $\alpha\beta\gamma\delta$ = e/a = 7 Now $\sum \frac{1}{\alpha\beta} = \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\delta} + \frac{1}{\delta\alpha}$ $=\frac{\alpha\beta+\beta\gamma+\gamma\delta+\delta\alpha}{\alpha\beta\gamma\delta} =\frac{\sum\alpha\beta}{\alpha\beta\gamma\delta} =\frac{-3}{7}$ ∴ Answer is option (d)



29. The number of solutions of the equation $x^2 + 3 |x| + 2 = 0$ is _



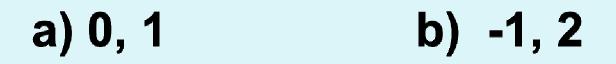
K MATHEMATICS Solution: Let y = |x| then $y^2 = x^2$ then y^2 + 3y + 2 = 0 \Rightarrow (y + 2) (y + 1) =0 \Rightarrow y = -2, -1 \Rightarrow | x | = -2, -1, not possible as $|x| \ge 0$ Hence no solution. .: Number of solutions =0 Hence Answer is option (d)



30. If 1 - p is a root of the equation

$$x^{2} + px + (1 - p) = 0$$
,

then the roots are

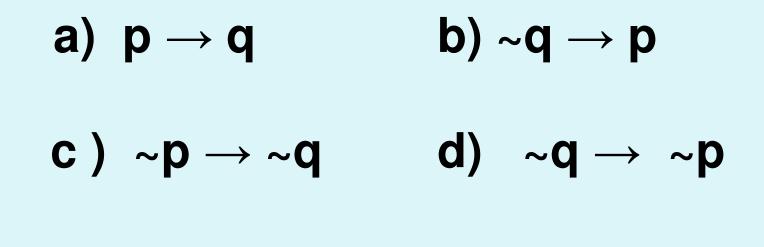


c) 0, -1 d) -1, 1 Ans c.

ΚΕΔ MATHEMATICS Solution: Since 1 – p is a root of $x^{2} + px + (1 - p) = 0$, we have $(1-p)^2 + p(1-p) + (1-p) = 0$. ⇒(1–p)[1–p+p+1]=0 \Rightarrow 2(1-p) =0 \Rightarrow p=1 \therefore when p= 1, the equation becomes $x^2 + x = 0 \implies x(x + 1) = 0 \implies x = 0,-1$ ∴ Answer is option (c)



31. The contrapositive of the inverse of $p \rightarrow \sim q$ is _____



Ans b.

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Solution:

We know that the inverse of $p \rightarrow q$ is $\sim p \rightarrow \sim q$ and contrapositive is $\sim q \rightarrow \sim p$ \therefore The inverse of $p \rightarrow \sim q$ is $\sim p \rightarrow q$. Its contrapositive is $\sim q \rightarrow p$ \therefore Answer is option (b)

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32. The proposition ($p \land \neg q$) \rightarrow ($r \lor \neg s$) is known to be false. Then the truth values of p, q, r & s are respectively,

> a) T, F, T, T b) T, T, T, F c) T, F, F, T d) T, T, F, F

> > Ans c.

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Solution: We know that
$$p \rightarrow q$$
 is false
when $p: T$ and $q: F$
Given $(p \land \neg q) \rightarrow (r \lor \neg s)$ is false
 $\therefore p \land \neg q: T$ and $r \lor \neg s: F$
 $\Rightarrow p: T$ and $\neg q: T; r: F$ and $\neg s: F$
 $\Rightarrow p:T, q:F; r:F; s: T$
Given options a) T, F, T, T b) T, T, T, F
 $c)$ T, F, F, T d) T, T, F, F

∴ Answer is option (c)



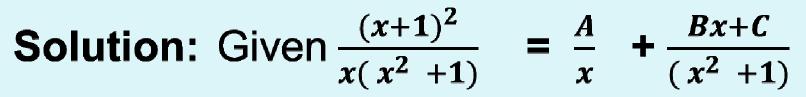
33. The negation of statement "If x = 4 and y = 6 then x + y = 10" is a) "if $x \neq 4$ and $y \neq 6$ then $x + y \neq 10$ " b) "if $x \neq 4$ or $y \neq 6$ then $x + y \neq 10$ " c) "if x = 4 and y = 6 then $x + y \neq 10$ " d) "x = 4 and y = 6 and $x + y \neq 10$ " Ans d. 66



