MATHEMATICS

1) $\int \sin ax \cos bx \, dx = + c$

1) $\dfrac{-1}{2}\left[\dfrac{\cos(a+b)x}{a+b} + \dfrac{\cos(a-b)x}{a-b}\right]$

2) $\dfrac{1}{2}\left[\dfrac{\cos(a+b)x}{a+b} + \dfrac{\cos(a-b)x}{a-b}\right]$

3) $\dfrac{1}{2}\left[\dfrac{\cos(a+b)x}{a+b} + \dfrac{\cos(a-b)x}{a-b}\right]$

4) *None*

MATHEMATICS

$$I = \frac{1}{2}\int 2 \sin ax \cos bx \, dx$$

$$= \frac{1}{2}\int \left[\sin (a+b)x + \sin (a-b)x\right] dx$$

$$= \frac{-1}{2}\left[\frac{\cos (a+b)x}{a+b} + \frac{\cos (a-b)x}{a-b}\right]$$

Ans (1)

MATHEMATICS

2) $$\int \frac{x\,dx}{(x-a)(x-b)} = \quad + c$$

1) $\frac{1}{a-b}\log\left[(x-a)^a/(x-b)^b\right]$

2) $\log\left[(x-a)^a\right] - \log\left[(x+b)^b\right]$

3) $\frac{1}{a+b}\log\left[(x+a)^a/(x-b)^b\right]$

4) *None*

MATHEMATICS

Use inspection method of partial fractions and then integrate.

Ans (1)

MATHEMATICS

3) $$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = ----+k$$

1) $\frac{1}{ab}\tan^{-1}\left(\frac{b\tan x}{a}\right)$ 2) $\tan^{-1}\left(\frac{b\tan x}{a}\right)$

3) $ab\ \tan^{-1}\left(\frac{\tan bx}{a}\right)$ 4) $\frac{1}{ab}\tan\left(\frac{\tan^{-1} bx}{a}\right)$

MATHEMATICS

Divide Nr. and Dr. by $\cos^2 x$

$$I = \int \frac{\sec^2 x\, dx}{a^2 + b^2 \tan^2 x} = \frac{1}{b^2} \int \frac{d(\tan x)}{(a/b)^2 + (\tan x)^2}$$

$$= \frac{1}{ab} \tan^{-1}\left(\frac{b \tan x}{a}\right)$$

Ans (1)

KEA

MATHEMATICS

4) $$\int \frac{dx}{x^2 + 8x + 20} = ----+c$$

1) $\frac{2}{\sqrt{16}} \tan^{-1}\left(\frac{x+4}{2}\right)$

2) $\frac{1}{\sqrt{16}} \tan^{-1}\left(\frac{x}{x+4}\right)$

3) $\frac{1}{\sqrt{16}} \tan^{-1}\left(\frac{x+2}{x-2}\right)$

4) $\frac{1}{\sqrt{16}} \tan^{-1}\left(\frac{x}{x-4}\right)$

MATHEMATICS

$$\int \frac{dx}{ax^2+bx+c}$$

$$= \frac{2}{\sqrt{4ac-b^2}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)+c$$

$$= \frac{2}{\sqrt{16}} \tan^{-1}\left(\frac{x+4}{2}\right)$$

Ans (1)

MATHEMATICS

5) $\int \frac{1 - x^2}{1 + x^2} dx = - - - - + c$

1) $x + 2 \tan^{-1} x$

2) $- x + \tan^{-1} x$

3) $- x + 3 \tan^{-1} x$

4) $- x + 2 \tan^{-1} x$

$$I = -\int \left(\frac{x^2 - 1}{x^2 + 1} \right) dx$$

$$= -\int \left(\frac{x^2 - 1}{x^2 + 1} - \frac{2}{x^2 + 1} \right) dx$$

$$= -x + 2\tan^{-1} x + c$$

Ans (4)

