MATHEMATICS

I PUC ALGEBRA

1. SET THEORY
2. LOGARITHMS
3. SUMMATION OF SERIES
4. THEORY OF EQUATIONS
5. MATHEMATICAL LOGIC
6. PARTIAL FRACTIONS
7. BINOMIAL THEOREM

1. Which of the following is not a singleton?

a) $\{ x : |x| < 1 , x \in Z \}$

b) $\{ x : |x| = 5 , x \in N \}$

c) $\{ x : x^3 + 27 = 0 , x \in R \}$

d) $\{ x : x^2 + 3x + 4 = 0, x \in R \}$

Ans: (d).

Solution :

[A singleton set is a set having only one element.]

a) : {0}

b) : { 5 }

c) : { -3}

d) : ϕ

$\therefore$ Answer is option (d)

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2. If $g(x) = 1 + \sqrt{x}$ and

$f(g(x)) = 3 + 2\sqrt{x} + x$,

then $f(4) =$

a) 3 b) 18

c) 11 d) 4

Ans b.

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Solution:

$$f(g(x)) = 3 + 2\sqrt{x} + x$$

$$= [1+2\sqrt{x} + \sqrt{x}^{2}] + 2$$

$$= (1 + \sqrt{x})^{2} + 2$$

$$= [g(x)]^{2} + 2$$

$\therefore f(g(x))=[g(x)]^{2} + 2$ put $g(x)=4$

$\therefore f(4) = (4)^{2} + 2 = 18$

$\therefore$ **Answer is option (b)**

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3. Let A={ -1, 0, 1} and B= { 0, 2} and a function f: A → B defined by $y = 2x^4$, then f is

a) one one onto

b) one one into

c) many one onto

d) many one into.

Ans c.

Solution:

f: A $\rightarrow$ B defined by y = 2 x^4 .

when x= -1 , y = 2 ;

x = 0, y =0 and x = 1 , y= 2

$\therefore$ -1 and 1 have 2 as their image .

$\therefore$ f is not one-one $\therefore$ f is many one.

$\therefore$ Answer is either option c or option d

Since both 0 and 2 are images, f is onto.

$\therefore$ f is many one and onto

$\therefore$ Answer is option (c)

4. Let R be a relation on the set of real numbers defined by a R b if $|a - b| \leq 1$, then R is

a) Reflexive and symmetric

b) Reflexive and transitive

c) symmetric and transitive

d) only reflexive.

Ans a.

Solution:

$|a - a| \leq 1 \Rightarrow 0 \leq 1$ is true. $\therefore a\,R\,a$

$\therefore$ R is reflexive.

We have $|x| = |-x| \Rightarrow |a - b| \leq 1$

$\Rightarrow |b - a| \leq 1$ $\therefore$ R is symmetric

$|3 - 2| \leq 1$ and $|2 - 1| \leq 1$ but $|3 - 1| \not\leq 1$

$\therefore$ R is not transitive .

$\therefore$ R is only reflexive and symmetric .

$\therefore$ Answer is option (a)

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5. If a function f: N → N such that
f(1) = 1, f(n+ 1) = 2 f(n) +1 ,
then f(n) =

a) 2^{n-1}

b) 2^{n} - 1

c) 2^{n-1} - 1

d) 2^{n+1} + 1

Ans b.

Solution:

Given f(1) =1, Now f(n+ 1) = 2 f(n) +1 .

∴ put n=1 ,

f(2) = 2 f(1) + 1 = 2 + 1 = 3 (∵ f(1) =1)

∴ f(2) = 3. When n= 2,

(a) → 2^{n-1} = 2 ; (b)→ 2^n - 1 = 4 - 1 = 3;

(c)→ 2^{n-1} - 1=1 ; (d)→ 2^{n+1} + 1 =9

∴ f(2) = 3 matches with option (b)

∴ Answer is option (b)

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6. The range of the function

$f(x) = {}^{7-x}P_{x-3}$ is

a) { 1, 2, 3, 4}

b) { 1, 2, 3, 4, 5, 6}

c) { 1, 2, 3 }

d) { 3, 4,5 }

Ans c.

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Solution: f(x) = ${}^{7-x}P_{x-3}$.

clearly 7- x ≥ x – 3 $\Longrightarrow$ 10 $\geq$ 2x $\Longrightarrow$ 5 ≥ x

$\Longrightarrow$ x ≤ 5

But x ≥ 3 ∴ x = 3,4,5.

∴ Domain is

$\{{}^{7-x}P_{x-3}$ \ x =3,4,5} = { ${}^{4}P_{0}$, ${}^{3}P_{1}$, ${}^{2}P_{2}$ } = { 1,3,2}

∴ Answer is option (c)

MATHEMATICS

7. If $f(x) = \frac{2x+1}{1-3x}$ then $f^{-1}(x) =$ ______

a) $\frac{x-1}{3x+2}$

b) $\frac{3x+2}{x-1}$

c) $\frac{x+1}{3x-2}$

d) $\frac{2x+1}{1-3x}$

Ans a.

