



CALCULUS LIMITS, CONTINUITY, DIFFERENTIABILITY **BLUE PRINT** 1. Limits - 1 question 2. Continuity – 1 question





The function f(x) is called an even function if f(-x) = f(x) for all x. The function f(x) is called an odd function if f(-x) = -f(x) for all x. The function f(x) is neither even nor odd if $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$ for all x.





Definition of Limit

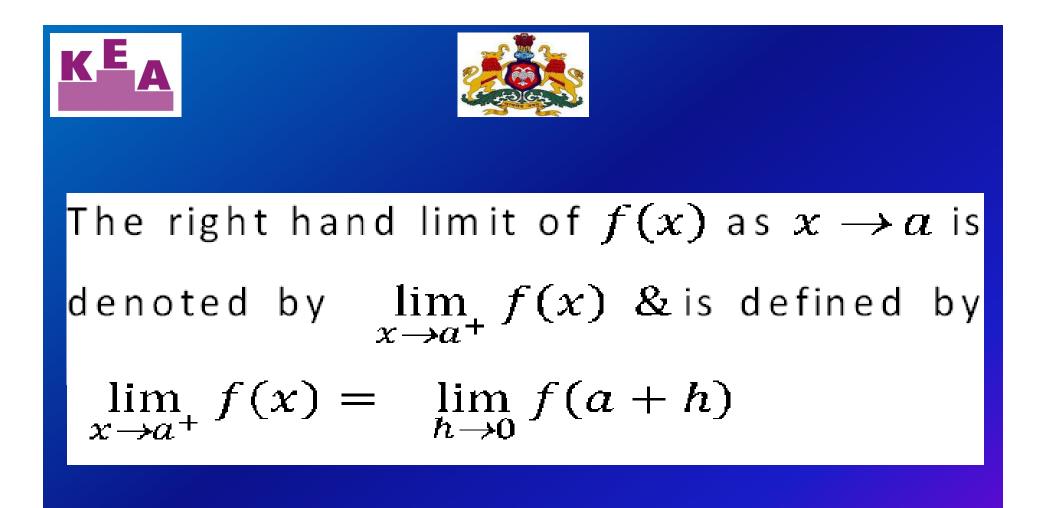
The function f(x) is said to tends to a limit I as x tends to a, if for any arbitrary $\varepsilon > 0$ and there exists a corresponding number $\delta > 0$ such that $|f(x) - I| < \varepsilon$ when ever $|x - a| < \delta$.

It is denoted by $\lim_{x \to a} f(x) = l$





The left hand limit of f(x) as $x \to a$ is denoted by $\lim_{x \to a^-} f(x)$ & is defined by $\lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a - h)$







When LHL = RHL, then we say $\lim_{x \to a} f(x)$ exists

The function f(x) is said to be continuous at x = a if $\lim_{x \to a} f(x)$ exists (i.e., LHL = RHL) & $\lim_{x \to a} f(x) = f(a)$





The function f(x) is said to be discontinuous at x = a if $\lim_{x \to a} f(x) \neq f(a)$ The function f(x) is said to be differentiable at x = a if $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ exists & is denoted by $f^{\dagger}(x)$.





Result: Every differentiable function is continuous, but the converse i.e., every continuous function is need not be differentiable. We can prove this by the example, f(x) = |x| at x = 0





L'Hospitals Rule : Let f(x) & g(x) are

the two real valued functions & if

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f''(x)}{g''(x)} \dots \& \text{ so}$$
on, if it is in $\frac{0}{0}$ form.





Results:

1)
$$\lim_{x \to a} \frac{x^{n} - a^{n}}{x - a} = n \ a^{n-1}$$

2)
$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \ \lim_{x \to 0} \ \frac{\tan x}{x} = 1 \quad \& \\ \lim_{x \to 0} \frac{\cos x}{x} = \frac{1}{0} = \infty$$

3)
$$\lim_{x \to 0} (1 + x)^{1/x} = e$$

4)
$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

5)
$$\lim_{x \to 0} \frac{a^{x} - 1}{x} = \log_{e} a$$





Examples: $\lim_{x \to 5} \frac{x^{K} - 5^{K}}{x - 5} = 500,$ 1) then the positive value of k is 1) 3 3) 5 2) 4 4) 6





Solution:

 $\left(\frac{x^K-5^K}{x-5}\right)=500$ $\lim_{x\to 5}$ $K.5^{K-1} = 500$ = 4X125 $= 4 X 5^3$ $=4X5^{4-1}$ K = 4Answer: 2



where n is a non zero positive integer, then a is equal to .

