MATHEMATICS

NUMBER THEORY

MATHEMATICS

1) If the number of positive divisors of the integer $2^{k+5} . 15^{3}, is\ 48$ then k=

1) 1   2) -1   3) -3   4) 4

MATHEMATICS

$$2^{k+5} . 15^{3} = 2^{k+6} . 3^{3} . 5^{3}$$

$$(k + 6).4.4 = 48$$

$$\Rightarrow k = -3$$

ANS: $(3) : -3$

MATHEMATICS

2) if p and q are two different primes, then which of the following is always not prime

1) p+q

2) p-q

3) pq

4) 2p-3q

MATHEMATICS

Since $p|pq$ and $q|pq$ and

$p \neq q \neq pq$, pq is always not

prime.

ANS: (3) : pq

MATHEMATICS

3) if a,b,x,y are integers ,and ax+by=1, then which of the following is not true

1) (a,b)=1

2) (a,y)=1

3) (x,b)=1

4) (a,x)=1

MATHEMATICS

ax+by is a linear combination

of a,b or a,y or x,b but not

a,x and hence $(a,x) \neq 1$

ANS:(4):$(a,x) = 1$

MATHEMATICS

4)If the sum of all positive divisors of the integer

$$2^{2k}.3, is, 2044\ , then\ , k =$$

1)8 2) 4) 3)10 4)12

MATHEMATICS

$$[\frac{2^{2k+1}-1}{2-1}][\frac{3^{1+1}-1}{3-1}]=2044$$

$$\Rightarrow (2^{2k+1}-1)4=2044$$

$$\Rightarrow 2^{2k+1}=512 \Rightarrow 2k+1=9 \Rightarrow k=4$$

ANS:(2): 4

MATHEMATICS

5)if p>1, is the least +ve divisor of any integer a, then which of the following is true

1) P is composite,p≤ $\sqrt{a}$
2) p is prime,p≤$\sqrt{a}$
3) P is prime ,p≤a
4) p is composite ,p>a

MATHEMATICS

ANS:(2):

P is always,prime and $\leq \sqrt{a}$

MATHEMATICS

6) If a is an integer such that, $a^2 - 4$ is prime, then, $a^2 - 4$ =

1) a+2 2) a-2 3) a 4) 2n $\forall n \in I$

MATHEMATICS

Since $a^2 - 4 = (a+2)(a-2)$
and it is prime,we have
either a+2=1 or a-2 =1
but a+2≠1.hence a-2=1

ANS:(1): a+2

MATHEMATICS

7) if a,b,r ,q are any integers such that $a<b$ and $b=aq+r$,then which of the following is true

1) $0 \le r \le a$

2) $0 \le r \le b$

3) $0 \le r < b$

4) $0 \le r < a$

MATHEMATICS

since $a<b$, when b is divided by a, the remainder r is always ≥ 0 and $<$ the divisor a

ANS:(4): $0 \leq r < a$

MATHEMATICS

8) if a|b and a|c, which of the following is not true.

1) $a \mid b^2 + c^2$

2) $b^2 - c^2 = aq + r, r = 0$

3) (a,b)=a

4) b+c=aq+r, r>0

MATHEMATICS

$a|b,\ a|c \Rightarrow a\ |b^2+c^2, a|b^2-c^2$

$a|b \Rightarrow (a,b)=a$

$a|b+c \Rightarrow b+c=aq$, since $r=0$

ANS:(4): $b=aq+r, r>0$

9)On the set of integers
the relation “divisibility”
is

1)reflexive and transitive
2)reflexive and symmetric
3)symmetric and transitive
4)transitive only

