CALCULUS

LIMITS, CONTINUITY, DIFFERENTIABILITY

BLUE PRINT

1. Limits - 1 question
2. Continuity – 1 question

The function $f(x)$ is called an even function if $f(-x) = f(x)$ for all x.

The function $f(x)$ is called an odd function if $f(-x) = -f(x)$ for all x.

The function $f(x)$ is neither even nor odd if $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$ for all x.

Definition of Limit

The function f(x) is said to tends to a limit l as x tends to a, if for any arbitrary $\varepsilon > 0$ and there exists a corresponding number $\delta > 0$ such that $|f(x) - l| < \varepsilon$ when ever $|x - a| < \delta$.

It is denoted by $\lim_{x \to a} f(x) = l$

The left hand limit of $f(x)$ as $x \to a$ is denoted by $\lim_{x \to a^-} f(x)$ & is defined by

$$\lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h)$$

The right hand limit of $f(x)$ as $x \to a$ is denoted by $\lim_{x \to a^+} f(x)$ & is defined by

$$\lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h)$$

When LHL = RHL, then we say $\lim_{x \to a} f(x)$ exists

The function $f(x)$ is said to be continuous at $x = a$ if $\lim_{x \to a} f(x)$ exists (i.e., LHL = RHL) & $\lim_{x \to a} f(x) = f(a)$

The function $f(x)$ is said to be discontinuous at $x = a$ if $\lim_{x \to a} f(x) \neq f(a)$

The function $f(x)$ is said to be differentiable at $x = a$ if $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ exists & is denoted by $f^{|}(x)$.

Result: Every differentiable function is continuous, but the converse i.e., every continuous function is need not be differentiable. We can prove this by the example, $f(x) = |x| at\ x = 0$

L' Hospitals Rule : Let $f(x)$ & $g(x)$ are the two real valued functions & if

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f''(x)}{g''(x)} \ldots \&$$

so on, if it is in $\frac{0}{0}$ form.

Results:

1) $$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\, a^{n-1}$$

2) $$\lim_{x \to 0} \frac{\sin x}{x} = 1, \quad \lim_{x \to 0} \frac{\tan x}{x} = 1 \quad \&$$

$$\lim_{x \to 0} \frac{\cos x}{x} = \frac{1}{0} = \infty$$

3) $$\lim_{x \to 0} (1 + x)^{1/x} = e$$

4) $$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

5) $$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$

Examples:

1) $\lim_{x \to 5} \frac{x^K - 5^K}{x-5} = 500,$

then the positive value of k is

1) 3 2) 4 3) 5 4) 6

Solution:

$$\lim_{x \to 5}\left(\frac{x^K - 5^K}{x - 5}\right) = 500$$

$$K.5^{K-1} = 500$$

$$= 4\,X\,125$$

$$= 4\,X\,5^3$$

$$= 4\,X\,5^{4-1}$$

$$.:. K = 4$$

Answer: 2

2) $$\lim_{x \to 0} \frac{\sin nx\,\{a-n)nx - \tan x\}}{x^2} = 500$$

where n is a non zero positive integer, then a is equal to .

1) $\frac{n+1}{2}$ **2) n^2+1** 3) $\frac{1}{n+1}$ 4) n+1/n

Solution:

$$\lim_{x \to 0} \frac{\sin nx}{x} \left\{ \frac{(a-n)nx - \tan x}{x} \right\} = 0$$

$$\lim_{x \to 0} \frac{\sin nx}{nx} . n \left\{ \frac{(a-n)nx}{x} - \frac{\tan x}{x} \right\} = 0$$

$$1 . n\{(a-n)n - 1\} = 0$$

$$an - n^2 - 1 = \frac{0}{n}$$

$$an = n^2 + 1 \quad \therefore a = \frac{n^2+1}{n}$$

$$a = n + \frac{1}{n}$$

Answer: 4

3) The function $f(x) = \log\left(\sqrt{1+x^2} + x\right)$ *is*

1) odd function
2) Even Function
3) Neither Even nor odd function
4) Periodic function

