MATHEMATICS

Group Theory

Already we are familiar with notations;

- N → Set of natural numbers
- Z → Set of integers
- Z_o → Set of non-zero integers
- Z+ → Set of positive integers
- Z_- → Set of negative integers
- Q → Set of rational numbers
- Q_o → Set of non-zero rational numbers
- Q+ → Set of positive rational numbers

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- Q-→ Set of negative rational numbers
- Q^1→ Set of irrational numbers
- R→ Set of real numbers
- R_o→ Set of non-zero real numbers
- R+→ Set of positive reals
- R_-→ Set of negative reals
- C→ Set of complex numbers
- C_o→ Set of non-zero complex numbers.

1) Which of the following is the binary operation.

1) Addition on the set Z_0.

2) Subtraction on the set N.

3) Division on the set Q.

4)Division on the set R^+.

Sol:

In (1) 2+ (– 2) = 0, $0 \notin Z_0$ $\therefore$ Addition is not the binary operation.

(2) 5 – 8 = -3, $-3 \notin N$ $\therefore$ Subtraction is not binary operation.

(3) $2 \div 0 = \frac{2}{0} = \infty$ $\infty \notin Q$ $\therefore$ Division is not binary operation.

(4) If we divide any two +ve real numbers then we get only a +ve real number.

$\therefore$ Division is the binary operation.

Ans is (4)

2) In the set R which of the following is not the binary operation under * $\forall\, a, b \in R$

1) $a * b = \sqrt{ab}$

2) $a * b = a + b - 10$

3) $a * b = a^2 + b^2$

4) $a * b = ab + 5$

Sol:

In (1) Let a = 4 & b = -1 then 4 * -1 = $\sqrt{4(-1)} = \sqrt{-4} = \pm 2i$

$$Here\ \pm 2i \notin R$$

$$\therefore * \ is\ not\ binary\ operation$$

Ans is (1)

3) Which of the following defines a binary operation on R ?

1) $a * b = a^b$

2) $a * b = \sqrt{ab}$

3) $a * b = \frac{a}{2b}$

4) $a * b = \frac{ab}{a^2 + b^2 + 1}$

Sol : $In(1), let a = 0, b = 0 then 0^0 is not defined, 0^0 \notin R$

In 4) $\forall$ a, b, $\in$ R, $a^2 + b^2$ *is always +ve real*

$\therefore a^2 + b^2 + 1$ *is always +ve real*

$\therefore \quad a * b \in R$

Ans is (4)

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4) **Which of the following is a binary operation.**

1) The cross product of 2- dimensional vectors.
2) The dot product of 3- dimensional vectors.
3) The cross product of 3-D vectors
4) The dot product of 3 vectors.

$Sol: \ In \ (1) \ If \ \vec{a} \ \& \ \vec{b} \ are vectors \ then \ \vec{a} \times \vec{b} \ is a vector$

$\perp^{r} \ to both \ \vec{a} \ \& \ \vec{b} \ and \ it \ is \ not \ in \ the \ plane \ of$

$\vec{a} \ \& \ \vec{b}.$

(2) *The dot product of* $\vec{a}$ & $\vec{b}$ *is a scalar*

$\therefore\ \vec{a}\,.\,\vec{b}$ *is not a vector.*

(3) $\vec{a} \times \vec{b}$ *is* $\perp^r$ *to both* $\vec{a}$ & $\vec{b}$ *and it belongs to* $3-D$ *space.*

4)

$\left(\vec{a}\,.\,\vec{b}\right).\,\vec{c}$ *is not defined and it is not a vector.*

Ans is (3)

5) In the set of irrationals $Q^{|}$ which one of the following signs satisfies closure law.

1) + 2) - 3) x 4) None of these

$$Sol: \quad In\ (1)\ \sqrt{2} + \left(-\sqrt{2}\right) = 0 \quad \&\ 0 \notin Q^{|}$$

$$In\left(2\right)\ \sqrt{2} - \sqrt{2} = 0 \quad \&\ 0 \notin Q^{|}$$

$$In\ \left(3\right) \sqrt{2} \times \sqrt{2} = 2 \quad \&\ 2 \notin Q^{|}$$

Ans is (4)

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6) **Which of the following satisfies the closure law under given condition on *.**

1) $a * b = \frac{a+b}{2} \quad on \quad N$

2) a* b = a-2b on N.

3) a * b = the smaller of (a + b) and (a –b) on N.