MATHEMATICS

Solution:

$$f(x) = \frac{2x+1}{1-3x}$$

put x = 0 , f(0) = 1 $\therefore$ $f^{-1}(1) = \{0\}$

Now put x = 1 in options.

a) $\frac{x-1}{3x+2}$ b) $\frac{3x+2}{x-1}$ c) $\frac{x+1}{3x-2}$ d) $\frac{2x+1}{1-3x}$

(a) $\rightarrow$ 0 (b) $\rightarrow$ ∞ (c) $\rightarrow$ 2 (d) $\rightarrow$ -3/2

$\therefore f^{-1}(1) = \{0\}$ matches with option (a) only

$\therefore$ Answer is option (a)

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8. If A = { 1, 2, 3, 4} Then which of the following is a function from A to itself

a) f_1 = { (x, y) \ y = x + 1}

b) f_2 = { (x, y) \ (x + y) > 4 }

c) f_3 = { (x, y) \ (y < x) }

d) f_4 = { (x, y) \ (x + y = 5) }

Ans d.

MATHEMATICS

Solution :

a) when x= 4 , y = x+1=5 $\notin$ A $\therefore$ 4 has no image

b) f_2 = { (x, y) \ (x + y) > 4 }

2 + 3 > 4 and 2 + 4 > 4 $\therefore$ 2 has two images #

c) f_3 = { (x, y) \ (y < x) }

when x = 1, y < 1 $\therefore$ y $\notin$ A

$\therefore$ options a, b , c are rejected .

Hence only possibility is option (d).

$\therefore$ Answer is option (d)

MATHEMATICS

9. If log_e^{2} , $log_e^{(2x-1)}$ and $log_e^{(2x+3)}$ are in A P then the value of x is___

a) $-\frac{1}{2}$ b) $\frac{5}{2}$

c) 1 d) $\frac{1}{2}$

Ans b.

MATHEMATICS

Solution :

when (a)⟶x= - $\frac{1}{2}$,

$log_e^{(2x-1)}$ = $log_e^{(-2)}$, meaningless

When (c)⟶ x = 1 ; log_e^{2} , $log_e^{(1)} = 0$

and $log_e^{(5)}$, which are not in AP

when (d)⟶ x = $\frac{1}{2}$,

$log_e^{(2x-1)}$ = $log_e^{(0)}$, meaningless

Hence only possibility is option (b)

∴ Answer is option (b)

MATHEMATICS

10. If x = $log_4^{\ 2}$, y = $log_6^{\ 4}$ and

z = $log_8^{\ 6}$,

then yz(2 – x) =

a) 2 b) -2

c) 1 d) 3

Ans c.

MATHEMATICS

Solution:

yz(2 – x) = 2 yz – xyz

$= 2log_6{}^4 log_8{}^6 - log_8{}^2$

$= 2log_8{}^4 - log_8{}^2$

$= log_8{}^{16} - log_8{}^2$

$= log_8{}^8 = 1$

Hence answer is option (c)

MATHEMATICS

11. If $\frac{1}{log_3^{\pi}} + \frac{1}{log_4^{\pi}} > k$ then the greatest integral value of k =

a) 3 b) 2

c) 1 d) 4

Ans b.

Solution:

Given log_{π}^{3} + log_{π}^{4} > k

$\Longrightarrow log_{\pi}^{12}$ > k

$\Longrightarrow$ 12 > π^{k} . (k greatest integer)

Now 12 > π^{2} and 12 $\ngtr \pi^{3}$

$\therefore$ k= 2

$\therefore$ Answer is option (b)

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MATHEMATICS

12. If $log_a^{ab} = x$ then $log_b^{ab} =$

a) $\frac{1}{x}$

b) $\frac{x}{x+1}$

c) $\frac{x}{x-1}$

d) $\frac{x}{1-x}$

Ans c.

MATHEMATICS

Solution:

We have log_a^{ab} = x $\therefore log_{ab}^{a} = \frac{1}{x}$

Now $log_{ab}^{a} + log_{ab}^{b} = log_{ab}^{ab} = 1$

$\Longrightarrow log_{ab}^{b} = 1 - log_{ab}^{a} = 1 - \frac{1}{x} = \frac{x-1}{x}$

$\therefore log_b^{ab} = \frac{1}{log_{ab}^{b}} = \frac{x}{x-1}$

$\therefore$ Answer is option (c).

MATHEMATICS

13. If x = 27 , y = $log_3{}^4$

then x^y = ____________

a) 64 b) 16

c) $\frac{3}{7}$ b) $\frac{1}{16}$

Ans a

Solution :

$$x^y = 27^{\log_3 4}$$

$$= 3^{3 \log_3 4}$$

$$= 3^{\log_3 64}$$

$$= 64 \quad (\because a^{\log_a x} = x)$$

$\therefore$ Answer is option (a)

MATHEMATICS

14. If 2 log_{10}^{a} - 3 log_{10}^{b} = 2 then 100 b^3 = ____________

a) a^2 b) a

c) a^3 d) 3a

Ans a.