Solution:

$$f(x) = \log\left(\sqrt{1+x^2} + x\right)$$

$$f(-x) = \log\left(\sqrt{1+x^2} - x\right)$$

$$f(-x) = \log\left(\frac{\left(\sqrt{1+x^2} - x\right)\left(\sqrt{1+x^2} + x\right)}{\sqrt{1+x^2} + x}\right)$$

$$f(-x) = \log\left(\frac{1+x^2 - x^2}{\sqrt{1+x^2} + x}\right)$$

$$f(-x) = \log 1 - \log\left(\sqrt{1+x^2} + x\right)$$

$$f(-x) = -f(x)$$

Answer: 1

4) If $f: R \to R$ is continuous function such that

$$f(x + y) = f(x) + f(y) \; \forall \; x, y \in R,$$

$$\& \; f(1) = 2 \quad \textit{then the value of } f(100) =$$

1) 0

2) 100

3) 200

4) 400

Solution:

$$f(x + y) = f(x) + f(y)$$

$$\text{put } y = 1, \quad f(x + 1) = f(x) + f(1)$$

$$\text{put } x = 1, \quad f(2) = f(1) + f(1) = 2f(1)$$

$$\text{put } x = 2, \quad f(3) = f(2) + f(1) = 2f(1) + f(1)$$

$$f(3) = 3f(1)$$

similarly $f(4) = 4f(1)$

$$\therefore f(100) = 100 f(1) = 100 X 2$$

$$= 200$$

Answer: 2

5) If $f(2) = 4 \; and \; f^{|}(2) = 4$

$$\lim_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2} =$$

1) 2

2) -2

3) -4

4) 3

Solution:

$$\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}, \qquad \frac{0}{0},$$ By L H rule

$$= \lim_{x\to 2}\frac{f(2)-2f^{|}(x)}{1-0}$$

$$= f(2)-2f^{|}(2)$$

$$= 4-2(4)$$

$$= -4$$

Answer: 3

6) $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b} =$

1) 1

2) e^{b-a}

3) e^{a-b}

4) e^{b}

Solution:

$$\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b} = \lim_{x\to\infty}\left(\frac{x+b+a-b}{x+b}\right)^{x+b}$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{a-b}{x+b}\right)^{\frac{x+b}{a-b}}\right]^{a-b}$$

$$= e^{a-b}$$

Answer: 3

7) $\lim_{x \to \infty} \left(\frac{x+6}{x+1}\right)^{x+4} =$

1) e^5

2) e^6

3) e^{10}

4) e^{-5}

Solution:

$$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x\to\infty}\left(\frac{x+1+5}{x+1}\right)^{x+4}$$

$$= \lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{x+1+3}$$

$$= \lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{x+1}.\left(1+\frac{5}{x+1}\right)^{3}$$

$$= \lim_{x\to\infty}\left[\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\right]^{5}\left(1+\frac{5}{\infty}\right)^{3}$$

$$= e^{5}.(1+0)^{3} = e^{5}$$

Answer: 1

8) The f unction $f(x) = |x| + \frac{|x|}{x}$ is

1) Continuous at the origin
2) Discontinuous at the origin because $|x|$ is discontinuous there.
3) Discontinuous at the origin because $|x| \& \frac{|x|}{x}$ is discontinuous there.
4) Discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there.

Solution:

$$LHL = \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0} \frac{-x}{x} = -1$$

$$RHL = \lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0} \frac{x}{x} = 1$$

$$LHL \neq RHL, \text{limit does not exists}$$

$\therefore \quad \frac{|x|}{x}$ is not continuous at the origin

Answer: 4

9) If $f(x) = \frac{\sin[x]}{[x]}$, when $[x] \neq 0$ and $f(x) = 0$, when $[x] = 0$ then $\lim_{x \to 0} f(x) =$

