MATHEMATICS

General solutions of Trigonometric Equations

1. General solution of $\sin x.\cos x = \frac{1}{4}$ is ....

1) $X = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4} \ \forall\, n \in I$

2) $X = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6} \ \forall\, n \in I$

3) $X = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12} \ \forall\, n \in I$

4) N.T.

Answer : 3

Soln., Given => $\sin x \cos x \ \frac{1}{2.2}$ => $2 \sin x \cos x = \frac{1}{2}$. $2 \sin A \cos A = \sin 2A$

$\sin 2x = \frac{1}{2}$ | $\sin \frac{1}{2} = \frac{\pi}{6}$

$\therefore \ \propto = \frac{\pi}{6}$ .i ,e G.S .is $2x = n\pi + (-1)^n \propto, \forall\, n \in I$

i ,e $2x = n\pi + (-1)^n \frac{\pi}{6}$ i ,e $X = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$.

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2. The General solution of $\cos\frac{3x}{5} = \frac{\sqrt{3}}{2}$ is x =......................

1) $\frac{5}{3}(2n\pi \pm \frac{\pi}{6})\ \forall\ n \in I$ 2) $\frac{3}{5}(n\pi \pm \frac{\pi}{6})\ \forall\ n \in I$

3) $\frac{5}{3}(n\pi \pm \frac{\pi}{6})\ \forall\ n \in I$ 4) N T

Answer : 1

Soln, Given => $\frac{3x}{5} = 2n\pi \pm \frac{\pi}{6}$ => $x = \frac{5}{3}(2n\pi \pm \frac{\pi}{6})\ \forall\ n \in I$

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3. The General solution of tan3x =1 is x =.................... ∀ n ∈I

1) $\frac{n\pi}{3} + \frac{\pi}{4}$ 2) = $n\pi + \frac{\pi}{4}$ 3) $\frac{n\pi}{3} + \frac{\pi}{12}$ 4) $\frac{n\pi}{3} - \pi$

Answer : 3

Solu . given => $\tan 3x = \tan\frac{\pi}{4}$ => $3x = n\pi + \propto$, $\forall\, n \in I$

Where $\propto = \frac{\pi}{4}$, => 3x= = $n\pi + \frac{\pi}{4}$

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4. If $\propto$ is a constant angle, then the G S Of $\sin^2\theta = \sin^2 \propto$ is ....

1) $\theta = n\pi + \propto \ \forall n \in I$ 2) $\theta = n\pi \pm \propto \forall n \in I$ 3) $\theta = n\pi - \propto \forall n \in I$ 4) $N.T$

Answer : 2

Soln. Given=> $2 \sin^2\theta = 2 \sin^2 \propto$

- ⇨ $1 - \cos 2\theta = 1 - \cos 2\propto$
- ⇨ $-\cos 2\theta = -\cos 2\propto$
- ⇨ $\cos 2\theta = -\cos 2\propto$
- ⇨ $2\theta = 2n\pi + \propto^l$
- ⇨ i.e $2\theta = 2n\pi + 2\propto$ ($\propto^l = 2\propto$) Divided by 2 on B S
- ⇨ $\theta = n\pi + \propto \ \forall n \in I$

5. The G.S of $\tan 2x \tan x = 1$ is x=.................$\forall$ n $\in$ I

1. $\frac{n\pi}{3}+\frac{\pi}{4}\forall$ n $\in$ I 2) $\frac{n\pi}{4}+\frac{\pi}{3}\forall$ n $\in$ I 3) $\frac{n\pi}{3}+\frac{\pi}{6}\forall$ n $\in$ I

Answer: 3

Soln. Given => $\tan 2x . \tan x = 1$

- ⇨ $\tan 2x = \frac{1}{\tan x}$ => $\tan 2x = \cot x$ =>$\tan 2x = tan \propto$
- ⇨ Where $\propto = (\frac{\pi}{2}$-$x)$, i.e $2x = (\frac{\pi}{2} - x)$
- ⇨ G.S is $2x = n\pi + \alpha \, \forall$ n $\in$ I
- ⇨ $2x = n\pi + \left(\frac{\pi}{2} - x\right)$ => $2x + x = n\pi + (\frac{\pi}{2})$
- ⇨ $3x = n\pi + \left(\frac{\pi}{2}\right)$ => $x = \frac{1}{3}(n\pi + \frac{\pi}{2})$
- ⇨ $i.e \; x = \frac{n\pi}{3} + \frac{\pi}{6}$

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6. The G.S of $\sin x + \cos x = 1$ is ....