MATHEMATICS

Solution :

Given $2\,log_{10}^{a}$ - $3\,log_{10}^{b} = 2$

$\Rightarrow log_{10}^{a^2}$ - $log_{10}^{b^3} = 2$

$\Rightarrow log_{10}^{\frac{a^2}{b^3}} = 2 \Rightarrow \frac{a^2}{b^3} = 100$

$\Rightarrow 100\,b^3 = a^2$

∴ Answer is option (a)

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MATHEMATICS

15. If x , y and z are any three odd consecutive odd positive integers then $\log_e(xz+4) =$

a) $log_e{}^{2y}$ b) $log_e{}^{y}$

c) $2log_e{}^{y}$ d) $4\ log_e{}^{y}$

Ans c.

Solution :

Take 3 consecutive odd positive integers as $x = 1 \quad y = 3 \quad z = 5$

then $xz + 4 = 9$

$\therefore \log_e (xz + 4) = \log_e 9$

$= 2 \log_e 3$

$= 2log_e{}^{y}$ ($\because y = 3$)

$\therefore$ Answer is option (c)

MATHEMATICS

16. If $S_n = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots\ldots\ldots$ to n terms , then $S_n =$

a) $\frac{2n}{2n+1}$

b) $\frac{n}{2n+1}$

c) $\frac{n}{n+2}$

d) $\frac{2n}{n+5}$

Ans b.

MATHEMATICS

Solution: By inspection method

a) $\frac{2n}{2n+1}$ b) $\frac{n}{2n+1}$ c) $\frac{n}{n+2}$ d) $\frac{2n}{n+5}$

Put n= 2 , Then LHS= $S_2 = \frac{1}{3} + \frac{1}{15} = \frac{6}{15} = \frac{2}{5}$

$\Rightarrow \boxed{S_2 = \frac{2}{5}}$

In options, a $\longrightarrow \frac{4}{5}$ $\boxed{b \longrightarrow \frac{2}{5}}$ c $\longrightarrow \frac{1}{2}$ d $\longrightarrow \frac{4}{7}$

∴ The value of S_2 matches with option (b).

∴Answer is option (b) .

Or $S_n = \frac{n}{a(a+nd)} = \frac{n}{1(1+2n)} = \frac{n}{2n+1}$

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MATHEMATICS

17. The sum 1.3 + 3.5 + 5. 7 +.... up to n terms is

a) $\frac{n}{5}$ [3 n^2 + 7n + 5)

b) $\frac{n}{2}$ [2 n^2 + 3n + 1)

c) $\frac{n}{3}$ [4 n^2 + 6n - 1)

d) $\frac{n}{3}$ [5 n^2 + 3n + 1)

Ans c.

MATHEMATICS

Solution: By inspection method:

instead of checking for n=1 or n= 2 , check for n=3 (fast) LHS = 1.3 + 3.5 + 5.7 =53

When n= 3 the values of the options are

a) $\frac{n}{5}$ [3 n^2 + 7n + 5) b) $\frac{n}{2}$ [2 n^2 + 3n + 1)

c) $\frac{n}{3}$ [4 n^2 + 6n - 1) d) $\frac{n}{3}$ [5 n^2 + 3n + 1)

a) $\frac{3}{5}$. 53 b) $\frac{3}{2}$. 28 = 42 **c) 53** d) 55

∴ **Answer is option (c) .**

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MATHEMATICS

18. The value of

$$1+\frac{2}{5}+\frac{3}{25}+\text{------------} \text{ to } \infty \text{ is}$$

a) $\frac{1}{25}$

b) $\frac{16}{25}$

c) $\frac{25}{16}$

d) $\frac{5}{4}$

Ans c.

Solution:

Given is in AG series

where a = 1 , d = 1 , r = $\frac{1}{5}$

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2} = \frac{1}{1-\frac{1}{5}} + \frac{\frac{1}{5}}{(1-\frac{1}{5})^2}$$

$$= \frac{5}{4} + \frac{5}{16} = \frac{25}{16}$$

∴ Answer is option (c)

19. The 25^{th} term of the series

$3 + 15 + 35 + 63 + \ldots\ldots\ldots$ is ________

a) 2500 b) 2499

c) 2501 d) 1249

Ans b.

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Solution:

summation by the method of differences.

Here I differences : 12, 20, 28;

II differences : 8,8,8,....

$\Delta = 12 \quad \Delta^2 = 8 \quad T_1 = 3$

$$T_n = T_1 + (n-1)\,\Delta + \frac{1}{2}\,(n-1)(n-2)\,\Delta^2$$

$$T_{25} = 3 + 24.\,12 + \frac{1}{2}.\,24.23.8$$

$$= 3 + 288 + 2208 = 2499$$

∴ Answer is option (b)

20. If

$2^3 + 4^3 + 6^3 + \ldots\ldots + (2n)^3 = k.\, n^2\,(n+1)^2$

then k =

a) $\frac{1}{2}$

b) 1

c) $\frac{3}{2}$

d) 2

Ans d.

Solution:

Inspection Method :

$2^3 + 4^3 + 6^3 + \ldots\ldots + (2n)^3 = k.\, n^2\,(n+1)^2$

Put n = 1

LHS = 8 RHS = k.4

$\therefore$ 4k = 8

$\Longrightarrow$ k = 2

$\therefore$ Answer is option (d)

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21. If the sum of n terms of an AP is $nA + n^2 B$ where A and B are constants. Then its common difference is

a) A- B

b) A + B

c) 2A

d) 2B

Ans d.