1) 1
2) 0
3) -1
4) limit does not exists

Solution:

$$\text{For } -1 < x < 0, \quad [x] = -1$$

$$\text{And for } 0 < x < 1, [x] = 0$$

$$\text{When } [x] \neq 0, f(x) = \frac{\sin[x]}{[x]}$$

$$LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin[x]}{[x]}$$

$$= \lim_{x \to 0} \frac{\sin(-1)}{-1} = \sin 1$$

$$RHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin[x]}{[x]}$$

$$= \lim_{x \to 0} \frac{\sin 0}{0} = 0 \qquad \therefore LHL \neq RHL$$

$$\therefore \text{limit does not exists.}$$

Answer: 4

10) $\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x} =$

1) $a + b$

2) $a - b$

3) e^{ab}

4) 1

Solution:

$$\lim_{x\to 0} \frac{e^{ax} - e^{bx}}{x}, \ \frac{0}{0}, \text{ by LH Rule}$$

$$= \lim_{x\to 0} \frac{e^{ax}.a - e^{bx}.b}{1}$$

$$= 1.a - 1.b = a - b$$

Answer: 2

11) $\lim_{x \to 0} \frac{1 - \cos 3x}{2x^2} =$

1) $\frac{9}{4}$

2) $-\frac{9}{4}$

3) $\frac{4}{9}$

4) $-\frac{4}{9}$

KEA

Solution:

$$\lim_{x\to 0}\frac{1-\cos 3x}{2x^2},\quad \frac{0}{0},\ \textit{by LH Rule}$$

$$=\lim_{x\to 0}\frac{0-(-\sin 3x).3}{4x}$$

$$=\lim_{x\to 0}\frac{\sin 3x}{3x}.\frac{3}{4}.3=\frac{9}{4}$$

Answer: 1

12) $\lim_{x \to \infty} \frac{(2+x)^{40}\ (4+x)^5}{(2-x)^5}\ is$

1) -1

2) 1

3) 16

4) 32

Solution:

$$= \lim_{x \to \infty} \frac{x^{40}\left(\frac{2}{x}+1\right)^{40} x^{5}\left(\frac{4}{x}+1\right)^{5}}{x^{45}\left(\frac{2}{x}-1\right)^{45}}$$

$$= \frac{(1+0)(0+1)}{(0-1)^{45}} = -1$$

Answer: 1

13) $$\lim_{n \to \infty} \frac{\sqrt{4n^2 - 3n + 5} + 4n}{4n + 3} =$$

1) $\frac{3}{2}$

2) $\frac{2}{3}$

3) $-\frac{3}{2}$

4) $-\frac{2}{3}$

Solution:

$$\lim_{n \to \infty} \frac{\sqrt{4n^2 - 3n + 5} + 4n}{4n + 3}$$

$$= \lim_{n \to \infty} \left(\frac{n\sqrt{4 - \frac{3}{n} + \frac{5}{n^2}} + 4n}{4n+3} \right)$$

$$= \lim_{n \to \infty} \frac{n\left(\sqrt{4 - \frac{3}{n} + \frac{5}{n^2}} + 4\right)}{n\left(4 + \frac{3}{n}\right)}$$

$$= \frac{\sqrt{4 - 0 + 0} + 4}{4 + 0} = \frac{6}{4} = \frac{3}{2}$$

Answer: 1

14) $$\lim_{n\to\infty} \frac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{(3n+1)(1-2n)(10-n)} =$$

1) $-\frac{1}{18}$

2) $\frac{1}{18}$

3) 18

4) -18

Solution:

$$\lim_{n\to\infty} \frac{1^2 + 2^2 + 3^2 + \dots\dots + n^2}{(3n+1)(1-2n)(10-n)}$$

$$= \lim_{n\to\infty} \left(\frac{n(n+1)(2n+1)}{6(3n+1)(1-2n)(10-n)}\right)$$

$$= \lim_{n\to\infty} \frac{n\, n\left(1+\frac{1}{n}\right) n\left(2+\frac{1}{n}\right)}{6n\left(3+\frac{1}{n}\right) n\left(\frac{1}{n}-2\right) n\left(\frac{10}{n}-1\right)}$$