MATHEMATICS

6) $\int (x-a)\left(x^{n-1}+ax^{n-2}+a^2x^{n-3}+\ldots\ldots+a^{n-1}\right)dx = ----+c$

1) $\frac{x^{n+1}}{n+1} - a^n x$

2) $\frac{x^n - ax}{a^n}$

3) $\frac{x^n - 1}{n-1} - a^{n-1} x$

4) $None$

MATHEMATICS

$$I = \int \left(x^n - a^n\right) dx$$

$$= \frac{x^{n+1}}{n+1} - a^n x + c$$

Ans (1)

KEA

MATHEMATICS

7) $$\int \frac{\sqrt{1-x^2}+\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = -----+c$$

1) $\sin^{-1} x - \sinh^{-1} x$ 2) $\sinh^{-1} x - \sin^{-1} x$

3) $\sinh^{-1} x + \sin^{-1} x$ 4) $2\sinh^{-1} x + x\sin^{-1} x$

MATHEMATICS

$$I = \int \left(\frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} + \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} \right) dx$$

$$= \int \left(\frac{1}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1-x^2}} \right) dx$$

$$= \sinh^{-1} x + \sin^{-1} x + c$$

Ans (3)

MATHEMATICS

8) $\int \frac{dx}{1+\sqrt{x}} dx = - - - - + c$

1) $2 \log \left(1+\sqrt{x}\right)$ 2) $\tan^{-1}\sqrt{x}$

3) $2\left[\sqrt{x}-\log\left(1+\sqrt{x}\right)\right]$ 4) $\sqrt{x}-\log\left(1+\sqrt{x}\right)$

KEA

MATHEMATICS

$$I = \int \frac{dx}{1 + \sqrt{x}}$$

$$= 2\int \frac{\sqrt{x}\;dx}{2\sqrt{x}\left(1 + \sqrt{x}\right)}$$

$$= 2\int \frac{t\;dt}{1 + t} = 2\int \left(\frac{t + 1 - 1}{1 + t}\right)dt$$

$$= 2\left(t - \log\left(t + 1\right)\right)$$

$$= 2\left[\sqrt{x} - \log\left(\sqrt{x} + 1\right)\right] + c$$

Ans (3)

MATHEMATICS

9) $\int \frac{dx}{(x+1)\left(1+\sqrt{x}\right)} = -----+c$

1) $\tan^{-1}\left[\sqrt{x+2}\right]$

2) $\log\left(\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right)$

3) $\log\left(\frac{\sqrt{x+2}-1}{\sqrt{x+2}-1}\right)$

4) $\frac{1}{2}\log\left(\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right)$

MATHEMATICS

$$I = \int \frac{2\,dx}{(x+2-1).2\sqrt{x+2}}$$

$$= 2\int \frac{dt}{t^{2}-1} \; where \quad t = \sqrt{x+2}$$

$$= 2 . \frac{1}{2} \log\left(\frac{t-1}{t+1}\right)$$

$$= \log\left(\frac{\sqrt{x+2}-1}{\sqrt{x+2}+1}\right) + c$$

Ans (2)

MATHEMATICS

10) $\displaystyle\int \frac{\sin x\, dx}{\sqrt{1+\sin^2 x}} = -----+c$

1) $\sin^{-1}\left(\dfrac{\cos x}{\sqrt{2}}\right)$ 2) $\cos^{-1}\left(\dfrac{\sin x}{\sqrt{2}}\right)$

3) $-\sin^{-1}\left(\dfrac{\sin x}{\sqrt{2}}\right)$ 4) $-\sin^{-1}\left(\dfrac{\cos x}{\sqrt{2}}\right)$

MATHEMATICS

$$I=\int \frac{\sin x\,dx}{\sqrt{2-\cos^2 x}}$$

$$=-\int \frac{d(\cos x)}{\sqrt{\left(\sqrt{2}\right)^2-(\cos x)^2}}$$

$$=-\sin^{-1}\left(\frac{\cos x}{\sqrt{2}}\right)+c$$

Ans (4)