1)
$$\frac{n+1}{2}$$
 2) n^2+1 3) $\frac{1}{n+1}$ 4) $n+1/n$





Solution: $\lim_{x \to 0} \frac{\sin nx}{x} \left\{ \frac{(a-n)nx - \tan x}{x} \right\} = 0$ $\lim_{x \to 0} \frac{\sin nx}{nx} \cdot n \left\{ \frac{(a-n)nx}{x} - \frac{\tan x}{x} \right\} = 0$ $1:n\{(a-n)n-1\}=0$ $an-n^2-1=\frac{0}{2}$ $an = n^2 + 1$: $a = \frac{n^2 + 1}{n^2}$ n $a = n + \frac{1}{-}$ n

Answer: 4





3) The function $f(x) = \log(\sqrt{1+x^2} + x)$ is

1) odd function
 2) Even Function
 3) Neither Even nor odd function
 4) Periodic function





Solution:
$$f(x) = \log(\sqrt{1 + x^2} + x)$$

 $f(-x) = \log(\sqrt{1 + x^2} - x)$
 $f(-x) = \log(\frac{(\sqrt{1 + x^2} - x)(\sqrt{1 + x^2} + x)}{\sqrt{1 + x^2} + x})$
 $f(-x) = \log(\frac{1 + x^2 - x^2}{\sqrt{1 + x^2} + x})$
 $f(-x) = \log 1 - \log(\sqrt{1 + x^2} + x)$
 $f(-x) = -f(x)$
Answer: 1





4) If $f: R \rightarrow R$ is continuous function such that $f(x + y) = f(x) + f(y) \forall x, y \in R,$ & f(1) = 2 then the value of f(100) =1) 0 2) 100 3) 200 4) 400





Solution: f(x + y) = f(x) + f(y)put y = 1, f(x + 1) = f(x) + f(1)put x = 1, f(2) = f(1) + f(1) = 2f(1)put x = 2, f(3) = f(2) + f(1) = 2f(1) + f(1)f(3) = 3 f(1)similarly f(4) = 4 f(1)f(100) = 100 f(1) = 100 XZ= 200Answer: 2





5) If f(2) = 4 and $f^{(2)} = 4$

 $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}=$ **x→**2

1) 22) -23) -44) 3





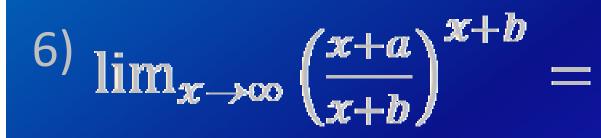
Solution:

$$\lim_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2}, \quad \frac{0}{0}, \text{ By L H rule}$$
$$= \lim_{x \to 2} \frac{f(2) - 2f^{\dagger}(x)}{1 - 0}$$
$$= f(2) - 2f^{\dagger}(2)$$
$$= 4 - 2(4)$$
$$= -4$$

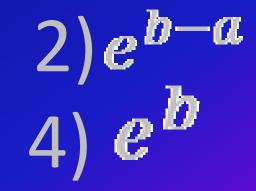
Answer: 3







1) **1** 3) *e^{a-b}*







Solution:

$$\lim_{x \to \infty} \left(\frac{x+a}{x+b}\right)^{x+b} = \lim_{x \to \infty} \left(\frac{x+b+a-b}{x+b}\right)^{x+b}$$
$$= \lim_{x \to \infty} \left[\left(1 + \frac{a-b}{x+b}\right)^{\frac{x+b}{a-b}} \right]^{a-b}$$
$$= e^{a-b}$$

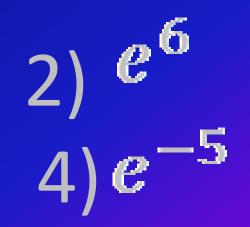
Answer: 3





7) $\lim_{x \to \infty} \left(\frac{x+6}{x+1}\right)^{x+4} =$

1) e⁵ 3) e¹⁰







Solution: $\lim_{x \to \infty} \left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x \to \infty} \left(\frac{x+1+5}{x+1}\right)^{x+4}$ $= \lim_{x \to \infty} \left(1 + \frac{5}{x+1} \right)^{x+1+3}$ $= \lim_{x \to \infty} \left(1 + \frac{5}{x+1} \right)^{x+1} \cdot \left(1 + \frac{5}{x+1} \right)^3$ $=\lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\left(1+\frac{5}{\infty}\right)$ $=e^{5}.(1+0)^{3}=e^{5}$ Answer: 1





8) The function $f(x) = |x| + \frac{|x|}{x}$ is

Continuous at the origin
 Discontinuous at the origin because x is discontinuous there.
 Discontinuous at the origin because x x is discontinuous there.
 Discontinuous at the origin because x is discontinuous there.





Solution:

$LHL = \lim_{x \to 0^{-}} \frac{|x|}{x} = \lim_{x \to 0^{-}} \frac{-x}{x} = -1$ $RHL = \lim_{x \to 0^{+}} \frac{|x|}{x} = \lim_{x \to 0^{+}} \frac{x}{x} = 1$ $LHL \neq RHL, limit does not exists$ |x|

is not continuous at the origin

Answer: 4

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9) If $f(x) = \frac{sin[x]}{[x]}$, when $[x] \neq 0$ and f(x) = 0, when [x] = 0 then $\lim_{x\to 0} f(x) =$ 1)1 2) 0 3) -1 4) limit does not exists



Solution:

For -1 < x < 0, [x] = -1And for 0 < x < 1, [x] = 0When $[x] \neq 0, f(x) = \frac{sin[x]}{[x]}$ $LHL = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{sin[x]}{[x]}$ $=\lim_{x\to 0}\frac{\sin(-1)}{-1}=\sin 1$ $RHL = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{sin[x]}{[x]}$ $= \lim_{x \to 0} \frac{\sin 0}{0} = 0 \qquad \therefore LHL \neq RHL$ Answer: 4 .: limit does not exists.