$$= \frac{(1+0)(2+0)}{6(3+0)(0-2)(0-1)} = \frac{1}{18}$$

Answer: 2

15) $\lim_{x \to 0} \dfrac{1-\cos mx}{1-\cos nx} =$

1) $\dfrac{m}{n}$

2) $\dfrac{n}{m}$

3) $\dfrac{m^2}{n^2}$

4) $\dfrac{n^2}{m^2}$

Solution:

$$\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx},\ \frac{0}{0},$$ *by L H rule*

$$= \lim_{x\to 0}\frac{0-(-\sin mx)\,m}{0-(-\sin nx)\,n},\ \frac{0}{0}$$

Again by L H Rule

$$= \lim_{x\to 0}\frac{m\cos mx.m}{n\cos nx.n} = \frac{m^2 X 1}{n^2 X\ 1} = \frac{m^2}{n^2}$$

Answer: 3

16) $f(x) = \cos(\log x)$ *then*

$$f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(x)\right]$$

has the value

1) -1

2) $\frac{1}{2}$

3) -2

4) 0

Solution:

$$f(x) = \cos(\log x)$$

$$f(xy) = \cos(\log(xy))$$

$$= \cos(\log x + \log y)$$

$$f\left(\frac{x}{y}\right) = \cos\left(\log\left(\frac{x}{y}\right)\right)$$

$$= \cos(\log x - \log y)$$

$$f\left(\frac{x}{y}\right) + f(xy) = \cos(\log x - \log y) \ + \cos(log x + log y)$$

$$= 2\cos(\log x)\cos(\log y)$$

$$= 2 f(x) f(y)$$

$$\therefore f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(x)\right] = 0$$

Answer: 4

17) $\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} =$

1) $2 \sin 2$

2) 0

3) $2 \cos 2$

4) 1

Solution:

$$\lim_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x}, \frac{0}{0}, \textit{ by LH rule}$$

$$= \lim_{x\to 0} \frac{\cos(2+x)(0+1)-\cos(2-x)(0-1)}{1},$$

$$= \cos 2 + \cos 2 = 2\cos 2$$

Answer: 3

18) $\lim_{x \to 0} (1 + ax)^{\frac{b}{x}} =$

1) e^{ab}

2) e^{a+b}

3) $e^{(a^b)}$

4) e

Solution:

$$\lim_{x \to 0} (1 + ax)^{\frac{b}{x}}$$

$$= \lim_{x \to 0} \left[(1 + ax)^{\frac{1}{ax}}\right]^{ab}$$

$$= e^{ab}$$

Answer: 1

19) $If\ f(x) = \begin{cases} \frac{x^5-32}{x-2} & when\ x \neq 2 \\ k & when\ x = 2 \end{cases}$ and

$hence\ f(x) is\ continuous\ at\ x = 2, \quad if\ k =$

1) 16
2) 80
3) 32
4) 8

❖ Solution: *Since* $f(x)$ *is continuous at* $x = 2$,

$$f(2) = \lim_{x \to 2} f(x)$$

$$K = \lim_{x \to 2} \frac{x^5 - 32}{x - 2}$$

$$= \lim_{x \to 2} \frac{x^5 - 2^5}{x - 2}$$

$$\therefore \quad K = 5\,(2)^{5-1}$$

$$K = 80$$

Answer: 2

20) The function $f(x) = \dfrac{\log(1+ax) - \log(1+bx)}{x}$

is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0 is

1) loga + logb

2) 2

3) a – b

4) a + b

Solution: *Since f(x) is continuous at x = 0*

$$f(0) = \lim_{x \to 0} f(x)$$

$$f(0) = \lim_{x \to 0} \frac{\log(1+ax) - \log(1+bx)}{x} \cdot \frac{0}{0}$$ By LH Rule

$$f(0) = \lim_{x \to 0} \frac{\frac{1}{(1+ax)}(0+a) - \frac{1}{(1+bx)}(0+b)}{1}$$