1. $x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}\ \forall\, \mathrm{n} \in \mathrm{I}$
2. $x = 2n\pi \pm \frac{\pi}{4} +\ \forall\, \mathrm{n} \in \mathrm{I}$
3. $x = 2n\pi + \frac{\pi}{4} - \frac{\pi}{4}\ \forall\, \mathrm{n} \in \mathrm{I}$
4. *None of these*

Soln. Given in the form $a\sin x + b\cos x = c$ where a, b,c ϵR

If $a = b = c = 1, r = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

By dividing $\sqrt{2}$ on B.S in the given eqn we have

i.e. $\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos\ x = \frac{1}{\sqrt{2}}$ => i.e. sin $\frac{\pi}{4}$ sin x + cos $\frac{\pi}{4}$ $\cos\ x = \frac{1}{\sqrt{2}}$

=>$\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4} = [2n\pi \pm \frac{\pi}{4}]\ \forall\, \mathrm{n} \in \mathrm{I}$ => $x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$

7. G Ss of $tan^2x + sec2x = 1$ is $x = \cdots \ldots \ldots \ldots \ldots \ldots$

1). $n\pi$ or $n \pm \frac{\pi}{3}\ \forall n \in I$ 2). $n\pi$ or $n - \frac{\pi}{3}\ \forall n \in I$

3). π or $n\pi \pm \frac{\pi}{3}\ \forall n \in I$ 4). N.T

Answer: 1 Given =>

$sec2x = 1 - tan^2x \Rightarrow \frac{1}{cos2x} = 1 - tan^2x \Rightarrow \frac{1+tan^2x}{1-tan^2x} = 1 - tan^2x$

$(1 + tan^2x) = (1 - tan^2x)^2 \Rightarrow 3tan^2x = tan^4x \Rightarrow 3tan^2x - tan^4x = 0$

$tan^2x(3 - tan^2x) = 0$ i,e $tan^2x = 0$

$\tan x = 0$, $\alpha = 0$

G , S $x = n\pi + \alpha\ \forall n \in I$ $3 - tan^2x = 0 \Rightarrow tan^2x = 3 \Rightarrow tanx = \pm\sqrt{3}$

$x = n\pi.$ $\alpha = \pm\frac{\pi}{3}$ G S $x = n\pi \pm \frac{\pi}{3}\ \forall n \in I$

8. The G.S of $9^{\sin x} - 2(3^{\sin x}) + 1 = 0$ is x= ....

1. $\frac{n\pi}{2}\ \forall\, n \in I$ 2) $n\pi\ \forall\, n \in I$ 3) $n\pi + (-1)^n \frac{\pi}{2}\ \forall\, n \in I$ 4) N.T

Answer : 2

Soln. Given => $(3^2)^{sinx} - 2(3^{sinx}) + 1 = 0$

=>$(3^{sinx})^2 - 2(3^{sinx}) + 1 = 0$------(1)

Put $3^{\sin x} = a\ in\ (1)\ i.e\ a^2 - 2a + 1 = 0$

- ⇨ $(a-1)^2 = 0$ => $(3^{\sin x} - 1)^2 = 0$
- ⇨ $3^{\sin x} = 3^0$ => $\sin x = 0$

$i.e\ G.S\ is\ x = n\pi + (-1)^n\ \propto,\ \forall\, n \in I$ where $\propto = 0$

$i.e\ x = n\pi + (-1)^n.0$ => $x = n\pi.$

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9. If $tan3x - tan2x = 1 + tan3x tan2x$ then the given soln x is..........

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{2}$ 3. No solution 4. N.T

Answer : 3

Solution given => $\frac{tan3x - tan2x}{1 + tan3x tan2x} = 1$

$i, e. \tan(3x - 2x) = 1.$ I,e $\tan x = 1.$

$but \tan x = 1, tan2x$ is not defined $\therefore$ there is no solution.