MATHEMATICS

11) $\int \frac{dx}{x(\log x)^n} dx = ----- + c$

1) $\frac{(\log\ x)^{1-n}}{1-n}$

2) $\frac{(\log\ x)^{n-1}}{n-1}$

3) $\frac{(\log\ x)^{n}}{n}$

4) $\frac{(\log\ x)^{n+1}}{n+1}$

MATHEMATICS

$$Sol: \quad Put \ \log x = t \ \therefore \ \frac{1}{x} dx = dt$$

$$I = \int t^{-n} \ dt$$

$$= \frac{t^{-n+1}}{-n+1} = \frac{(\log x)^{1-n}}{1-n}$$

Ans (1)

MATHEMATICS

12) $\int \frac{\cos^2 x}{1-\sin x}\, dx = -----+c$

1) $x + \cos x$

2) $x - \cos x$

3) $\sin x - x$

4) $None$

MATHEMATICS

$$I = \int \frac{1 - \sin^2 x}{1 - \sin x}\, dx$$

$$= \int \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)}\, dx$$

$$= \int (1 + \sin x)\, dx \;=\; x - \cos x$$

Ans (2)

13) $$\int \frac{\left[\sqrt{f(x)}+1\right] f^{1}(x)}{f(x)}\, dx = - - - - + c$$

1) $2\sqrt{f(x)}-\log f(x)$

2) $2\sqrt{f(x)}+\log f(x)$

3) $\sqrt{f(x)}+\log f(x)$

4) $\sqrt{f(x)}-2\log f(x)$

$$\int \frac{\sqrt{f(x)}\ f^{1}(x)}{f(x)} + \int \frac{\sqrt{f(x)}\ dx}{f(x)}$$

$$\int \frac{f^{1}(x)}{\sqrt{f(x)}} + \log\ f(x)$$

$$= 2\sqrt{f(x)} + \log\ f(x)$$

Ans (2)

MATHEMATICS

14) $$\int \frac{(1+x)e^{x}\,dx}{1-\sin^{2}(xe^{x})} = ----+c$$

1) $\sec(xe^{x})$

2) $\tan(xe^{x})$

3) $\sec^{2}(xe^{x})$

4) $-\tan^{2}(xe^{x})$

MATHEMATICS

Put $xe^x = t$ and use $1-\sin^2 t = \cos^2 t$.

$$I = \int \sec^2 t \, dt$$

$$= \tan t$$

$$= \tan\left(xe^n\right)$$

Ans (2)

MATHEMATICS

15) $$\int \frac{e^{\sqrt{x}}\ \sin\left(e^{\sqrt{x}}\right)}{\sqrt{x}}\,dx = - - - - + c$$

1) $2\left(\cos\sqrt{x} - 2\sin\sqrt{x}\right)$

2) $2\left(\sin\sqrt{x} - x\cos\sqrt{x}\right)$

3) $2\left(\sin e^{\sqrt{x}} - e^{\sqrt{x}}\cos e^{\sqrt{x}}\right)$

4) $-2\cos e^{\sqrt{x}}$

MATHEMATICS

$$Put \quad e^{\sqrt{x}} = t$$

$$\therefore \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$$

$$\therefore \quad I = 2\int t \, \sin t \; dt$$

$$= \left(- t \, \cos t - \int - \cos t \; dt\right)$$

$$= \left(- t \, \cot t + \sin \; dt\right)$$

$$= 2\left(- e^{\sqrt{x}} \, \cos \sqrt{x} + \sin e^{\sqrt{x}}\right)$$

Ans (3)