MATHEMATICS

Divisibility is transitive only, since

$a|b$ and $b|c \Rightarrow a|c$

ANS:(4): Transitive only

10) If $(a+b,a-b)=1$, then $(a,b)=$

1) 1 2) 2 3) a 4)4

MATHEMATICS

$$(a+b, a-b) = 1$$

$$\Rightarrow (a+b)x + (a-b)y = 1, \quad x, y \in I$$

$$\Rightarrow (x+y)a + (x-y)b = 1$$

$$\Rightarrow (a,b) = 1$$

ANS:(1): 1

MATHEMATICS

11) Which of the following is not true

1) $a|b \Rightarrow ac|bc$

2) $ac|bc \Rightarrow a|b$

3) $a|b$ and $b|a \Rightarrow a = \pm b$

4) $a|a \; \forall \; a \in I$

MATHEMATICS

$a|b \Rightarrow b = ak \Rightarrow bc = ack \Rightarrow ac|bc.$

$a|a$ only when $a \neq o.$

ANS :(4): $a|a \; \forall \; a \in I$

MATHEMATICS

12) The product of “r” consecutive integers is divisible by,

1) $(r+1)!$

2) $r!$

3) $(r-1)!$

4) $3r!$

MATHEMATICS

The product is always divisible by r!

ANS:(2): r!

13) If n is any integer, then, $(n^2-9)(n)(n^2-4)(n^2-1)$, is divisible by

1) 7!   2) 8!   3) 10!   4) 11!

MATHEMATICS

$(n-3)(n-2)(n-1)n(n+1)(n+2)(n+3)$
this is a product of 7 consecutive integers, hence divisible by 7!

ANS:(1): 7!

MATHEMATICS

14) $\forall n$, $n(2n-1)(2n+1)(2n+2)(2n+3)$ is divisible by,

1) 60   2) 120   3) 40   4) $2n \; \forall \; n$

MATHEMATICS

(2n-1) 2n(2n+1)(2n+2)(2n+3) is divisible by 5!=120 and hence, the given integer is divisible by 60

ANS:(1): 60

MATHEMATICS

15) The integer, $\left(49^{2}-4\right)\left(49^{3}-49\right)$ is divisible by,

1) 6! 2) 7! 3) 5! 4) 9!

MATHEMATICS

$$=49\left(49^2-4\right)\left(49^2-1\right)$$

$$=49(49-2)(49+2)(49-1)(49+1)$$

$$=49.47.51.48.50$$

= which is divisible by 5!

ANS:(3): 5!

MATHEMATICS

16)If a is the unit digit and b is the ten's digit in 1!+3!+5!+-----, then (ab,a)=

1)7 2)14 3)28 4)1

we know that $\forall\, n \geq 5$
the unit digit in n! is 0,
and $\forall\, n \geq 10$,the ten's digit
in n! is also 0. Hence a=7,b=4
$\Rightarrow$ (ab,a)=(28,7)=7

ANS:(1): 7

MATHEMATICS

17) On the set of integers the congruence relation is

1) not reflexive but symmetric
2) not symmetric but reflexive
3) transitive only
4) equivalence relation

MATHEMATICS

congruence is an equivalence relation on the set of integers, since it is reflexive,symmetric and transitive.

ANS:(4): is an eq relation

MATHEMATICS

18) If $xa \equiv xb \pmod{n}$, then which of the following is true.

1) $a \equiv b \pmod{n}$
2) $a \equiv b \pmod{n}$, if $(x, ab) = 1$
3) $a \equiv b \pmod{n}$, if $(ab, n) = 1$
4) $a \equiv b \pmod{n}$, if $(x, n) = 1$

$xa \equiv xb \pmod{n}$

$\Rightarrow n|xa-xb \Rightarrow n|x(a-b)$

but $n|a-b$, only when

$(x,n)=1$

ANS:(4): $a \equiv b \pmod{n}$, if $(x,n)=1$

19) The remainder, when $(1!)^2+(2!)^2+(3!)^2+---(100!)^2$ is divided by (10^2) is,

1) 28 2) 17 3) 14 4) 23

MATHEMATICS

$5! = 120, (5!)^2 = 14400$

$\forall n \geq 5$, the unit and ten's digits in $(n!)^2$ are 0.

hence,the last two digits will be 17,which is the remainder.