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10. *If* $cos^2x + cos2x = 2$ *then the G.solution is* $x = \cdots$ ……………

1. $n\pi \ \forall \text{ n} \in \text{I}$
2. $\frac{n\pi}{2} \ \forall \text{ n} \in \text{I}$
3. $\frac{\pi}{2} \ \forall \text{ n} \in \text{I}$
4. N.T

Answer : 1

Solution , given => $\cos^2 x + (2cos^2x - 1) = 2$

,=> $\cos^2 x + 2\cos^2 x = 2+1$

,=> $3cos^2x = 3$

=> $cos^2x = 1$

,=> $cos^2x = cos^20, \ \propto = 0$ $\therefore cos^2\theta = cos^2 \propto$

,=> given solution $x = n\pi \ \forall \text{ n} \in \text{I}$. $i,e \ \theta = n\pi \pm \propto$

Inverse Trigonometric Functions

MATHEMATICS

11. The value of $cos^{-1}(-1) - sin^{-1}(1)$ is ....

1. π
2. $\frac{\pi}{2}$
3. $-\frac{\pi}{2}$
4. $None\ of\ these$

Answer : 2

Soln given => $cos^{-1}(-1) - sin^{-1}(1)$ => $\left(\pi - \frac{\pi}{2}\right) = \frac{\pi}{2}$

12. If sin $[tan^{-1}\left(\frac{1-x^2}{2x}\right) + cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)]$ **= ....**

1. 0 2) -1 3) 1 4) $N.T$

Answer : 3

Soln. Given =>$\sin\left[tan^{-1}\left(\frac{1-x^2}{2x}\right) + \boldsymbol{cos}^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = \sin(\frac{\pi}{2}) = 1$.

By using $(\ tan^{-1}x + cot^{-1}x = \frac{\pi}{2}\ )$

Where $cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = cot^{-1}\left(\frac{1-x^2}{2x}\right)$

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13. The value of $\cos\left[2\tan^{-1}\left(\frac{1}{3}\right)\right]$ is ....

1. $\frac{1}{3}$ 2). $\frac{2}{3}$ 3). $\frac{3}{2}$ 4). $\frac{4}{5}$

Answer : 4

Soln. Let $\tan^{-1}\frac{1}{3} = \alpha \Rightarrow \tan\alpha = \frac{1}{3}$, Given=> $\cos[2\alpha] = \frac{1-\tan^2\alpha}{1+\tan^2\alpha} = \frac{1-(\frac{1}{3})^2}{1+(\frac{1}{3})^2} = \frac{8/9}{10/9} = \frac{4}{5}$

14. If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$, then $\sum x =$ ………………

1. $\frac{xy}{z}$ 2). xyz 3). $\frac{xz}{y}$ 4). $yz - x$

Answer : 2

Soln. Given=> $tan^{-1}\left[\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right] = \pi$

=> $\left[\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right] = \tan\pi$

=> $\left[\frac{\sum x-xyz}{1-(\sum xy)}\right] = 0$ $(tan\pi = 0)$

=> $\sum x - xyz = 0$ $i.e$ $\sum x = xyz$

KEA

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15. The value of $\left[sec^2(tan^{-1}5) + cosec^2(cot^{-1}(3)\right]$ *is* ……………………………

1) 8 2) 34 3) 36 4) N.T

Answer :3

Soln. Given => $[1 + tan^2\theta_1 + 1 + cot^2\theta_2]$ $\theta_1 = tan^{-1}5 \; and \; \theta_2 = cot^{-1}3$

=>$[1 + \tan(\tan^{-1} 5^2) + 1 + \cot(cot^{-1}3^2)]$

=>$[1 + 25 + 1 + 9]$

=36

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16. The value of $2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is =.................

1) $\frac{\pi}{4}$ 2) $\frac{2\pi}{3}$ 3) $\frac{3\pi}{4}$ 4) *None of these.*

Answer : 2

Soln. Given=> $\tan^{-1}\frac{1}{\sqrt{3}}+\left(\tan^{-1}\frac{1}{\sqrt{3}}+\cot^{-1}\frac{1}{\sqrt{3}}\right)=$

$\tan^{-1}\frac{1}{\sqrt{3}}+\frac{\pi}{2}$ => $\frac{\pi}{6}+\frac{\pi}{2}=\frac{4\pi}{6}=\frac{2\pi}{3}$. $((\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2})$

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17. The domain of $y = \sqrt{sin^{-1}2x + \frac{\pi}{6}}$ then $x \in$............