MATHEMATICS

16) $$\int \frac{\cos^3 dx}{\sin^2 x + \sin x} = - - - - + c$$

1) $\sin x + \log \sin x$

2) $\cos x - \log \sin x$

3) $\log \sin x - \sin x$

4) $\log \cos x - 2 \sin x$

MATHEMATICS

$$I = \int \frac{\cos x(1 - \sin x)(1 + \sin x)dx}{\sin x(1 + \sin x)}$$

$$= \int \cos x\left(\frac{1}{\sin x} - 1\right)dx$$

$$= \int\left(\frac{1}{\sin x} - 1\right)d(\sin x)$$

$$= \log \sin x - \sin x$$

Ans (3)

MATHEMATICS

17) $\int \frac{dx}{(\sin x + 2\cos x)(2\sin x + 2\cos x)} = -----+c$

1) $\frac{1}{2}\log\left(\frac{2\tan x + 1}{\tan x + 2}\right)$

2) $\frac{1}{3}\log\left(\frac{3\tan x + 1}{\tan x + 2}\right)$

3) $\frac{1}{3}\log\left(\frac{2\tan x + 2}{2\tan x + 1}\right)$

4) *None*

$$I=\int\frac{\sec^2 x\,dx}{(\tan x+2)(2\tan x+1)}$$

($\div$ Dr. & Nr. by $\cos^2 x$)

$$=\int\frac{dt}{(t+2)(2t+1)}$$

$$by\ \ t=\tan x$$

use inspection method of partial fractions and integrate.

Ans (2)

MATHEMATICS

18) $\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x} = ----+c$

1) $\tan^{-1}\left(\frac{2\tan x + 1}{2}\right)$

2) $-4\tan^{-1}\left(\tan x + \frac{1}{2}\right)$

3) $\tan^{-1}\left(\frac{2\tan x - 1}{2}\right)$

4) $\frac{1}{4}\tan^{-1}\left(\frac{2\tan x + 1}{2}\right)$

MATHEMATICS

$$I = \int \frac{\sec^2 x\, dx}{4\tan^2 x + 4\tan x + 5}$$

(by $\div$ each term by $\cos^2 x$)
and proceed.

Ans (4)

MATHEMATICS

19) $\int \frac{\sin 2x \, dx}{\sin^4 x + \cos^4 x} = - - - - + c$

1) $\tan^{-1}(\sin 2x)$

2) $\sin^{-1}(\tan 2x)$

3) $-\tan^{-1}\cos 2x$

4) $\tan^{-1}(\cos 2x)$

MATHEMATICS

$$I = \int \frac{\sin 2x \, dx}{\left(\sin^2 x + \cos^2 x\right)^2 - \frac{4\sin^2 x \cos^2 x}{2}}$$

$$= \int \frac{2\sin 2x \, dx}{2 - \sin^2 2x} = \int \frac{2\sin 2x \, dx}{1 + \cos^2 2x}$$

$$= -\int \frac{d\left(\cos 2x\right)}{1 + \left(\cos 2x\right)^2}$$

$$= -\tan^{-1}\left(\cos 2x\right) + c$$

Ans (3)

20) $\int \frac{x^2+1}{x^4+1}dx = - - - - + c$

1) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{2x}\right)$

2) $\frac{1}{2}\tan^{-1}\left(\frac{x^2-1}{2x}\right)$

3) $\frac{1}{2}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)$

4) *None*

MATHEMATICS

$$i=\int\frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x^2+\frac{1}{x^2}\right)}\quad put\; x-\frac{1}{x}=t$$

$$\therefore\ \left(1+\frac{1}{x^2}\right)dx = dt \quad and$$

$$\therefore\ x^2+\frac{1}{x^2}=t^2+2$$

$$\therefore\ I=\int\frac{dt}{2+t^2}=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right)+c$$

Ans (3)