MATHEMATICS

Solution:

Given S_n = nA + n^2 B. Put n=1 , 2

$\therefore$ T_1 = S_1 = A + B

And S_2 = 2A + 4B But S_2 = T_1 +T_2

$\therefore$ T_2 = S_2 - T_1 = (2A + 4B) – (A + B)

=A + 3B .

Now T_1 = A + B and T_2 = A + 3B

$\therefore$ Common difference = T_2 - T_1 = 2B

$\therefore$ Answer is option (d)

MATHEMATICS

22. If the roots of the quadratic equation x^2 + px + q = 0 are $\tan 30^0$ and $\tan 15^0$, Then q= ___________

a) 1 – p

b) p - 1

c) p +1

d) $\sqrt{3}$ p

Ans c.

MATHEMATICS

Solution: Consider x^2 + px + q = 0

Let $\propto$ = $\tan 30^0$ and β = $\tan 15^0$.

The $\propto + \beta$ = -b/a =-p and $\propto\beta$ = c/a =q.

Now $\tan 45^0 = \tan(30^0 + 15^0)$

$$\Rightarrow 1 = \frac{tan30^0 + tan15^0}{1 - tan30^0 \ .tan15^0} = \frac{\propto + \beta}{1 - \propto \beta} = \frac{-p}{1-q}$$

$$\Rightarrow 1 = \frac{-p}{1-q} \Rightarrow 1 - q = -p$$

$$\Rightarrow q - p = 1 \Rightarrow q = 1 + p$$

$\therefore$ Answer is option (c)

23. The roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$ are in AP, then the roots are __________

a) 3, 4, 5
b) 2, 4, 6
c) 1, 4, 7
d) -1, -4, -7

Ans c.

MATHEMATICS

Solution: x^3 - 12 x^2 + 39x – 28 = 0

The sum of the roots = -b/a = 12. ⟶(m)

Now product of the roots=-d/a = 28⟶(n)

(a) 3, 4, 5 (b) 2, 4, 6

(c) 1, 4, 7 (d) -1, -4, -7

(m) and (n) matches with option (c) only.

∴ Answer is option (c)

24. If two roots of x^3 + p x^2 + qx + r = 0 are connected by the relation $\alpha\beta + 1 = 0$, then the condition is ____

a) $r^2 - pr + q + 1 = 0$

b) $r^2 + pr + q + 1 = 0$

c) $p^2 + pr + q + 1 = 0$

d) $q^2 + pr + q + 1 = 0$

Ans c.

MATHEMATICS

Solution: Consider x^3 + p x^2 + qx + r = 0

Let the roots be ∝,β and γ .

Now by data ∝ β = - 1

Then the sum = ∝+ β + γ = -b/a = -p ;

product = ∝βγ = -d/a = -r

Now ∝βγ = -r ⟹ (-1) γ = -r ⟹ γ = r

Now γ = r satisfies x^3 + p x^2 + qx + r = 0

⟹r^3 + p r^2 + qr + r = 0

⟹ r[r^2+ p r + q + 1]=0 ⟹ r^2 + p r + q + 1 =0

∴ Answer is option (c)

MATHEMATICS

25. If the roots of the equation $3x^3$ - k x^2 + 52x – 24 =0 are in GP, then k=

a) 21 b) -21

c) -26 d) 26

Ans d.

Solution:

Since the roots are in GP ,

x = $\sqrt[3]{\frac{-d}{a}}$ = $\sqrt[3]{\frac{24}{3}}$ = $\sqrt[3]{8}$ = 2 is a root.

Put x = 2 in $3x^3$ - k x^2 + 52x – 24 = 0

we have 24 – 4k + 104 – 24 = 0

$\Longrightarrow$ 4k = 104 $\Longrightarrow$ k = 26

∴ Answer is option (d)

MATHEMATICS

26. Two roots of the equation x^3 - 7 x^2 + kx + m = 0 are related by β = 2 ∝ and the third root being -2 , then k and m are respectively,

a) 1 and 36

b) -1 and-36

c) 0 and 36

d) 36 and 0

Ans c.

MATHEMATICS

Solution: x^3 - 7x^2+ kx + m =0 →(*)

Let the roots be ∝ , β and γ.

Then by data γ = -2 and β = 2 ∝

Sum of the roots = ∝ + β + γ = - b/a = 7

⟹∝ + 2 ∝ + (-2) = 7 ⟹ 3 ∝ = 9 ⟹ ∝ = 3

Now ∝ = 3 satisfies (*) ∴ 27 - 63 +3k + m = 0

⟹ 3k + m = 36 which is satisfied by option (c) only, i.e. (c) 0 and 36 where k = 0 and m = 36, by inspection.

∴ Answer is option (c)

MATHEMATICS

27. If the equation x^3 +ax +1 =0 and x^4 + a x^2 + 1 =0 have a root in common , then a =

a) 2 b) -2

c) 1 d) -1

Ans b.