10) $\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x}$

1) *a* + *b* 3) *e^{ab}* 2) **a** – **b** 4) 1

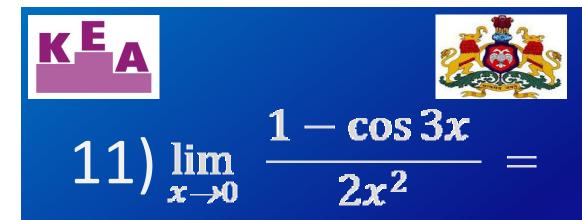


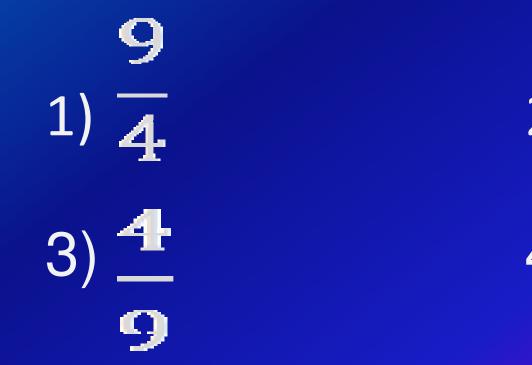


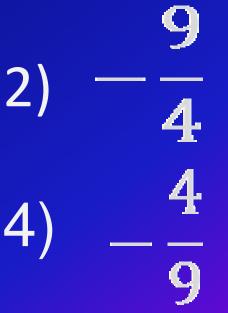
Solution:

$\frac{e^{ax}-e^{bx}}{x}, \frac{0}{0}, by LH Rule$ $x \rightarrow 0$ e^{ax}.a-e^{bx}.b = lim 1 **x**→0 = 1.a - 1.b = a - b

Answer: 2







KEA



Solution:

 $\lim_{x \to 0} \frac{1 - \cos 3x}{2x^2}, \quad \frac{0}{0}, by LH Rule$ $= \lim_{x \to 0} \frac{0 - (-\sin 3x) \cdot 3}{4x}$ $= \lim_{x \to 0} \frac{\sin 3x}{3x} \cdot \frac{3}{4} \cdot 3 = \frac{9}{4}$







12) $\lim_{x\to\infty} \frac{(2+x)^{40} (4+x)^5}{(2-x)^5}$ is

1) - 12)13)164)32

KEA Solution:



$$= \lim_{x \to \infty} \frac{x^{40} \left(\frac{2}{x} + 1\right)^{40} x^5 \left(\frac{4}{x} + 1\right)^5}{x^{45} \left(\frac{2}{x} - 1\right)^{45}}$$

$$=\frac{(1+0)(0+1)}{(0-1)^{45}}=-1$$





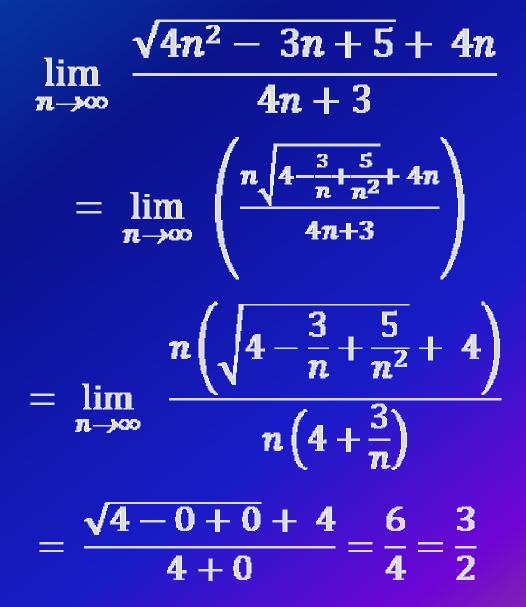


 $\sqrt{4n^2-3n+5}+4n$ 13) lim 4n + 3 $n \rightarrow \infty$

2) $\frac{2}{3}$ 1) $\frac{3}{2}$ 3 2 3) 4 3

KEA Solution:





Answer: 1





 $\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{(3n+1)(1-2n)(10-n)} =$ 14)







$$\lim_{n \to \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{(3n+1)(1-2n)(10-n)}$$

$$= \lim_{n \to \infty} \left(\frac{n(n+1)(2n+1)}{6(3n+1)(1-2n)(10-n)} \right)$$

$$\lim_{n \to \infty} \frac{nn\left(1 + \frac{1}{n}\right)n\left(2 + \frac{1}{n}\right)}{6n\left(3 + \frac{1}{n}\right)n\left(\frac{1}{n} - 2\right)n\left(\frac{10}{n} - 1\right)}$$