$$f(0) = \frac{\frac{a}{1+0} - \frac{b}{1+0}}{1}$$

$$f(0) = a - b$$

Answer: 3

21) $$\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$$

1) 1

2) 0

3) $\frac{1}{2}$

4) 2

Solution:

$$\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3} \cdot \frac{0}{0} \quad \textit{by LH Rule}$$

$$= \lim_{\tan x \to 3} \frac{2\tan x \sec^2 x - 2\sec^2 x - 0}{2\tan x \sec^2 x - 4\sec^2 x + 0}$$

$$= \lim_{\tan x \to 3} \frac{2\sec^2 x \; [\tan x - 1]}{2\sec^2 x \; [\tan x - 2)}$$

$$= \frac{3-1}{3-2} = \frac{2}{1} = 2$$

Answer: 4

22) If the function $f(x)=\begin{cases}\dfrac{1-\cos x}{x^2} & if \quad x\neq 0\\ k & if \quad x=0\end{cases}$

is continuous at x = 0 then the value of k is ______

1) 1

2) 0

3) $\frac{1}{2}$

4) -1

Solution:

Since f(x) is continuous at x = 0

$$\therefore f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1 - \cos x}{x^2} \left(\frac{0}{0}\right)$$ ***by LH Rule***

$$K = \lim_{x \to 0} \frac{0 - (-\sin x)}{2x} = \lim_{x \to 0} \frac{1}{2} \cdot \frac{\sin x}{x} = \frac{1}{2}$$

Answer: 3

23) $If\ f(x) = \begin{cases} \dfrac{\sin 5x}{x^2 + 2x} & when\ x \neq 0 \\ k + \dfrac{1}{2} & when\ x = 0 \end{cases}$

$is\ continuous\ at\ x = 0\ \ then\ the\ value\ of\ K\ is$

1) $\frac{1}{2}$

2) 2

3) $-\ 2$

4) 1

Solution: *Since* $f(x)$ *is continuous at* $x = 0$,

$$f(0) = \lim_{x \to 0} f(x)$$

$$K + \frac{1}{2} = \lim_{x \to 0} \frac{\sin 5x}{x^2 + 2x}, \frac{0}{0} \text{ by LH Rule}$$

$$= \lim_{x \to 0} \frac{\cos 5x . 5}{2x + 2}$$

$$K + \frac{1}{2} = \frac{1 X 5}{0+2} \quad \therefore K = \frac{5}{2} - \frac{1}{2}$$

$$K = 2$$

Answer: 2

24) $If\ f(x) = \begin{cases} \frac{\log(1+ax)-\log(1-bx)}{x} & when\ x \neq 0 \\ k & when\ x = 0 \end{cases}$

$and\ f(x)\ is\ continuous\ at\ x = 0, then\ the\ value\ of\ K\ is$

1) $a - b$

2) $a + b$

3) $log\ a + log\ b$

4) $log\ a - log\ b$

Solution: *Since $f(x)$ is continuous at $x = 0$,*

$$f(0) = \lim_{x \to 0} f(x)$$

$$K = \lim_{x \to 0} \frac{\log(1+ax) - \log(1-bx)}{x}, \frac{0}{0} \text{ by LH Rule}$$

$$K = \lim_{x \to 0} \frac{\frac{1}{(1+ax)}(0+a) - \frac{1}{(1-bx)}(0-b)}{1}$$

$$K = \frac{\frac{a}{1+0} + \frac{b}{1-0}}{1} \qquad \therefore K = a + b$$

Answer: 2

25) $If\ f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & for\ x < 0 \\ K & for\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16-\sqrt{x}-4}} & for\ x > 0 \end{cases}$

then the value of K *for which* $f(x)$ *is continuous at* $x = 0$ *is*

1) 5

2) 8

3) 4

4) 3

Solution: *Since $f(x)$ is continuous at $x = 0$,*

$$LHL = RHL = f(0)$$

$$\therefore f(0) = LHL$$

$$K = \lim_{x \to 0} \frac{1 - \cos 4x}{x^2}, \quad \frac{0}{0}, \quad \text{by LH rule}$$