MATHEMATICS

21) $\int \frac{\sqrt{x}}{\sqrt{1+x^3}}\,dx = - - - - + c$

1) $\frac{-2}{3}\log\left(x^{3/2}+\sqrt{1+x^3}\right)$

2) $\frac{2}{3}\sinh^{-1}\left(x^{3/2}\right)$

3) $\frac{2}{3}\log\left(x+\sqrt{1+x^3}\right)$

4) *None*

MATHEMATICS

$$put\ \ x^{3/2} = t \Rightarrow x^3 = t^2$$

$$\therefore 3/2\sqrt{x}\ dx = dt$$

$$\therefore I = \frac{2}{3}\int \frac{dt}{\sqrt{1+t^2}}$$

$$= \frac{2}{3}\sinh^{-1} t$$

$$= \frac{2}{3}\sinh^{-1}\left(x^{3/2}\right) + c$$

Ans (2)

MATHEMATICS

22) $\int \frac{(1+x)^2}{x+x^3}dx = ----+c$

1) $\log x + \log(1+x^2)$

2) $\log x + \tan^{-1} x$

3) $\log x + 2\tan^{-1} x$

4) *None*

MATHEMATICS

$$I = \int \left[\frac{1 + x^2}{x\left(1 + x^2\right)} + \frac{2x}{x\left(1 + x^2\right)} \right] dx$$

$$= \log x + 2\tan^{-1} x + c$$

Ans (3)

MATHEMATICS

23) $\int_0^1 \frac{2^{x+1} - 3^{x-1}}{6^x} dx =$

1) $\frac{4}{5}\log_3^e + \frac{1}{6}\log_2^e$

2) $\frac{3}{4}\log_2^e - \frac{1}{6}\log_2^e$

3) $\frac{4}{5}\log_3^e - \frac{1}{6}\log_2^e$

4) $\frac{4}{3}\log_3^e - \frac{1}{6}\log_2^e$

$$I = 2\int_0^1 3^{-x}\,dx - \frac{1}{3}\int_0^1 2^{-x}\,dx$$

$$= 2 \cdot \frac{3^{-x}}{(-\log 3)} - \frac{1}{3} \cdot \frac{2^{-x}}{-\log 2}\Bigg|_0^1$$

$$= -\log_3^e\left[\frac{1}{3^1} - \frac{1}{3^0}\right] + \frac{1}{3}\log_2^e\left[\frac{1}{2^1} - \frac{1}{2^0}\right]$$

$$= \frac{4}{3}\log_3^e - \frac{1}{6}\log_2^e$$

Ans (4)

MATHEMATICS

24) $$\int_0^1 \left(x+\frac{1}{x}\right)^{3/2}\left(\frac{x^2-1}{x^2}\right)dx =$$

1) $\frac{\sqrt{3}}{2}$

2) $\frac{\sqrt{3}}{4}$

3) $\frac{8}{\sqrt{3}}$

4) $\frac{25\sqrt{5}-32}{10\sqrt{2}}$

MATHEMATICS

$$I = 2\int_1^2 \left(x + \frac{1}{x}\right)^{3/2} .d\left(x + \frac{1}{x}\right)$$

$$= \left.\frac{\left(x + \frac{1}{x}\right)^{5/2}}{5/2}\right|_1^2 = \frac{2}{5}\left[\left(\frac{5}{2}\right)^{5/2} - 2^{5/2}\right]$$

$$= \frac{25\sqrt{5} - 32}{10\sqrt{2}}$$

Ans (4)

MATHEMATICS

25) $$\int_0^{\pi/4} \frac{\sin x \cos x \, dx}{\cos^2 x + 3\cos x + 2} =$$

1) $\log \dfrac{8}{9}$

2) $\log \dfrac{5}{6}$

3) $\log \dfrac{2}{3}$

4) $\log\left[\dfrac{2\sqrt{2}\left(\sqrt{2}+1\right)}{\left(2\sqrt{2}+1\right)^2}\right]$