MATHEMATICS

Solution:

Given x^3 + ax +1 =0 $\Longrightarrow x^3$ + ax = - 1

multiply by x , x^4 + a x^2 = -x

Now given eq2 (x^4 + a x^2) + 1 =0

$\Longrightarrow$ -x + 1 = 0 $\Longrightarrow$ x = 1

Put x = 1 in x^3 + ax + 1 =0 $\Longrightarrow$ **a = -2**

Given options (a) 2 (b) -2 (c) 1 (d) -1

∴ Answer is option (b)

28. If α, β , γ and δ are the roots of the equation x^4 - 3 x^2+7 =0 ,
then $\sum \frac{1}{\alpha\beta}$ = __________

$a)$ $\frac{3}{7}$

b) $\frac{7}{3}$

c) - $\frac{7}{3}$

d)- $\frac{3}{7}$

Ans d.

MATHEMATICS

Solution:

Consider $x^4 - 3x^2 + 7 = 0$

here a= 1, b= 0 , c= -3, d= 0 , e = 7

$\sum \alpha$ = -b/a = 0 , $\sum \alpha\beta$ = c/a = -3

product = $\alpha\beta\gamma\delta$ = e/a = 7

Now $\sum \frac{1}{\alpha\beta} = \frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\delta} + \frac{1}{\delta\alpha}$

$= \frac{\alpha\beta+\beta\gamma+\gamma\delta+\delta\alpha}{\alpha\beta\gamma\delta} = \frac{\sum \alpha\beta}{\alpha\beta\gamma\delta} = \frac{-3}{7}$

$\therefore$ Answer is option (d)

MATHEMATICS

29. The number of solutions of the equation $x^2 + 3|x| + 2 = 0$ is ______

a) 2 b) 1

c) 4 d) 0

Ans d.

Solution:

Let y = | x| then y^2 = x^2 then

y^2 + 3y + 2 = 0

⟹ (y + 2) (y + 1) =0

⟹ y = -2, -1 ⟹ | x | = -2, -1,

not possible as | x | ≥ 0

Hence no solution .

∴ Number of solutions =0

Hence Answer is option (d)

30. If 1 – p is a root of the equation

$x^2 + px + (1 - p) = 0$,

then the roots are

a) 0, 1 b) -1, 2

c) 0, -1 d) -1, 1

Ans c.

MATHEMATICS

Solution:

Since 1 – p is a root of

$x^2 + px + (1 - p) = 0$,

we have $(1 - p)^2 + p(1 - p) + (1 - p) = 0$.

$\Rightarrow (1 - p)[1 - p + p + 1] = 0$

$\Rightarrow 2(1 - p) = 0 \Rightarrow p = 1$

$\therefore$ when p= 1 , the equation becomes

$x^2 + x = 0 \Rightarrow x(x + 1) = 0 \Rightarrow x = 0, -1$

$\therefore$ Answer is option (c)

MATHEMATICS

31. The contrapositive of the inverse of $p \rightarrow {\sim}q$ is __________

a) $p \rightarrow q$

b) ${\sim}q \rightarrow p$

c) ${\sim}p \rightarrow {\sim}q$

d) ${\sim}q \rightarrow {\sim}p$

Ans b.

Solution:

We know that the inverse of

$p \rightarrow q$ is $\sim p \rightarrow \sim q$

and contrapositive is $\sim q \rightarrow \sim p$

$\therefore$ The inverse of $p \rightarrow \sim q$ is $\sim p \rightarrow q$.

Its contrapositive is $\sim q \rightarrow p$

$\therefore$ Answer is option (b)

MATHEMATICS

32. The proposition $(p \wedge \sim q) \rightarrow (r \vee \sim s)$ is known to be false. Then the truth values of p, q, r & s are respectively,

a) T, F, T, T

b) T, T, T, F

c) T, F, F, T

d) T, T, F, F

Ans c.

MATHEMATICS

Solution: We know that $p \rightarrow q$ is false when p: T and q: F

Given $(p \wedge \sim q) \rightarrow (r \vee \sim s)$ is false

$\therefore p \wedge \sim q$: T and $r \vee \sim s$: F

$\Rightarrow p$: T and $\sim q$: T ; r: F and $\sim s$: F

$\Rightarrow$ $\boxed{p\text{:T}, \quad q\text{ : F} \quad ; \quad r\text{: F} ; \quad s\text{: T}}$

Given options a) T, F, T, T b) T, T, T, F

c) T, F, F, T d) T, T, F, F

$\therefore$ Answer is option (c)

33. **The negation of statement**
" If $x = 4$ and $y = 6$ then $x + y = 10$ " is

a) " if $x \neq 4$ and $y \neq 6$ then $x + y \neq 10$ "

b) " if $x \neq 4$ or $y \neq 6$ then $x + y \neq 10$ "

c) " if $x = 4$ and $y = 6$ then $x + y \neq 10$ "

d) " $x = 4$ and $y = 6$ and $x + y \neq 10$ "

Ans d.

Solution:

Let p: (x = 4 and y = 6) & q: (x + y = 10)

Then given statement is $p \rightarrow q$.