$$= \lim_{x \to 0} \frac{0 - (-\sin 4x).4}{2x}$$

$$= \lim_{x \to 0} \frac{\sin 4x}{4x} X\, 4\, X \frac{4}{2}$$

$$= 1\, X\, 4\, X\, 2 = 8$$

$$K = 8$$

Answer: 2

26) *The function* $f(x) = \frac{(2^x - 1)^2}{\sin x \,.\log(1+x)}$

is not defined at $x = 0$ *what value should be assigned to* $f(0)$ *so that* $f(x)$ *becomes continuous at* $x = 0$

1) $\log 4$

2) 2

3) 1

4) $(\log 2)^2$

Solution:

$$Since\ f(x)\ is\ continuous\ at\ x = 0,$$

$$\therefore f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(2^x - 1)^2}{\sin x . \log(1+x)}$$

$$= \lim_{x \to 0} \frac{(2^x - 1)^2}{x . \frac{\sin x}{x} . \log(1+x)}$$

$$= \lim_{x \to 0} \frac{(2^x - 1)^2}{x^2 . 1 . \frac{1}{x} \log(1 + x)}$$

$$= \lim_{x \to 1} \left(\frac{2^x - 1}{x}\right)^2 , \frac{1}{\log(1+x)^{\frac{1}{x}}}$$

$$= (\log 2)^2 . \frac{1}{\log_e e}$$

$$= (\log 2)^2$$

Answer: 4

27) $\lim_{x \to 0} \dfrac{6^x - 3^x - 2^x + 1}{x^2} =$

1) $\log\left(\frac{2}{3}\right)$

2) $\log 2 + \log 3$

3) $(\log 2)(\log 3)$

4) $\dfrac{\log 2}{\log 3}$

Solution:

$$\lim_{x\to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{(3 X 2)^x - 3^x - 2^x + 1}{x^2}$$

$$= \lim_{x\to 0} \frac{3^x . 2^x - 3^x - 1(2^x - 1)}{x . x}$$

$$= \lim_{x\to 0} \frac{3^x(2^x - 1) - 1(2^x - 1)}{x . x}$$

$$= \lim_{x\to 0} \left(\frac{2^x - 1}{x}\right)\left(\frac{3^x - 1}{x}\right)$$

$$= (\log 2).(\log 3)$$

Answer: 2

28) $$\lim_{x\to 2}\frac{2x^2-5x+2}{x^2-3x+2}=$$

1) 1

2) 2

3) 3

4) 8

Solution:

$$\lim_{x\to 2} \frac{2x^2-5x+2}{x^2-3x+2}, \quad \frac{0}{0}, \quad \textit{by L H Rule}$$

$$= \lim_{x\to 2} \frac{4x-5}{2x-3}$$

$$= \frac{8-5}{4-3}$$

$$= 3$$

Answer: 3

29) $\lim\limits_{\theta \to 0} \dfrac{\sin 2\theta \,.\, \sin 3\theta}{3\,\theta .\ \tan 4\theta} =$

1) $\dfrac{2}{3}$

2) $\dfrac{3}{2}$

3) 2

4) $\dfrac{1}{2}$

Solution:

$$\lim_{\theta \to 0} \frac{\sin 2\theta \,.\, \sin 3\theta}{3\,\theta.\ \tan 4\theta}$$

$$= \lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta} \,.\, 2\theta \,.\, \frac{\sin 3\theta}{3\theta} \,.\, \frac{4\theta}{\tan 4\theta}\ X\ \frac{1}{4\theta}$$

$$= 1\ X\ 2\ X\ 1\ X\ 1\ X \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Answer: 4

30) *If* $f(x)$ *is an even function* & $f'(x)$ *exists then* $f'(e) + f'(-e)$ *is*