Solution:

Let p: (x = 4 and y = 6) & q: (x + y = 10)Then given statement is $p \rightarrow q$. Now $\sim (p \rightarrow q) \equiv p^{\wedge} \sim q$ The negation of the given statement is "x = 4 and y = 6 and $x + y \neq 10$ " \therefore Answer is option (d)





$$(x + 1)^2 = A(x^2 + 1) + (Bx + c)x$$

 $x^2 + 2x + 1 = A(x^2 + 1) + (Bx^2 + cx)$
Put x = 0 $\therefore A = 1$

Compare coefficient of x, we have C = 2 $\therefore \sin^{-1} \left[\frac{A}{C}\right] = \sin^{-1} \left[\frac{1}{2}\right] = 30^{0} = \frac{\pi}{6}$ $\therefore \text{ Answer is option (a)}$

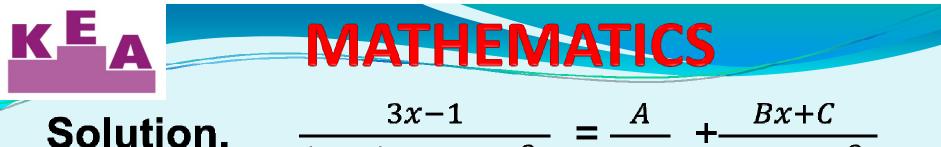


35.
$$\frac{3x-1}{(x+2)(1-x+x^2)}$$
 is resolved into

partial fractions, then it is equal to ____?

a)
$$\frac{1}{x+2}$$
 + $\frac{x}{(1-x+x^2)}$ b) $\frac{1}{x+2}$ + $\frac{x-1}{(1-x+x^2)}$
c) $\frac{-1}{x+2}$ + $\frac{x}{(1-x+x^2)}$ d) $\frac{-1}{x+2}$ + $\frac{x-1}{(1-x+x^2)}$

Ans c.



$$\frac{1}{(x+2)(1-x+x^2)} = \frac{1}{x+2} + \frac{1}{(1-x+x^2)}$$

Put x= -2 except at x+2 on LHS: A = $\frac{-7}{7}$ = -1

Hence answer is either option (c) or (d)

c)
$$\frac{-1}{x+2}$$
 + $\frac{x}{(1-x+x^2)}$ d) $\frac{-1}{x+2}$ + $\frac{x-1}{(1-x+x^2)}$

But after comparing the constant,

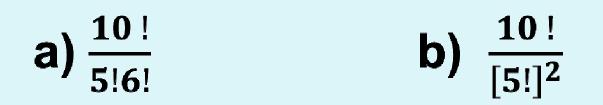
$$2C + A = -1 \implies C=0$$
 (:: $A = -1$)

C=0 holds good in option (c) only.

∴ Answer is option (c)



36. The greatest coefficient in the expansion of $[1 + x]^{10}$ is



10 !	-l\	10!
c) $\frac{10}{5!7!}$	d)	5 !4!

Ans b.

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Solution:

The greatest coefficient is the coefficient of the middle term.

- In $(x + a)^n = [1 + x]^{10}$, there are 11 terms.
 - $:T_6$ is the middle term.
 - $n=10, \ r=5, \ x \to 1, \ a \to x$
 - $T_6 = {}^n C_r x^{n-r} a^r = {}^{10} C_5 .1 .x^5$.
 - : The coefficient is ${}^{10}C_5 = \frac{10!}{[5!][5!]} = \frac{10!}{[5!]^2}$

Hence Answer is option (b)



37. In the expansion of $\left[x^2 - \frac{1}{x}\right]^{18}$, the constant term is _____

a)
$${}^{18}C_4$$
 b) ${}^{18}C_6$

c) ${}^{18}C_5$ d) ${}^{18}C_7$

Ans b.