Now $\sim(p \rightarrow q) \equiv p \wedge \sim q$

$\therefore$ The negation of the given statement is " x = 4 and y = 6 and x + y ≠ 10 "

$\therefore$ Answer is option (d)

MATHEMATICS

34. If $\frac{(x+1)^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$,

then $sin^{-1}\left[\frac{A}{C}\right]$ =

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

Ans a.

MATHEMATICS

Solution: Given $\frac{(x+1)^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$

$(x+1)^2$ = A(x^2+1) + (B x + c) x

x^2 + 2 x + 1 = A(x^2+1) + (B x^2 + c x)

Put x =0 ∴ A = 1

Compare coefficient of x , we have C = 2

$\therefore sin^{-1}\left[\frac{A}{C}\right] = sin^{-1}\left[\frac{1}{2}\right] = 30^0 = \frac{\pi}{6}$

∴ Answer is option (a)

35. $\frac{3x-1}{(x+2)(1-x+x^2)}$ is resolved into partial fractions , then it is equal to _____?

a) $\frac{1}{x+2}+\frac{x}{(1-x+x^2)}$

b) $\frac{1}{x+2}+\frac{x-1}{(1-x+x^2)}$

c) $\frac{-1}{x+2}+\frac{x}{(1-x+x^2)}$

d) $\frac{-1}{x+2}+\frac{x-1}{(1-x+x^2)}$

Ans c.

Solution. $$\frac{3x-1}{(x+2)(1-x+x^2)} = \frac{A}{x+2} + \frac{Bx+C}{(1-x+x^2)}$$

Put x= -2 except at x+2 on LHS: A = $\frac{-7}{7}$ = -1

Hence answer is either option (c) or (d)

c) $\frac{-1}{x+2} + \frac{x}{(1-x+x^2)}$ **d)** $\frac{-1}{x+2} + \frac{x-1}{(1-x+x^2)}$

But after comparing the constant,

2C + A = -1 $\Longrightarrow$ C=0 ($\because$ A = -1)

C=0 holds good in option (c) only .

$\therefore$ Answer is option (c)

MATHEMATICS

36. The greatest coefficient in the expansion of $[1+x]^{10}$ is

a) $\frac{10\,!}{5!6!}$

b) $\frac{10\,!}{[5!]^2}$

c) $\frac{10\,!}{5!7!}$

d) $\frac{10\,!}{5\,!4!}$

Ans b.

Solution:

The greatest coefficient is the coefficient of the middle term.

In .(x + a)ⁿ = $[1+x]^{10}$, there are 11 terms.

$\therefore T_6$ is the middle term.

n= 10, r = 5, x → 1 , a → x

$T_6 = {}^nC_r x^{n-r} a^r = {}^{10}C_5 \,.1\,.x^5$.

∴ The coefficient is $^{10}C_5 = \frac{10\,!}{[5!]\,[5!]} = \frac{10\,!}{[5!]^2}$

Hence Answer is option (b)

MATHEMATICS

37. In the expansion of $\left[x^2 - \frac{1}{x}\right]^{18}$,

the constant term is ________

a) $^{18}C_4$

b) $^{18}C_6$

c) $^{18}C_5$

d) $^{18}C_7$

Ans b.

Solution:

$$T_{r+1} = {}^{n}C_r x^{n-r} a^r$$

$$= {}^{18}C_r [x^2]^{18-r} \left[\frac{-1}{x}\right]^r$$

$$= {}^{18}C_r [x^{36-3r}] (-1^r)$$

For constant term 36 – 3r =0 $\Longrightarrow$ r = 12

$$T_{13} = {}^{18}C_{12}[x^0] (-1^{12}) = {}^{18}C_{12} = {}^{18}C_6$$

which is option (b).

Hence Answer is option (b)

38. The sum of the coefficients in the expansion of $\left[1+2x-4x^2\right]^{173}$ is ____

a) 0

b) 1

c) -1

d) 2

Ans c.

Solution:

Consider $\left[1+2x-4x^2\right]^{173}$

To find the sum of the coefficients,

Put x= 1

Sum of the coefficients is -1

∴Answer is option (c)

MATHEMATICS

39. In the expansion of $[1+x]^n\,[1+\frac{1}{x}]^n$, the term independent of x is ________

a) $C_0{}^2+2.C_1{}^2+3.C_2{}^2+\ldots\ldots.+(\mathrm{n}+1).C_n{}^2$

b) $(C_0+C_1+C_2+\ldots\ldots\ldots+C_n)^2$

c) $C_0{}^2+C_1{}^2+C_2{}^2+\ldots\ldots..+C_n{}^2$

d) $C_0C_1+C_1C_2+C_2C_3+\ldots\ldots+C_{n-1}C_n$

Ans c.

Solution:

$$[1+x]^n \ [1+\frac{1}{x}]^n =$$

$$[C_0 + C_1 x + C_2\, x^2 + \ldots\ldots\ldots + C_n x^n\,] \,.$$

$$[C_0 + C_1 \frac{1}{x} + C_2\, \frac{1}{x^2} + \ldots\ldots\ldots + C_n \frac{1}{x^n}\,]$$

∴The term independent of x in RHS is

$$= C_0{}^2 + C_1{}^2 + C_2{}^2 + \ldots\ldots\ldots + C_n{}^2$$

which is option (c)

∴ Answer is option (c)

MATHEMATICS

40. In the expansion of $[1 + x]^{50}$, the sum of the coefficients of odd powers of x is ________

a) 0

b) 2^{49}

c) 2^{50}

d) 2^{51}

Ans b.