MATHEMATICS

$$I = -\int_0^{\pi/4} \frac{\cos x(-\sin x dx)}{(\cos x)^2 + 3(\cos x) + 2}$$

$$= -\int_0^{1/\sqrt{2}} \frac{t\ dt}{t^2 + 3t + 2}$$

$$= -\int_0^{1/\sqrt{2}} \frac{t\ dt}{(t+1)(t+2)}$$

$$= -\int_1^{1/\sqrt{2}} \left[\frac{-1}{t+1} + \frac{2}{t+2}\right] dt$$

$$= \left[\log(t+1) - 2\log(t+2)\right]\Big|_0^{1/\sqrt{2}}$$

$$= \left[\log \frac{1+t}{(2+t)^2}\right]_0^{1/\sqrt{2}}$$

$$= \log \frac{\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)}{\left(\frac{2\sqrt{2}+1}{\sqrt{2}}\right)^2} - \log\frac{1}{2}$$

$$= \log\left[\frac{2\sqrt{2}(\sqrt{2}+1)}{(2\sqrt{2}+1)^2}\right]$$

Ans (4)

MATHEMATICS

26) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$ then

1) $I_n + I_{n-2} = \dfrac{1}{n-1}$

2) $I_n - 2I_{n-2} - n = 0$

3) $I_n + I_{n-2} = n + 1$

4) *None*

MATHEMATICS

$$I_n = \int_0^{\pi/4} \tan^{n-2}\theta \ \tan^2\theta \, d\theta$$

$$= \int_0^{\pi/4} \tan^{n-2}\theta \ \left(\sec^2\theta - 1\right) d\theta$$

$$= \int_0^{\pi/4} \tan^{n-2}\theta \ d\left(\tan\theta\right) - \int_0^{\pi/4} \tan^{n-2}\theta \ \ d\theta$$

$$= \left.\frac{\tan^{n-2}\theta}{n-1}\right|_0^{\pi/4} - I_{n-2}$$

$$\Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$$

Ans (1)

MATHEMATICS

27) If $I_n = \int_0^{\pi/2} \frac{f(\sin x)dx}{f(\sin x) + f(\cos x)} =$

1) $\frac{\pi}{2}$

2) $\frac{\pi}{3}$

3) $\frac{\pi}{4}$

4) $\frac{2\pi}{3}$

Use

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

Ans (3)

MATHEMATICS

28) $I_n = \int_{-\pi/2}^{\pi/2} x^2 \sin x \; dx =$

1) 0

2) π

3) $\frac{\pi}{2}$

4) $\frac{3\pi}{7}$

MATHEMATICS

$$f(x) = x^2 \sin x \ is \ odd$$

$$\because f(-x) = -x^2 \sin x = -f(x)$$

$$\& \int_{-a}^{a} f(x)dx = 0 \ if \ f \ is \ odd \ .$$

Ans (1)

MATHEMATICS

29) $\int_{a}^{b} f(x)\,dx - \int_{0}^{a} f(a+b-x)\,dx = ----$

where f is continuous on $[a, b]$

1) 0
2) $a\pi$
3) $5a\pi/2$
4) $3a/\pi$

MATHEMATICS

By a property of definite integral

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\therefore \int_a^b f(x)\,dx - \int_a^b f(a+b-x)\,dx = 0$$

Ans (1)

MATHEMATICS

30) $\int_{0}^{2} \sqrt{\frac{2+x}{2-x}}\, dx =$

1) $\frac{\pi}{2} + 1$

2) $\pi + 3/2$

3) $\pi + 1$

4) $\frac{\pi}{3} + 2$

MATHEMATICS

Multiply Nr. and Dr. by $\sqrt{2+x}$

$$I = \int_0^2 \frac{2+x}{\sqrt{4-x^2}}$$

$$= 2\sin^{-1}\frac{x}{2}\Big|_0^2 - \frac{1}{2}\int_0^2 \frac{-2xdx}{\sqrt{4-x^2}}$$

$$= \left(2\sin^{-1}\frac{1}{2} - 0\right) - \sqrt{4-x^2}\Big|_0^2$$

$$= 2\frac{\pi}{6} - (0-2) = \frac{\pi}{3} + 2$$

Ans (4)