ANS:(2): 17

MATHEMATICS

20)The unit digit in (194)! + 7^{194} is

1) 2   2) 6   3)8   4)4

MATHEMATICS

The unit digit in 194! is 3.

$$7^2 = 49 \equiv -1 (\text{mod } 10)$$

$$7^{194} \equiv 1 (\text{mod } 10)$$

$\therefore$ the remainder and hence the unit digit is 4.

ANS:(4): 4

MATHEMATICS

21) If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then the maximum value of n is,

1) a-b+c-d 2) ac-bd

3) (ac,bd) 4) (a-b,c-d)

MATHEMATICS

$n|(a-b)$ and $n|(c-d)$, and

n is maximum

$\Rightarrow n=(a-b, c-d)$

ANS:(4): $(a-b, c-d)$

22) The least +ve integer x, such that, $2^{2010} \equiv 3x \pmod{5}$,is

1) 1 2) 2 3) 3 4) 4

MATHEMATICS

both 2^{2010} and 3x, will leave the same remainder,when divided by 5.

$$2^2 = 4 \equiv -1 (\mathrm{mod} 5) \Rightarrow 2^{2010} \equiv -1 (\mathrm{mod} 5)$$

$$\therefore 2^{2010} \equiv 4 (\mathrm{mod} 5) \Rightarrow 3x \equiv 4 (\mathrm{mod}\, 5)$$

hence least x=3. ANS:(4):3

MATHEMATICS

23) The incongruent solutions of $ax \equiv b \pmod{n}$, if exist, belong to the interval,

1) $[0, n-1]$
2) $(0, n-1)$
3) $(1, n)$
4) $[0, n]$

The incongruent solutions will lie in the closed inteval [0,n-1]

i.e $0 \leq x \leq n-1$

ANS:(1): [0,n-1]

MATHEMATICS

24) if p,q,r are the only solutions of $ax \equiv b \pmod{12}$, which lie in [0,11], and $at \equiv b \pmod{12}$, is also true, then which of the following is not true.

1) $(a,12) \neq t$ 2) $p \equiv q \pmod{12}$

3) $q \equiv r \pmod{12}$ 4) $t \equiv p$ or q or $r \pmod{12}$

MATHEMATICS

$(a,12)=3$ and $3|12$.

Incongruent solutions are not

Congruent among themselves.

Since $t \notin [0,11]$,

$t \equiv p$ or q or $r \pmod{12}$

ANS:(3): $q \equiv r \pmod{12}$

MATHEMATICS

25) The linear congruence $ax \equiv b \pmod{n}$, has 'd' number of incongruent solutions if,

1) $(a,b)=d, d|n$ 2) $(b,n)=d, d|a$

3) $(a,n)=d, d|b$ 4) $(a,n)=(b,n)=d$

MATHEMATICS

(a,n)=d and d|b

$\Rightarrow$ there are ‘d’ number incongruent solutions.

ANS:(3): (a,n)=d,d|b

MATHEMATICS

26) Which of the following has no solution.

1) $3x \equiv 7 \pmod{9}$ 2) $5x \equiv 3 \pmod{2}$

3) $12x \equiv 8 \pmod{4}$ 4) $6x \equiv 10 \pmod{5}$

MATHEMATICS

$3x \equiv 7 \pmod 9$ has no solution,

$\because$ $(3,9)=3$ and $3 \nmid 7$.

ANS:(1): $3x \equiv 7 \pmod 9$

27)The total number of incongruent solutions of $45x \equiv 30 \pmod{10}$,is

1)3 2)5 3)10 4)8

MATHEMATICS

(45,10)=5 and 5|30

hence there are 5

incongruent solutions in [0,9].

ANS:(2): 5

28)The number of solutoins of $6x \equiv 5(\text{mod}12)$, which lie in [0,11] is,

1)0 2)1 3)2 4)6

$(6,12)=6$ and $6 \nmid 5$. Hence there is no solution.

ANS:(1): 0

MATHEMATICS

29) If $a \equiv b \pmod{12}$, which of the following is not true.