4) $a * b = \frac{a}{b} \quad on \quad Q^{+}$

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Sol : In (1) Let $a = 2$ & $b = 1$ then $2 * 1 = \frac{2+1}{2} = \frac{3}{2}$

$\frac{3}{2} \notin N$ $\therefore$ *Closure law fails*

In 2) Let a = 2 & b= 3 then 2 * 3 =2-(2)3 =-4 and -4 is not in the set N

$\therefore$ *Closure law fails*

In (3) Let a = 2 & b = 5 then 2 + 5 =7, 2 – 5 = -3
There fore, 2 * 5 = smaller of (2 + 5) and (2 -5) = -3

$\therefore$ 5 * 2 = -3 & $-3 \notin N$ $\therefore$ *Closure law fails*

In (4) For any a & b, $\frac{a}{b}$ is always +ve rationals

Ans is (4).

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7) Which of the following is not commutative under * in N.

1) a * b= 3a + 5b

2) a * b = a + b – 1

3) a * b = ab + 2

4) a * b = $a^2 + b^2$

$$Sol: \quad In\ (1)\ a*b = 3a + 5b, \qquad b*a = 3b + 5a$$

$$\therefore 3a + 5b \neq 3b + 5a$$

Ans is (1)

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8) On the set R which of the following is commutative under *

1) $a * b = |a - b|$

2) $a * b = a - b$

3) $a * b = \dfrac{a}{2b}$

4) $a * b = \dfrac{a+2}{b}$

$Sol: In(1)\ \ a * b = |a - b|, \qquad b * a = |b - a| = |a - b|$

$\therefore \ \ a * b = b * a$

Ans is (1)

9) Which of the following defines a commutative binary operation?

1) Scalar product of vectors
2) Division on the set of all square roots of unity.
3) In Z, $a * b = 4a - 3b + 10$
4) Cross product of two vectors.

$Sol:$ $In\ 1)$ $dot\ product\ of\ two\ vectors\ is\ not\ the\ binary\ operation$

$In\ 2)$

$\div$	1	-1
1	1	-1
-1	-1	1

Binary operation definition and commutative laws are satisfied.

Ans is (2)

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10) On the set R^+ under * which of the following are not associative.

1) $a * b = a^b$

2) $a * b = a + b + 3$

3) $a * b = \frac{ab}{10}$

4) $a * b = a+b-100$

$$Sol: \quad In\ 1)\ (a*b)*c = a^b * c = \left(a^b\right)^c = a^{bc} \ldots\ldots (1)$$

$$a*(b*c) = a*b^c = a^{\left(b^c\right)} \ldots\ldots (2)$$

$$But\ a^{bc} \neq a^{\left(b^c\right)} \quad \therefore * \text{ is not associative}$$

Ans. is (1)

11) In set Q which one is associative under *

1) $a * b = a - 2b$

2) $a * b = a$

3) $a * b = a - b + 10$

4) $a * b = \frac{2a}{3b}$

$$Sol: In(2), (a * b) * c = a * c = a ----(1)$$

$$a * (b * c) = a * b = a ----(2)$$

$$(a * b) * c = a * (b * c) \ \forall a, b, c \in Q$$

$$\therefore * \text{ is associative}$$

Ans. is (2)

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12) In the group G= $\left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix}, \; x \in R_0 \right\}$ **Identity element (matrix) under multiplication is**

1) $\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$ 2) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 3) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ 4) $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

$$Sol: \quad Let \quad X = \begin{pmatrix} x & x \\ x & x \end{pmatrix} \; Y = \begin{pmatrix} y & y \\ y & y \end{pmatrix} \quad E = \begin{pmatrix} e & e \\ e & e \end{pmatrix}$$

From definition of identity axiom

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$X \times E = X$ then E identity matrix

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix} \begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$\begin{pmatrix} xe + xe & xe + xe \\ xe + xe & xe + xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

$$\begin{pmatrix} 2xe & 2xe \\ 2xe & 2xe \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$$

equating $2xe = x \quad \therefore e = \frac{1}{2}$

Substituting $E = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

Ans is (1)

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13) In the group Q_0 under * if $a * b = \frac{2ab}{3} \forall\ a, b \in Q_0$ **then the inverse of 4 is**

1) $\frac{16}{9}$ 2) $\frac{3}{2}$ 3) $\frac{2}{3}$ 4) $\frac{9}{16}$

Sol: From identity axiom; a * e = a

we get $\frac{2ae}{3} = a \Rightarrow e = \frac{3}{2}$

From Inverse axiom $a * a^{-1} = e \Rightarrow \frac{2aa^{-1}}{3} = \frac{3}{2} \Rightarrow a^{-1} = \frac{9}{4a}$

$put\ \ a = 4\ \ \ then\ \ \ 4^{-1} = \frac{9}{4(4)} = \frac{9}{16}$

ans is (4)

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14) In the Group $\left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix}, \ x \in R_0 \right\}$ under multiplication of matrices inverse of $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$ is