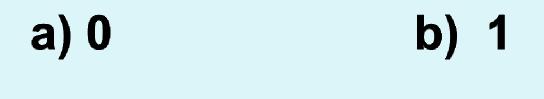
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$$T_{r+1} = {}^{n}C_{r}x^{n-r}a^{r}$$

= ${}^{18}C_{r}[x^{2}]^{18-r}\left[\frac{-1}{x}\right]^{r}$
= ${}^{18}C_{r}[x^{36-3r}](-1^{r})$
For constant term $36 - 3r = 0 \implies r = 12$
 $T_{13} = {}^{18}C_{12}[x^{0}](-1^{12}) = {}^{18}C_{12} = {}^{18}C_{6}$
which is option (b).
Hence Answer is option (b)



38. The sum of the coefficients in the expansion of $[1 + 2x - 4x^2]^{173}$ is _____



c) -1 d) 2



Consider $[1 + 2x - 4x^2]^{173}$ To find the sum of the coefficients, Put x= 1 Sum of the coefficients is -1

∴Answer is option (c)

KEA MATHEMATICS 39. In the expansion of $[1 + x]^n [1 + \frac{1}{r}]^n$, the term independent of x is _ a) $C_0^2 + 2 C_1^2 + 3 C_2^2 + \dots + (n+1) C_n^2$ **b)** $(C_0 + C_1 + C_2 + \dots + C_n)^2$ c) $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2$ d) $C_0C_1 + C_1C_2 + C_2C_3 + \dots + C_{n-1}C_n$ Ans c. 78



$$[1+x]^{n} [1+\frac{1}{x}]^{n} = [C_{0} + C_{1}x + C_{2}x^{2} + \dots + C_{n}x^{n}].$$
$$[C_{0} + C_{1}\frac{1}{x} + C_{2}\frac{1}{x^{2}} + \dots + C_{n}\frac{1}{x^{n}}]$$

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. The term independent of x in RHS is

$$= C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2$$

which is option (c)

∴ Answer is option (c)



40. In the expansion of $[1 + x]^{50}$, the sum of the coefficients of odd powers of x is _____

a) 0 b) 2⁴⁹

c) 2^{50} d) 2^{51}



The sum of the coefficients of odd powers of x in $[1+x]^n$ is 2^{n-1} Hence required sum in $[1 + x]^{50}$ is $2^{50-1} = 2^{49}$ **c)** 2⁵⁰ **b**) 2⁴⁹ d) 2^{51} options a) 0 \therefore answer is option (b)



41. If $[1 + x]^n = a_0 + a_1 x + a_2 x^2 + ...$+ $a_n x^n$ then the values of $a_1 + 2 a_2 + 3a_3 + 4 a_4$=

a) 0 b) 2^n

c) n.2ⁿ⁻¹ d) 2ⁿ⁻¹

Ans c.

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 $[1 + x]^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$ **Differentiating w.r.t. x**, n $[1 + x]^{n-1} = a_1 + a_2 2x + \dots + a_n n x^{n-1}$ Put x= 1, we have $a_1+2 a_2 + 3a_3 + 4 a_4 + \dots + na_n = n \cdot 2^{n-1}$ Hence answer is option (c).

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42. If $[1 + x - 2x^2]^6$ = 1 + a₁ x + a₂x² +..... + a₁₂x¹², then the value of a₂ + a₄++ a₁₂ is

a) 31 b) 32

c) 64 d) 1024

Ans a.



 $[1 + x - 2x^2]^6 = 1 + a_1 x + a_2 x^2 + \dots + a_{12} x^{12}$ put x = 1, $1 + a_1 + a_2 + a_3 + \dots + a_{12} = 0. \rightarrow (1)$ $put \ x = -1$, $1 - a_1 + a_2 - a_3 + \dots + a_{12} = 2^6 = 64. \rightarrow (2)$ $(1) + (2) \implies 2 [1 + a_2 + a_4 + \dots + a_{12}] = 64$ \Rightarrow 1+ a_2 + a_4 ++ a_{12} = 64/2 = 32 $\Rightarrow a_2 + a_4 + \dots + a_{12} = 32 - 1 = 31$ Hence Answer is option (a)



43. The resolution of
$$\frac{3x-7}{x^3-x}$$
 into partial fractions yields

a)
$$\frac{2}{x} - \frac{7}{(x-1)} - \frac{5}{(x+1)}$$

b) $\frac{7}{x} - \frac{2}{(x-1)} - \frac{5}{(x+1)}$
c) $\frac{7}{x} + \frac{2}{(x-1)} - \frac{5}{(x+1)}$
d) $\frac{7}{x} - \frac{5}{(x-1)} - \frac{2}{(x+1)}$
Ans b.