$$\frac{(1+0)(2+0)}{6(3+0)(0-2)(0-1)} = \frac{1}{18}$$

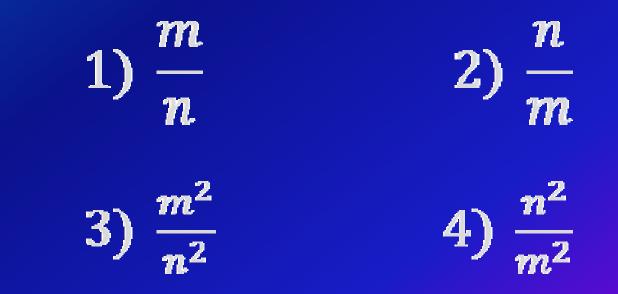
Answer: 2

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15) $\lim_{x\to 0} \frac{1-\cos m x}{1-\cos n x} =$







$$\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}, \frac{0}{0}, \quad by \ L \ H \ rule$$

$$= \lim_{x \to 0} \frac{0 - (-\sin mx) m}{0 - (-\sin nx) n}, \frac{0}{0}$$

Again by L H Rule

=	lim	m cos mx.m	m^2X1	m^2
		n cos nx. n	$\frac{1}{n^2 X 1}$	$\overline{n^2}$







16) $f(x) = \cos(\log x)$ then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(x)\right]$ has the value 2) $\frac{1}{2}$ 1) -1 4)0 3) – 2





 $f(x) = \cos(\log x)$ $f(xy) = \cos(\log(xy))$ $= \cos (\log x + \log y)$ $f\left(\frac{x}{y}\right) = \cos\left(\log\left(\frac{x}{y}\right)\right)$ $= \cos(\log x - \log y)$





$f\left(\frac{x}{y}\right) + f(xy) = \cos(\log x - \log y) + \cos(\log x + \log y)$

$= 2\cos(\log x)\cos(\log y)$

= 2 f(x)f(y):: $f(x)f(y) - \frac{1}{2} \left[f\left(\frac{x}{y}\right) + f(x) \right] = 0$







 $\frac{\sin(2+x)-\sin(2-x)}{2}$ 17) $\lim_{x\to 0}$ X

1) 2 sin 2
2) 0
3) 2 cos 2
4) 1





$$\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x}, \frac{0}{0}, by LH rule$$

$$= \lim_{x\to 0} \frac{\cos(2+x)(0+1) - \cos(2-x)(0-1)}{1},$$

 $=\cos 2 + \cos 2 = 2\cos 2$







18) $\lim_{x\to 0} (1+ax)^{\frac{b}{x}} =$

1) e^{ab}

2) e^{a+b}

4) e

3) $e^{(a^b)}$

KEA Solution:



 $\lim_{x \to 0} (1 + ax)^{\frac{b}{x}}$ $= \lim_{x \to 0} \left[(1 + ax)^{\frac{1}{ax}} \right]^{ab}$ $= e^{ab}$







19) If
$$f(x) = \begin{cases} \frac{x^5-32}{x-2} & \text{when } x \neq 2 \\ k & \text{when } x = 2 \end{cases}$$

hence
$$f(x)$$
 is continuous at $x = 2$, if $k =$

1) 16	2) 80
3) 32	4) 8





Solution: Since f(x) is continuous at x = 2,

$f(2) = \lim_{x \to \infty} f(x)$	f(x)
$K = \lim_{x \to 2}$	$\frac{x^5-32}{x-2}$
$= \lim_{x \to 2}$	$\frac{x^5-2^5}{x-2}$
	= <mark>5 (2)</mark> ^{5–1}
K =	= <mark>80</mark>

Answer: 2





20) The function $f(x) = \frac{\log(1 + ax) - \log(1 + bx)}{x}$

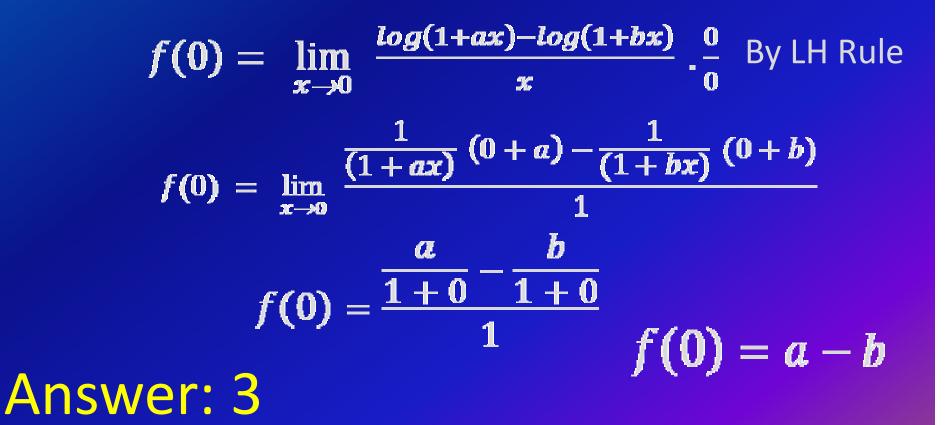
is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0 is