1) > 0

2) $= 0$

3) ≥ 0

4) < 0

Solution:

$$f(x) \text{ is an even function}$$

$$\therefore f(-x) = f(x)$$

$$f'(-x)(-1) = f'(x)$$

$$\therefore f'(x) + f'(-x) = 0$$

$$\therefore f'(e) + f'(-e) = 0$$

Answer: 2

31) $If\ f(x)\ is\ a\ function\ such\ that$

$f(x) + f''(x) = 0 \quad and$

$g(x) = [f(x)]^2 + [f'(x)]^2$ &

$g(3) = 8 \quad then\ g(8) =$

1) 0

2)5

3) 8

4) 3

Solution: $g(x) = [f(x)]^2 + [f'(x)]^2$

$$g'(x) = 2[f(x)]f'(x) + 2[f'(x)]f''(x)$$

$$= 2f'(x)[f(x) + f''(x)]$$

$$= 2f'(x).0$$

$$g'(x) = 0$$

$$\therefore g(x) \text{ is constant}$$

$$\therefore g(x) = 8, \qquad g(3) = 8,$$

$$\text{Thus } g(8) = 8,$$

Answer: 3

KEA

32) $Let\ f(x+y) = f(x)f(y)$ $for\ all\ x\ and\ y.$ $Suppose$ $f(5) = 2,\ f'(0) = 3\ then\ f'(5) is$

1) 4
2) 3
3) 8
4) 6

Solution:

$$f(x+y) = f(x)f(y)$$

$$Put\ y = 5,\ f(x+5) = f(x)f(5)$$

$$f'(x+5)(1+0) = f'(x)f(5)$$

$$Put\ x = 0,\ f'(0+5) = f'(0)f(5)$$

$$f'(5) = 3\ X\ 2$$

$$= 6$$

Answer: 4

33) $\lim_{n\to\infty}\left(3^n + 4^n\right)^{\frac{1}{n}} =$

1) 4

2) 3

3) e

4) ∞

Solution: $\lim_{n\to\infty} (3^n + 4^n)^{1/n}$

$$= \lim_{n\to\infty} \left(4^n \left(\frac{3^n}{4^n} + 1\right)\right)^{1/n}$$

$$= \lim_{n\to\infty} 4\left(\left(\frac{3}{4}\right)^n + 1\right)^{\frac{1}{n}}$$

$$= 4\,(0+1)^0 = 4$$

Answer: 1

34) $If\ f(1) = 1\ \&\ f(n+1) = 2f(n) + 1$

$for\ n \geq 1\ then\ f(n) =$

1) $3^n - 2$

2) $2^n - 1$

3) $\frac{1}{2}(2^n + 1)$

4) $2n - 1$

Solution:

$$f(1) = 1, \quad f(n+1) = 2f(n) + 1$$

$$f(1+1) = 2f(1) + 1$$

$$f(2) = 2 + 1 = 3 = 2^2 - 1$$

$$Similarly\ f(3) = 2^3 - 1$$

$$\therefore f(n) = 2^n - 1$$

Answer: 2

35) $If\ f(9) = 9\ and\ f'(9) = 4$

$$then \lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} =$$

1) 2

2) 3

3) 4

4) $\frac{1}{2}$

Solution: $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3}, \frac{0}{0}$ *By LH Rule*

$$= \lim_{x \to 9} \frac{\frac{1}{2\sqrt{f(x)}} f'(x) - 0}{\frac{1}{2\sqrt{x}} - 0}$$

$$= \frac{f'(9)}{2\sqrt{f(9)}} \; X \; \frac{2\sqrt{9}}{1}$$

$$\frac{4}{\sqrt{9}} \; X \; 3 = \frac{4 \; X \; 3}{3} = 4$$

Answer: 3

36) $\lim_{n \to \infty} \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} \dots \dots \frac{1}{4^n} \right) =$