 $\frac{3x-7}{x^3-x} = \frac{3x-7}{x(x^2-1)} = \frac{3x-7}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

To find A, B, C put x= 0, 1, -1 on LHS except at x, x - 1 and x+1 resp.

Then A = 7, B = -4/2 = -2, C = -10/2 = -5which matches with option b

∴ Answer is option (b)



44. The domain of the function

$$\sqrt{x-2} + \sqrt{1-x}$$
 is _____

a) $x \ge 2$ b) set of real numbers

c) $x \le 2$ d) { $x \setminus x \in N : x^2 < 1$ }

Ans: d.



 $\sqrt{x-2}$ is defined when $x \ge 2$.

but not defined when $x \le 1$

$$\sqrt{1-x}$$
 is defined when $x \le 1$.

but not defined when $x \ge 2$

Hence options a, b, c are rejected as the domain is an empty set, which matches with option (d). Hence Answer is option (d)

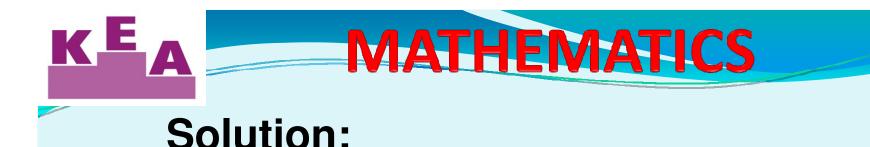
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45. The correct statement of the following is

a) The relation " is less than " on Z is antisymmetric

- b) The relation " is sister of " on the members of the family is transitive
- c) The relation " is relatively prime " on N is reflexive.
- d) The relation " is perpendicular " on the set of lines in a plane is transitive.

Ans: b.



 a) on Z, a< b and b< a ⇒ a = b hence R is not antisymmetric

b) If A is a sister of B and B is a sister of C, then clearly A is a sister of C. Hence relation is transitive. Hence (b) is true.

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c) since GCD of 2, $2 = (2, 2) = 2 \neq 1$

∴The relation " is relatively prime " is not reflexive [for a, $b \in Z$ if (a, b) = 1 then a and b are relatively prime.]

d) On L, the set of lines if L1 ⊥ L2 and L2 ⊥ L3 then L1 ⊥ L3 is wrong.
Hence only option (b) is true.
∴ Answer is option (b)

EXAMPLATE ITEMPARTIES
46. If
$$t_n = \frac{1}{4} (n+1) (n+2)$$
 for
 $n = 1,2,3,.....then$
 $\frac{1}{t_1} + \frac{1}{t_2} + + \frac{1}{t_{100}} =$
a) $\frac{51}{100}$ **b)** $\frac{51}{50}$
c) $\frac{100}{51}$ **d)** $\frac{50}{51}$
Ans c.

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Solution:

$$\frac{1}{t_n} = \frac{4}{(n+1)(n+2)} = 4 \left[\frac{1}{(n+1)(n+2)} \right]$$

$$\therefore \sum_{n=1}^{100} \frac{1}{t_n} = 4 \sum_{n=1}^{100} \frac{1}{(n+1)(n+2)}$$

$$= 4 \left[\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{101.102} \right]$$

$$= 4 \left[\frac{n}{a(a+nd)} \right] = 4 \left[\frac{100}{2(2+100)} \right] = \frac{4.100}{4.51}$$

$$= \frac{100}{51} \text{ which is option (c)}$$

Hence Answer is option (c)



47. If 1, a_1 , a_2 , a_3 , a_{n-1} are the nth roots of unity, then $(1 - a_1) (1 - a_2) (1 - a_3) \dots (1 - a_{n-1}) =$ a) 0 b) 1 d) n^2 **c)** n

Ans c.