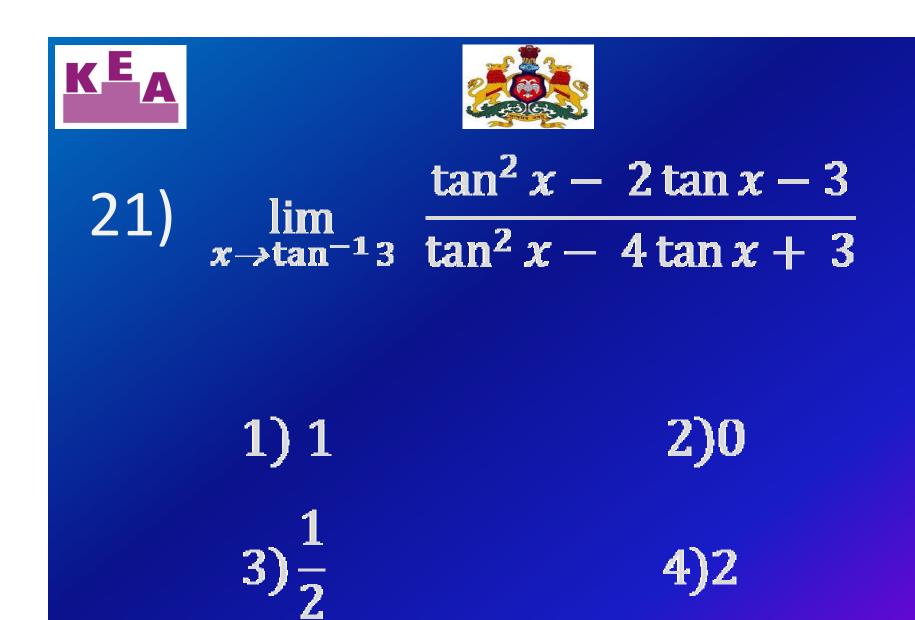
1) loga + logb	2) 2
3) a – b	4) a + b





Solution: Since f(x) is continuous at x = 0 $f(0) = \lim_{x \to 0} f(x)$









$$\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3} \cdot \frac{0}{0} by LH Rule$$

$$= \lim_{\tan x \to 3} \frac{2\tan x \sec^2 x - 2\sec^2 x - 0}{2\tan x \sec^2 x - 4\sec^2 x + 0}$$

$$= \lim_{\tan x \to 3} \frac{2\sec^2 x [\tan x - 1]}{2\sec^2 x [\tan x - 2]}$$

$$= \frac{3-1}{3-2} = \frac{2}{1} = 2$$

Answer: 4





22) If the function $f(x) = \begin{cases} \frac{1 - \cos x}{x^2} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$

is continuous at x = 0 then the value of k is _____







Since f(x) is continuous at x = 0

$$\therefore f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1 - \cos x}{x^2} \left(\frac{0}{0}\right) \text{ by LH Rule}$$
$$K = \lim_{x \to 0} \frac{0 - (-\sin x)}{2x} = \lim_{x \to 0} \frac{1}{2} \cdot \frac{\sin x}{x} = \frac{1}{2}$$

Answer: 3





23) If
$$f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x} & \text{when } x \neq 0\\ k + \frac{1}{2} & \text{when } x = 0 \end{cases}$$

is continuous at x = 0 then the value of K is

1)
$$\frac{1}{2}$$
 2) 2
3) - 2 4) 1





Solution: Since f(x) is continuous at x = 0, $f(0) = \lim_{x \to 0} f(x)$ $K + \frac{1}{2} = \lim_{x \to 0} \frac{\sin 5 x}{x^2 + 2x}, \frac{0}{0}$ by LH Rule $= \lim_{x \to 0} \frac{\cos 5x.5}{2x+2}$ $K + \frac{1}{2} = \frac{1 \times 5}{0+2}$.: $K = \frac{5}{2} - \frac{1}{2}$ K=2Answer: 2





24) If
$$f(x) = \begin{cases} \frac{\log(1+ax)-\log(1-bx)}{x} & \text{when } x \neq 0\\ k & \text{when } x = 0 \end{cases}$$

and f(x) is continuous at x = 0, then the value of K is

a - b
 a + b
 log a + log b
 log a - log b





Solution: Since f(x) is continuous at x = 0, $f(0) = \lim_{x \to 0} f(x)$ $K = \lim_{x \to 0} \frac{\log(1+ax) - \log(1-bx)}{x} , \frac{0}{0} by LH Rule$ $K = \lim_{x \to 0} \frac{\frac{1}{(1+ax)} (0+a) - \frac{1}{(1-bx)} (0-b)}{1}$ $K = \frac{\frac{a}{1+0} + \frac{b}{1-0}}{\ldots} K = a + b$ Answer: 2





$$25)If f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & \text{for } x < 0\\ K & \text{for } x = 0\\ \frac{\sqrt{x}}{\sqrt{16-\sqrt{x}-4}} & \text{for } x > 0 \end{cases}$$

then the value of K for which f(x)is continuous at x = 0 is

 1) 5
 2) 8

 3) 4
 4) 3





Solution: Since f(x) is continuous at x = 0, LHL = RHL = f(0) $\therefore f(0) = LHL$ $K = \lim_{x \to 0} \frac{1 - \cos 4x}{x^2}, \frac{0}{0}, by LH rule$ $= \lim_{x \to 0} \frac{0 - (-\sin 4x) \cdot 4}{2x}$ $= \lim_{x \to 0} \frac{\sin 4x}{4x} X 4 X \frac{4}{2}$ $x \rightarrow 0$ = 1 X 4 X 2 = 8K = 8Answer: 2