1) $a \equiv b \pmod{6}$ 2) $a \equiv b \pmod{4}$

3) $a \equiv b \pmod{3}$ 4) $a \equiv b \pmod{10}$

MATHEMATICS

$a \equiv b \pmod{12} \Rightarrow 12 | a-b$. Also any +ve divisor of 12 also divides $(a-b)$. But $10 \nmid 12$.

$\therefore 10 \nmid a-b \Rightarrow a \not\equiv b \pmod{10}$

ANS: (4): $a \not\equiv b \pmod{10}$

MATHEMATICS

30) The digit in the unit place of $11^{121} \times 3^{21}$ is,

1)1 2)2 3)3 4)4

MATHEMATICS

$$11 \equiv 1 (\text{mod} 10) \Rightarrow 11^{121} \equiv 1 (\text{mod } 10)$$

$$3^2 \equiv -1 (\text{mod} 10) \Rightarrow 3^{20} \equiv 1 (\text{mod} 10)$$

$$\Rightarrow 3^{21} \equiv 3 (\text{mod} 10)$$

$$\therefore 11^{121} . 3^{21} \equiv 3 (\text{mod} 10)$$

ANS :(3):3

MATHEMATICS

31)If a|b+c and a|b-c,then which of the following is not true.

1) $b^2 \equiv c^2 (\mathrm{mod}\, a^2)$ 2) $b^4 \equiv c^4 (\mathrm{mod}\, a^2)$

3) $b^8 \equiv c^8 (\mathrm{mod}\, a^2)$ 4) $b^3 \equiv c^3 (\mathrm{mod}\, a^2)$

MATHEMATICS

$$a^2 \mid b^2 - c^2 \Rightarrow b^2 \equiv c^2 \pmod{a^2}$$

$$\Rightarrow b^4 \equiv c^4 \pmod{a^2}$$

$$\Rightarrow b^8 \equiv c^8 \pmod{a^2}$$

$$but, \not\Rightarrow b^3 \equiv c^3 \pmod{a^2}$$

ANS: (4) : $b^3 \equiv c^3 \pmod{a^2}$

MATHEMATICS

32) If $1415! - 2^{99} \equiv x \pmod{7}$, then, the least +ve value of x is,

1) 1 2) 6 3) 0 4) 3

MATHEMATICS

$7|1415! \Rightarrow 1415! \equiv 0 \pmod 7.$

$2^3 = 8 \equiv 1 \pmod 7 \Rightarrow 2^{99} \equiv 1 \pmod 7$

$\therefore 1415! - 2^{99} \equiv -1 \pmod 7$

$\equiv 6 \pmod 7$

ANS : (2) : 6

MATHEMATICS

33) The remainder when 5^{75} is divided by 126 is,

1) 125   2) 121   3) 123   4) 122

MATHEMATICS

$$5^3 = 125 \equiv -1 \pmod{126}$$

$$\Rightarrow 5^{75} \equiv -1 \pmod{126}$$

$$\equiv 125 \pmod{126}$$

ANS : (1) : 125

MATHEMATICS

34)Find the unit digit in 17^{50}

1)3 2)9 3)1 4)7

MATHEMATICS

$$17 \equiv 7(\bmod 10) \Rightarrow 17^{50} \equiv 7^{50}(\bmod 10)$$

$$but, 7^{2} \equiv -1(\bmod 10) \Rightarrow 7^{50} \equiv -1(\bmod 10)$$
$$\equiv 9(\bmod 10)$$

$$\therefore 17^{50} \equiv 9(\bmod 10)$$

ANS : (2) : 9

35)The number of solutions of $5x \equiv 8 \pmod{6}$, in [0,5], is

1)1 2)2 3)3 4)4

MATHEMATICS

(5,6)=1.Hence there is only one solution in [0,5].

ANS:(1): 1

MATHEMATICS

36) If $385 \equiv 21 (\bmod n)$ and $587 \equiv 167 (\bmod n)$, then the maximum value of n is,

1) 156 2) 56 3) 32 4) 28

MATHEMATICS

$n|(385-21)$ and $n|(587-167)$

$n|364$ and $n|420$

$n=(364,420)=28$

ANS:(4): 28