Solution:

The sum of the coefficients of odd powers of x in $[1+x]^n$ is 2^{n-1}

Hence required sum in $[1+x]^{50}$ is

$2^{50-1} = 2^{49}$

options a) 0 b) 2^{49} c) 2^{50} d) 2^{51}

∴ answer is option (b)

MATHEMATICS

41. If $[1+x]^n = a_0 + a_1 x + a_2 x^2 + \ldots \ldots \ldots \ldots + a_n x^n$ then the values of $a_1 + 2a_2 + 3a_3 + 4a_4 \ldots \ldots \ldots \ldots$ = _____

a) 0

b) 2^n

c) $n.2^{n-1}$

d) 2^{n-1}

Ans c.

Solution:

$[1+x]^n = a_0 + a_1x + a_2x^2 + \dots\dots\dots + a_nx^n$

Differentiating w.r.t. x,

$n[1+x]^{n-1} = a_1 + a_2 2x + \dots\dots\dots + a_n n x^{n-1}$

Put x= 1, we have

$a_1 + 2a_2 + 3a_3 + 4a_4 + \dots\dots + na_n = n \cdot 2^{n-1}$

Hence answer is option (c).

42. If $\left[1 + x - 2x^2\right]^6 = 1 + a_1 x + a_2 x^2 + \ldots\ldots + a_{12} x^{12}$, then the value of $a_2 + a_4 + \ldots\ldots + a_{12}$ is

a) 31

b) 32

c) 64

d) 1024

Ans a.

MATHEMATICS

Solution:

$[1 + x - 2x^2]^6 = 1 + a_1 x + a_2 x^2 + \dots + a_{12} x^{12}$

$put\ x = 1$,

$1 + a_1 + a_2 + a_3 + \dots + a_{12} = 0. \longrightarrow (1)$

$put\ x = -1$,

$1 - a_1 + a_2 - a_3 + \dots + a_{12} = 2^6 = 64. \longrightarrow (2)$

$(1) + (2) \Longrightarrow 2[1 + a_2 + a_4 + \dots + a_{12}] = 64$

$\Longrightarrow 1 + a_2 + a_4 + \dots + a_{12} = 64/2 = 32$

$\Longrightarrow a_2 + a_4 + \dots + a_{12} = 32 - 1 = 31$

Hence Answer is option (a)

43. The resolution of $\frac{3x-7}{x^3 - x}$ into partial fractions yields

a) $\frac{2}{x} - \frac{7}{(x-1)} - \frac{5}{(x+1)}$

b) $\frac{7}{x} - \frac{2}{(x-1)} - \frac{5}{(x+1)}$

c) $\frac{7}{x} + \frac{2}{(x-1)} - \frac{5}{(x+1)}$

d) $\frac{7}{x} - \frac{5}{(x-1)} - \frac{2}{(x+1)}$

Ans b.

Solution :

$$\frac{3x-7}{x^3 - x} = \frac{3x-7}{x\,(x^2 - 1)} = \frac{3x-7}{x\,(x - 1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

To find A , B, C put x= 0, 1, -1 on LHS except at x, x – 1and x+1 resp.

Then A = 7 , B = -4/ 2 = -2 , C= -10/2 =-5 which matches with option b

∴ Answer is option (b)

44. The domain of the function $\sqrt{x-2} + \sqrt{1-x}$ is ______________

a) $x \geq 2$ b) set of real numbers

c) $x \leq 2$ d) $\{ x \setminus x \in N : x^2 < 1 \}$

Ans: d.

Solution:

$\sqrt{x-2}$ is defined when x ≥ 2.

but not defined when x ≤ 1

$\sqrt{1-x}$ is defined when x ≤ 1.

but not defined when x ≥ 2

Hence options a, b, c are rejected as the domain is an empty set, which matches with option (d). Hence Answer is option (d)

45. The correct statement of the following is

a) The relation “ is less than “ on Z is antisymmetric

b) The relation “ is sister of “ on the members of the family is transitive

c) The relation “ is relatively prime “ on N is reflexive.

d) The relation “ is perpendicular “ on the set of lines in a plane is transitive.

Ans: b.

MATHEMATICS

Solution:

a) on Z , a< b and b< a $\nRightarrow$ a = b hence R is not antisymmetric

b) If A is a sister of B and B is a sister of C, then clearly A is a sister of C. Hence relation is transitive. Hence (b) is true.

c) since GCD of 2 ,2 = (2, 2) = 2 ≠ 1

∴The relation " is relatively prime " is not reflexive [for a, b ∈ Z if (a , b) = 1 then a and b are relatively prime.]

d) On L, the set of lines if $L1 \perp L2$ and $L2 \perp L3$ then $L1 \perp L3$ is wrong.

Hence only option (b) is true.