26) The function
$$f(x) = \frac{(2^x - 1)^2}{\sin x \cdot \log(1 + x)}$$

is not defined at x = 0 what value should be assigned to f(0) so that f(x) becomes continuous at x = 0

 1) log 4 2) 2

 3) 1
 4) $(log 2)^2$





Since f(x) is continuous at x = 0, $f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(2^x - 1)^2}{\sin x \cdot \log(1 + x)}$ $= \lim_{x \to 0} \frac{(2^{x} - 1)^{2}}{x \cdot \frac{\sin x}{x} \cdot \log(1 + x)}$





$$= \lim_{x \to 0} \frac{(2^{x} - 1)^{2}}{x^{2} \cdot 1 \cdot \frac{1}{x} \log(1 + x)}$$
$$= \lim_{x \to 1} \frac{(2^{x} - 1)^{2}}{x}, \frac{1}{\log(1 + x)^{2}}$$

$$= (\log 2)^2 \cdot \frac{1}{\log_e e}$$

 $= (\log 2)^2$

Answer: 4

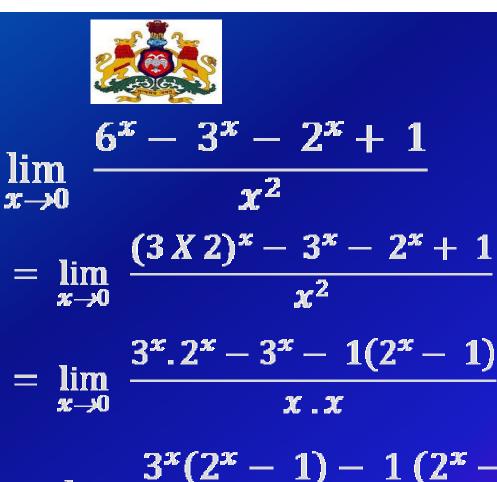




 $\frac{6^x - 3^x - 2^x + 1}{x^2}$ 27) $\lim_{x \to 0}$

1) $\log\left(\frac{2}{3}\right)$ 2) $\log 2 + \log 3$ log 2 log 3 4) 3) $(\log 2) (\log 3)$





$$= \lim_{x \to 0} \frac{3^{x}(2^{x}-1)-1(2^{x}-1)}{x \cdot x}$$
$$= \lim_{x \to 0} \left(\frac{2^{x}-1}{x}\right) \left(\frac{3^{x}-1}{x}\right)$$

 $= (\log 2). (\log 3)$

Answer: 2





28) $\lim_{x \to 2} \frac{2x^2 - 5x + 2}{x^2 - 3x + 2} =$



2)2

4)8





$$\lim_{x \to 2} \frac{2x^2 - 5x + 2}{x^2 - 3x + 2}, \quad \frac{0}{0}, \quad by \ L \ H \ Rule$$
$$= \lim_{x \to 2} \frac{4x - 5}{2x - 3}$$
$$= \frac{8 - 5}{4 - 3}$$
$$= 3$$
Answer: 3





29) $\lim_{\theta \to 0} \frac{\sin 2\theta \cdot \sin 3\theta}{3 \theta \cdot \tan 4\theta} =$

1) $\frac{2}{3}$ 2) $\frac{3}{2}$ 3) 2 4) $\frac{1}{2}$





 $\lim_{\theta \to 0} \frac{\sin 2\theta \cdot \sin 3\theta}{3\theta \cdot \tan 4\theta}$

 $= \lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta} \cdot 2\theta \cdot \frac{\sin 3\theta}{3\theta} \cdot \frac{4\theta}{\tan 4\theta} \times \frac{1}{4\theta}$ $= 1 \times 2 \times 1 \times 1 \times \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$







30) If f(x) is an even function & f'(x)exists then f'(e) + f'(-e) is

1) > 02) = 03) ≥ 0 4) < 0</td>





f(x) is an even function $\therefore f(-x) = f(x)$ f'(-x)(-1) = f'(x)f'(x) + f'(-x) = 0f'(e) + f'(-e) = 0







31) If f(x) is a function such that f(x) + f'(x) = 0 and $g(x) = [f(x)]^2 + [f'(x)]^2 \&$ g(3) = 8 then g(8) =1)0 2)5 3) 8 4) 3





Solution: $g(x) = [f(x)]^2 + [f'(x)]^2$ g'(x) = 2[f(x)]f'(x) + 2[f'(x)]f''(x)= 2f'(x)[f(x) + f''(x)]= 2f'(x).0g'(x)=0 $\therefore g(x)$ is constant $\therefore g(x) = 8, \quad g(3) = 8,$ Thus g(8) = 8, Answer: 3





32) Let f(x + y) = f(x)f(y)for all x and y. Suppose f(5) = 2, f'(0) = 3 then f'(5) is 1)4 2) 3 3)8 4)6





Solution: f(x+y) = f(x)f(y)Put y = 5, f(x+5) = f(x)f(5)f'(x+5)(1+0) = f'(x)f(5)Put x = 0, f'(0+5) = f'(0)f(5)f'(5) = 3X2= 6Answer: 4