Let n= 3. then we know that cube roots of unity are 1, ω and ω^2

Then

$$\begin{array}{l} (1 - a_1) \left(1 - a_2 \right) \left(1 - a_3 \right) \dots \left(1 - a_{n-1} \right) \\ = (1 - a_1) \left(1 - a_2 \right) \\ = (1 - \omega) \left(1 - \omega^2 \right) \\ = 1 - (\omega + \omega^2) + \omega^3 \\ = 1 - (-1) + 1 = 3 = n \\ (\because \omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0) \\ \therefore \text{ Answer is option (c) .} \end{array}$$



48. If two roots of the equation $x^4 + x^3 - 25 x^2 + 41x + 66 = 0$ are $3 \pm i \sqrt{2}$, then the other two roots satisfies the equation

a)
$$x^2$$
+ 7x + 6 = 0 b) x^2 - 7x + 6 = 0

c)
$$x^2$$
 + 7x - 6 = 0 d) x^2 - 7x - 6 = 0

Ans a.

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Solution: Let the roots be \propto , β , γ and δ Let \propto , $\beta = 3 \pm i\sqrt{2}$ then $\propto + \beta = 6$ and $\propto \beta = (3 + i\sqrt{2})(3 - i\sqrt{2}) = 11$ sum of the roots=($\propto + \beta$)+ $\gamma + \delta = -b/a = -1$ \Rightarrow 6 + (γ + δ) = -1 \Rightarrow (γ + δ) = -7 product of the roots = $\propto \beta \gamma \delta$ = e/a = 66 $\Rightarrow \gamma \delta = 66 / \propto \beta = 66 / 11 = 6$ Now $(\gamma + \delta) = -7$ and $\gamma \delta = 6$ satisfies option a only. \therefore Answer is option (a)



49. The coefficient of x in the expansion of $\left[x^2 + \frac{c}{x}\right]^5$ is _____

a) 20c b) 10c

c) 10 c^3 d) 20 c^2

Ans c.

KEA MATHEMATICS Solution: Consider $\left[x^2 + \frac{c}{r}\right]^5$ compare with $[x + a]^n$ Here $\mathbf{x} \to x^2$, $\mathbf{n} \to \mathbf{5}$, $\mathbf{a} \to \frac{c}{r}$, $\therefore \mathbf{r} = \mathbf{3}$ $T_{r+1} = {}^{n}C_{r}x^{n-r}a^{r} = {}^{5}C_{r}(x^{2})^{5-r}(\frac{c}{r})^{r}$ $={}^{5}C_{r}x^{10-3r}$. $c^{r}={}^{5}C_{r}$. $c^{r}x^{10-3r}$ -(*) For coefficient of x, $10-3r = 1 \Rightarrow r=3$ (*) ⇒ $T_4 = T_{3+1} = {}^5C_3$. c^3 .x = 10 c^3 x \therefore The coefficient of x is $10c^3$ ∴ Answer is option (c)



50. The number of solutions of $log_4^{(x-1)} - log_2^{(x-3)} = 0$ is _

a) 3
b) 1
c) 2
d) 0



$$log_{4}^{(x-1)} = log_{2}^{(x-3)}$$

$$\Rightarrow \frac{1}{2} log_{2}^{(x-1)} = log_{2}^{(x-3)}$$

$$\Rightarrow log_{2}^{(x-1)} = 2log_{2}^{(x-3)}$$

$$\Rightarrow log_{2}^{(x-1)} = log_{2}^{(x-3)^{2}}$$

$$\Rightarrow x - 1 = (x - 3)^{2}$$





$$\Rightarrow x - 1 = x^2 - 6x + 9$$

$$\Rightarrow x^2 - 7x + 10 = 0$$

$$\Rightarrow (x - 5) (x - 2) = 0$$

$$\Rightarrow x = 5, 2$$

but x = 2 is not a solution

since $log_2^{(x-3)}$ is not defined when x = 2

 \therefore x= 5 is the only solution.

: Answer is option (b)



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