∴ Answer is option (b)

46. If $t_n = \frac{1}{4}$ (n+1) (n+2) for n= 1,2,3,.......then

$$\frac{1}{t_1} + \frac{1}{t_2} + ...+ \frac{1}{t_{100}} = \_\_\_\_\_\_$$

a) $\frac{51}{100}$

b) $\frac{51}{50}$

c) $\frac{100}{51}$

d) $\frac{50}{51}$

Ans c.

Solution:

$$\frac{1}{t_n} = \frac{4}{(n+1)(n+2)} = 4\left[\frac{1}{(n+1)(n+2)}\right]$$

$$\therefore \sum_{n=1}^{100} \frac{1}{t_n} = 4 \sum_{n=1}^{100} \frac{1}{(n+1)(n+2)}$$

$$= 4\left[\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{101.102}\right]$$

$$= 4\left[\frac{n}{a(a+nd)}\right] = 4\left[\frac{100}{2(2+100)}\right] = \frac{4.\ 100}{4.51}$$

$$= \frac{100}{51}$$ **which is option (c)**

Hence Answer is option (c)

47. If 1, a_1, a_2, a_3 .... , a_{n-1} are the nth roots of unity , then $(1 - a_1)(1 - a_2)(1 - a_3).....(1 - a_{n-1}) =$

a) 0

b) 1

c) n

d) n^2

Ans c.

Solution :

Let n= 3 . then we know that cube roots of unity are 1, ω and ω^2

Then

$$(1 - a_1)(1 - a_2)(1 - a_3)........(1 - a_{n-1})$$

$$=(1 - a_1)(1 - a_2)$$

$$= (1 - \omega)(1 - \omega^2)$$

$$= 1 - (\omega + \omega^2) + \omega^3$$

$$= 1 - (-1) + 1 = 3 = n$$

$$(\because \omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0)$$

$\therefore$ Answer is option (c) .

48. If two roots of the equation $x^4 + x^3 - 25x^2 + 41x + 66 = 0$ are $3 \pm i\sqrt{2}$, then the other two roots satisfies the equation

a) $x^2 + 7x + 6 = 0$

b) $x^2 - 7x + 6 = 0$

c) $x^2 + 7x - 6 = 0$

d) $x^2 - 7x - 6 = 0$

Ans a.

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Solution: Let the roots be α, β, γ and δ

Let $\alpha, \beta = 3 \pm i\sqrt{2}$ then $\alpha + \beta = 6$

and $\alpha\beta = (3 + i\sqrt{2})(3 - i\sqrt{2}) = 11$

sum of the roots $= (\alpha + \beta) + \gamma + \delta = -b/a = -1$

$\Longrightarrow 6 + (\gamma + \delta) = -1 \Longrightarrow (\gamma + \delta) = -7$

product of the roots $= \alpha\beta\gamma\delta = e/a = 66$

$\Longrightarrow \gamma\delta = 66/\alpha\beta = 66/11 = 6$

Now $(\gamma + \delta) = -7$ and $\gamma\delta = 6$

satisfies option a only.

$\therefore$ Answer is option (a)

KEA

MATHEMATICS

49. The coefficient of x in the expansion of $\left[x^2 + \frac{c}{x}\right]^5$ is ______

a) 20c

b) 10c

c) 10 c^3

d) 20 c^2

Ans c.

MATHEMATICS

Solution:

Consider $\left[x^2 + \frac{c}{x}\right]^5$ compare with $[x + a]^n$

Here x $\rightarrow x^2$, n $\rightarrow$ 5 , a $\rightarrow \frac{c}{x}$, $\therefore$ r = 3

T_{r+1} = ${}^nC_r x^{n-r} a^r = {}^5C_r (x^2)^{5-r} (\frac{c}{x})^r$

$= {}^5C_r x^{10-3r}.\ c^r = {}^5C_r.\ c^r\ x^{10-3r}$ -(*)

For coefficient of x, 10-3r =1$\Rightarrow$ r=3

(*) $\Rightarrow T_4 = T_{3+1} = {}^5C_3.\ c^3$.x = 10$c^3$ x

$\therefore$ The coefficient of x is 10c^3

$\therefore$ Answer is option (c)

50. The number of solutions of $log_4^{(x-1)}$ - $log_2^{(x-3)}$ = 0 is ________

a) 3 b) 1

c) 2 d) 0

Ans b.

Solution:

$$log_4^{(x-1)} = log_2^{(x-3)}$$

$$\Longrightarrow \frac{1}{2}\, log_2^{(x-1)} = log_2^{(x-3)}$$

$$\Longrightarrow log_2^{(x-1)} = 2log_2^{(x-3)}$$

$$\Longrightarrow log_2^{(x-1)} = log_2^{(x-3)^2}$$

$$\Longrightarrow x - 1 = (x-3)^2$$

MATHEMATICS

$\Longrightarrow$ x -1 = x^2 - 6x + 9

$\Longrightarrow$ x^2 - 7x + 10 =0

$\Longrightarrow$(x- 5) (x – 2) = 0

$\Longrightarrow$ x = 5, 2

but x = 2 is not a solution

since $\boldsymbol{log_2^{(x-3)}}$ is not defined when x = 2

$\therefore$ x= 5 is the only solution.

$\therefore$ **Answer is option** (b)

MATHEMATICS

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