 $\lim_{n\to\infty} (3^n + 4^n)^{\frac{1}{n}} =$

1) 4 **2)**3

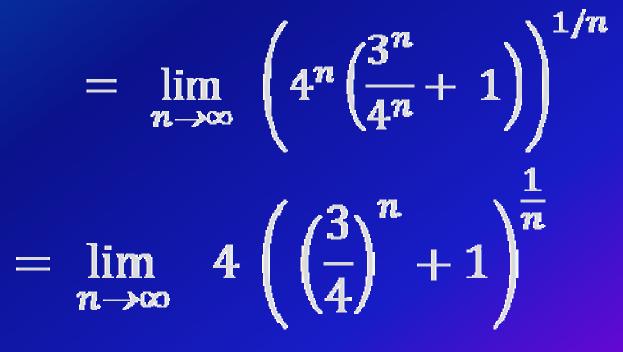
3) e

4)∞





Solution: $\lim_{n \to \infty} (3^n + 4^n)^{1/n}$



 $= 4 (0+1)^0 = 4$







34) If f(1) = 1 & f(n+1) = 2f(n) + 1for $n \geq 1$ then f(n) =1) $3^n - 2$ $(2)2^{n}-1$ 3) $\frac{1}{2}(2^n + 1)$ 4) 2n - 1





f(1) = 1, f(n+1) = 2 f(n) + 1f(1+1) = 2 f(1) + 1 $f(2) = 2 + 1 = 3 = 2^2 - 1$ Similarly $f(3) = 2^3 - 1$ $\therefore f(n) = 2^n - 1$

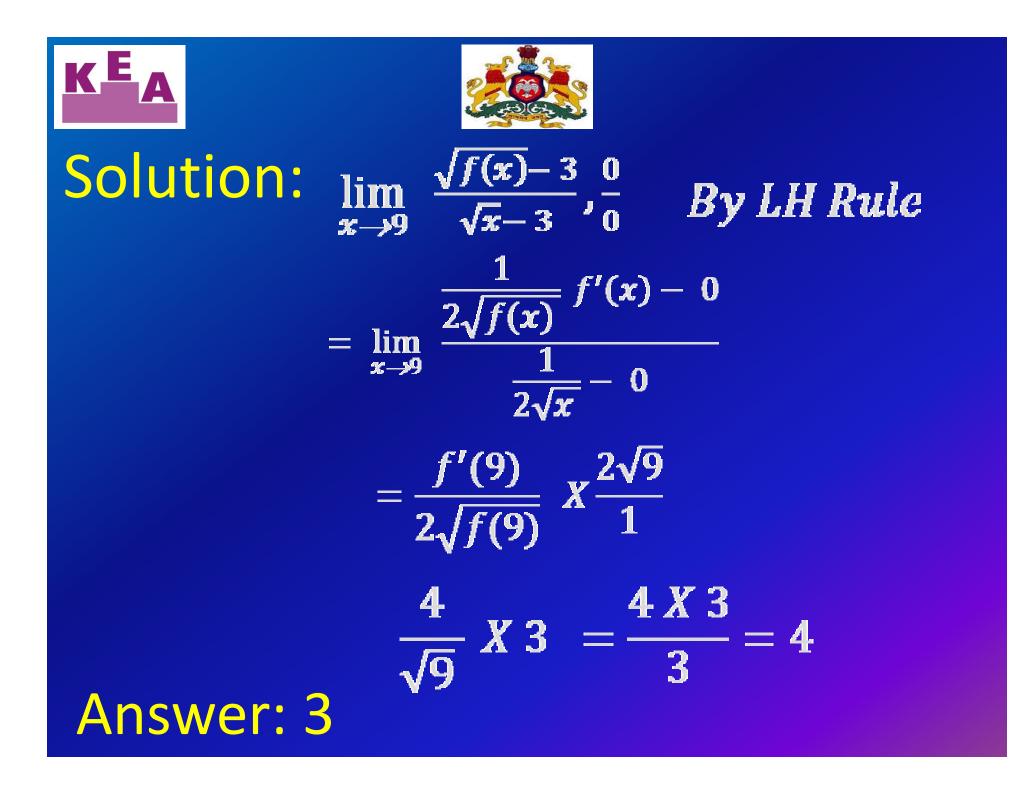






35) If f(9) = 9 and f'(9) = 4then $\lim_{x \to 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} =$

1) 2 2)3 3) 4 4) $\frac{1}{2}$







 $\lim_{n \to \infty} \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} - \dots - \frac{1}{4^n} \right)$ 36) E







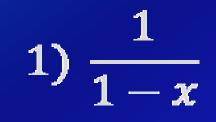
Solution: $\left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} - \frac{1}{4^n}\right)_{is in G.P}$ lim $n \rightarrow \infty$ $a = -\overline{4}$ $r = \overline{4}$ $\frac{4}{1} = \frac{4}{3} = \frac{3}{3}$ $=s_{\infty}=\frac{a}{1-r}$ 1