1) $\frac{1}{3}$

2) $\frac{1}{2}$

3) 2

4) $\frac{1}{4}$

Solution:

$$\lim_{n\to\infty}\left(\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}\cdots\cdots\frac{1}{4^n}\right)\ is\ in\ G.P$$

$$a=\frac{1}{4}\quad r=\frac{1}{4}$$

$$=s_\infty=\frac{a}{1-r}=\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$$

Answer: 1

37) $\lim_{n \to \infty} \left(1 - \frac{1}{x} + \frac{1}{x^2} + \ldots\ldots \text{to } n \text{ terms}\right) =$

1) $\frac{1}{1-x}$

2) $\frac{1}{1+x}$

3) $\frac{x}{1+x}$

4) $\frac{x}{1-x}$

Solution:

$$\lim_{n\to\infty}\left(1-\frac{1}{x}+\frac{1}{x^2}+\ \ldots\ldots to\ n\ terms\right)$$

$$=\lim_{n\to\infty}\left(1+\left(\frac{-1}{x}\right)+\left(\frac{-1}{x}\right)^2+\ \ldots\ \ldots .to\ n\ terms\right)$$

Is in G P, $a=1, r=\frac{-1}{x}$

$$=s_\infty=\frac{a}{1-r}=\frac{1}{1-\left(-\frac{1}{x}\right)}=\frac{1}{1+\frac{1}{x}}$$

$$=\frac{x}{1+x}$$

Answer: 3

38) $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2}{4^x + 4^{-x} - 2} =$

1) $\dfrac{1}{\log 16}$

2) $-2\log 4$

3) $(\log 4)^{-2}$

4) $2\log 4$

Solution:

$$\lim_{x \to 0} \frac{e^{x} + e^{-x} - 2}{4^{x} + 4^{-x} - 2}, \quad \frac{0}{0} \quad \textit{by LH rule}$$

$$= \lim_{x \to 0} \frac{e^{x} + e^{-x}\ (-1)}{4^{x} \log 4 + 4^{-x}.\log 4(-1)}, \quad \frac{0}{0}$$

Again by L H Rule

$$= \lim_{x \to 0} \frac{e^{x} - e^{-x}\ (-1)}{4^{x} \log 4 \,.\log 4 - 4^{-x} \log 4 \,.\log 4(-1)}$$

$$= \frac{1+1}{(log4)^2 + (\log 4)^2}$$

$$= \frac{2}{2\,(\log 4)^2} = (\log 4)^{-2}$$

Answer: 3

39) $\lim_{n\to\infty} \dfrac{3\,.\,2^{n+1} - 4.5^{n+1}}{5.2^{n} + 7.5^{n}} =$

1) $\dfrac{3}{5}$

2) $-\dfrac{4}{5}$

3) $-\dfrac{20}{7}$

4) $\dfrac{20}{7}$

Solution:

$$\lim_{n \to \infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^{n} + 7.5^{n}}$$

$$= \lim_{n \to \infty} \frac{3.2^{n}.2^{1} - 4.5^{n}.5}{5^{n}\left(5.\frac{2^{n}}{5^{n}} + 7\right)}$$

$$= \lim_{n \to \infty} \frac{5^{n}\left(6.\frac{2^{n}}{5^{n}} - 20\right)}{5^{n}\left(5.\frac{2^{n}}{5^{n}} + 7\right)}$$

$$= \frac{6 X 0 - 20}{0 + 7} = -\frac{20}{7}$$

Answer: 3

40) $If\ f(a) = 2,\ f'(a) = 1,$

$g(a) = -3, g'(a) = -1$ then

$$\lim_{x \to a} \frac{f(a)g(x) - f(x)g(a)}{x - a} =$$

1) 6 2) 1

3) -1 4) -5

Solution:

$$\lim_{x \to a} \frac{f(a)g(x) - f(x)g(a)}{x-a}, \frac{0}{0}$$

By LH rule

$$= \lim_{x \to a} \frac{f(a)g'(x) - f'(x)g(a)}{1-0}$$

$$= f(a)g'(a) - f'(a)g(a)$$

$$= 2(-1) - 1(-3) = -2 + 3 = 1$$

Answer: 2