1) $[\frac{1}{2}, \frac{1}{4}]$ 2). $[-\frac{1}{4}, \frac{1}{2}]$ 3). $[-\frac{1}{2}, \frac{1}{4}]$ 4) *None of these*

Answer : 2

Soln. Given => $sin^{-1}2x + \frac{\pi}{6} \geq 0$

$=> -\frac{\pi}{6} \leq sin^{-1}2x \leq \frac{\pi}{2}$

=> *Apply* sin *in each inequality*

$=> \sin(-\frac{\pi}{6}) \leq 2x \leq sin\frac{\pi}{2}$

$=> -\sin(-\frac{\pi}{6}) \leq 2x \leq sin\frac{\pi}{2}$

$=> -\frac{1}{2} \leq 2x \leq 1$

$=> i.e \; -\frac{1}{4} \leq x \leq \frac{1}{2}$

$=> x \in [-\frac{1}{4}, \frac{1}{2}]$.

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18. The value of $\sin\left(2\sin^{-1}\frac{4}{5}\right) = \sin\left(2\cos^{-1}\frac{3}{5}\right) = \sin\left(2\tan^{-1}\frac{4}{3}\right)$ is ....

1) $\frac{48}{225}$ 2) $\frac{4}{5}$ 3) $\frac{24}{25}$ 4) N.T

Answer : 3

Soln , By using the result, from $x^2 + y^2 = r^2$

I.e. $\sin\left(2\sin^{-1}\frac{y}{r}\right) = \sin\left(2\sin^{-1}\frac{x}{r}\right) = \sin\left(2\tan^{-1}\frac{y}{x}\right) = \left(\frac{2xy}{x^2+y^2}\right)$

When x=3 y=4 $\therefore \Rightarrow \left[\frac{2.3.4}{3^2+4^2}\right] = \left[\frac{24}{25}\right]$.

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19. The value of $\cos\left(2\sin^{-1}\frac{1}{\sqrt{5}}\right) = \cos\left(2\cos^{-1}\frac{2}{\sqrt{5}}\right) = \cos\left(2\tan^{-1}\frac{1}{2}\right)$ is ....

1) $\frac{3}{5}$ 2) $\frac{5}{3}$ 3) $\frac{1}{\sqrt{5}}$ 4) *None of these*

Answer: 1

Soln. By using formula $\cos\left(2\sin^{-1}\frac{y}{r}\right) = \cos\left(2\sin^{-1}\frac{x}{r}\right) = \cos\left(2\tan^{-1}\frac{y}{x}\right) = \frac{x^2-y^2}{x^2+y^2}$

when $x = 2, y = 1$ Given => $\frac{4-1}{2^2+1^2} = \left(\frac{3}{5}\right)$

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20. The value of $\cos\left[2\,tan^{-1}\frac{1}{7}\right]$ is ....

1) $\frac{2}{7}$ 2) 7 3) $\frac{24}{25}$ 4) *None of these*

Answer: 3

Soln. By using formula $\cos\left[2\,tan^{-1}\frac{y}{x}\right]=\frac{x^2-y^2}{x^2+y^2}=\frac{7^2-1^2}{7^2+1^2}=\frac{49-1}{50}=\frac{48}{50}=\frac{24}{25}$

where $x=7, y=1$

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21. The value of cos $\left[\frac{1}{2}cos^{-1}\left(\frac{7}{25}\right)\right]$ is……….

1) $\frac{3}{4}$ 2) $\frac{4}{5}$ 3) $\frac{7}{5}$ 4) *None of these*

Answer : 2

Soln. By using formula cos $\left[\frac{1}{2}\ cos^{-1}\left(\frac{x}{r}\right)\right] = \frac{\sqrt{r+x}}{2r}$

When $x = 7, r = 25$ Given => $\sqrt{\frac{25+7}{2*25}} = \sqrt{\frac{32}{50}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$.