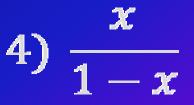


$37) \lim_{n\to\infty} \left(1-\frac{1}{x}+\frac{1}{x^2}+\dots to \ n \ terms\right) =$



2) $\frac{1}{1+x}$

3) $\frac{x}{1+x}$







$$\lim_{n \to \infty} \left(1 - \frac{1}{x} + \frac{1}{x^2} + \dots \text{ to n terms} \right)$$
$$= \lim_{n \to \infty} \left(1 + \left(\frac{-1}{x} \right) + \left(\frac{-1}{x} \right)^2 + \dots \text{ to n terms} \right)$$

Is in G P,
$$a = 1, r = \frac{-1}{x}$$

$$= s_{\infty} = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{x}\right)} = \frac{1}{1+\frac{1}{x}} = \frac{1}{1+\frac{1}{x}} = \frac{x}{1+\frac{1}{x}}$$

Answer: 3





38)
$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{4^x + 4^{-x} - 2} =$$

1) $\frac{1}{\log 16}$

 $2) - 2 \log 4$

3) $(log 4)^{-2}$ 4) 2 log 4

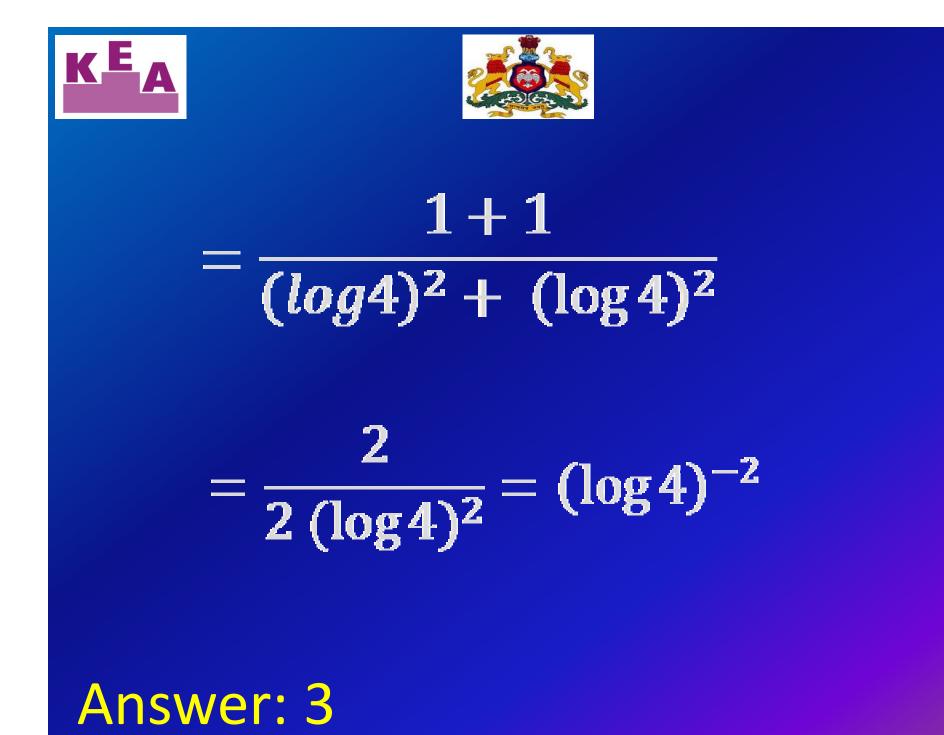




$$\lim_{x \to 0} \frac{e^{x} + e^{-x} - 2}{4^{x} + 4^{-x} - 2}, \quad \frac{0}{0} \quad by \ LH \ rule$$

$$= \lim_{x \to 0} \frac{e^{x} + e^{-x} (-1)}{4^{x} \log 4 + 4^{-x} \log 4(-1)}, \frac{0}{0}$$
Again by L H Rule

$$= \lim_{x \to 0} \frac{e^x - e^{-x} (-1)}{4^x \log 4 \cdot \log 4 - 4^{-x} \log 4 \cdot \log 4 (-1)}$$







 $\frac{3.2^{n+1}-4.5^{n+1}}{5.2^n+7.5^n}$ 39) lim $n \rightarrow \infty$

1) $\frac{3}{5}$ 2) $-\frac{4}{5}$ 3) $-\frac{20}{7}$ 4) $\frac{20}{7}$





$$\lim_{n \to \infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n}$$

$$= \lim_{n \to \infty} \frac{3.2^n \cdot 2^1 - 4.5^n \cdot 5}{5^n \left(5.\frac{2^n}{5^n} + 7\right)}$$

$$= \lim_{n \to \infty} \frac{5^n \left(6.\frac{2^n}{5^n} - 20\right)}{5^n \left(5.\frac{2^n}{5^n} + 7\right)}$$

$$= \frac{6 \times 0 - 20}{0 + 7} = -\frac{20}{7}$$

Answer: 3





40) If f(a) = 2, f'(a) = 1, g(a) = -3, g'(a) = -1 then $\lim_{x \to a} \frac{f(a)g(x) - f(x)g(a)}{x - a} =$ $x \rightarrow a$ 1) 6 2)1 $(3) - 1 \quad (4) - 5$