1) $\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

2) $\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$

3) $\begin{pmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}$

4) $\begin{pmatrix} 1/8 & 1/8 \\ 1/8 & 1/8 \end{pmatrix}$

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$$Sol:\ Let\quad X=\begin{pmatrix} x & x \\ x & x \end{pmatrix} \qquad X^{-1}=\begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$Already\ w.k.t.\quad E=\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$From\ \ defn.\ under\ \times,\ \ X\times X^{-1}=E$$

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} y & y \\ y & y \end{pmatrix}=\begin{pmatrix} e & e \\ e & e \end{pmatrix}$$

$$\begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}=\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$\text{equating} \quad 2xy = \frac{1}{2}, \qquad y = \frac{1}{4x}$$

$$\text{Put} \quad x = 2 \qquad y = \frac{1}{4(2)} = \frac{1}{8}$$

$$\text{Sub in } X^{-1} = \begin{pmatrix} y & y \\ y & y \end{pmatrix}$$

$$X^{-1} = \begin{pmatrix} 1/8 & 1/8 \\ 1/8 & 1/8 \end{pmatrix}$$

Ans is (4)

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15) In the group {1, 3, 5, 7} x mod 8 the value of

$$\left(3 \times 5^{-1}\right)^{-1} \text{ is}$$

1) 1 2) 3 3) 5 4) 7

Sol:

$\otimes_8$	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

$Identity\ is\ 1$

$5^{-1} = 5$

$\left(3\ X\ 5^{-1}\right)^{-1} = \left(3\ X\ 5\right)^{-1} = 7^{-1} = 7$

Ans is (4)

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16) *In the group* $\left\{ A(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} , \quad \theta \in R \right\}$ *the inverse of* $A(\pi/7)$ *under matrix multilication is*

1) $A(\pi/7)$

2) $A(-\pi/7)$

3) $A(-\pi/2)$

4) $A(6\pi/7)$

$Sol: \quad w.k.t. \ |A| = 1$

$$A\left(\frac{\pi}{7}\right) = \begin{pmatrix} \cos\pi/7 & \sin\pi/7 \\ -\sin\pi/7 & \cos\pi/7 \end{pmatrix}$$

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$$\therefore\ A^{-1}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\therefore\ A^{-1}\left(\frac{\pi}{7}\right) = \begin{pmatrix} \cos\pi/7 & -\sin\pi/7 \\ \sin\pi/7 & \cos\pi/7 \end{pmatrix} ---(1) \neq A\left(\frac{\pi}{7}\right)$$

$$\&\ A\left(\frac{-\pi}{7}\right) = \begin{pmatrix} \cos(-\pi/7) & -\sin(-\pi/7) \\ \sin(-\pi/7) & \cos(-\pi/7) \end{pmatrix}$$

$$A\left(\frac{-\pi}{7}\right) = \begin{pmatrix} \cos\pi/7 & \sin\pi/7 \\ -\sin\pi/7 & \cos\pi/7 \end{pmatrix} ----(2) = A^{-1}\left(\frac{\pi}{7}\right)$$

Ans is (2)

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17) In a group $\{Q - \{1\}\}$, if a * b = a + b - ab and $(x * 3^{-1} * 2) = 10$ then x =

1) 9 2) 19 3) -19 4) -9

Sol : Let us find 3^{-1}, From Identity axiom

$$a * e = a \Rightarrow \quad a + e - ae = a$$

$$e(1-a) = 0 \qquad \therefore \quad e = 0$$

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$$From\ \ Inverse\ \ axiom\ \ a * a^{-1} = e \Rightarrow a + a^{-1} - aa^{-1} = e$$

$$a + a^{-1} - aa^{-1} = 0 \qquad a^{-1}(1-a) = -a$$

$$\therefore a^{-1} = -\frac{a}{1-a} \qquad put\ \ a = 3,$$

$$3^{-1} = -\frac{3}{1-3} = \frac{3}{2} \qquad \therefore 3^{-1} = \frac{3}{2}$$

$$Now\ \ x * \left(3^{-1} * 2\right) = 10$$

$$\Rightarrow \quad x * \left(3/2 * 2\right) = 10 \quad Simplifying\ \ we\ \ get \qquad \therefore x = 19$$

Ans is(2)

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18) In the group { e, a, b} where e is an identity then $a^4\ b^5$ =

1) a   2) b   3) a^{-1} 4) b^{-1}

A group with 3 elements is always abelian.

$$a^4\ b^5 = a^4\ b^4\ b = (a\,b)^4\ b$$

$$Here\quad ab = e$$

$$= e^4 b = eb = b$$

.	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

Ans is (2)

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19) In an abelian group G, $\left(a\, b\, c^{2}\, b^{-1}\, a^{-1}\right)^{-1}$ **is**

1) c^{-2} 2) c^{-1} 3) acb 4) c

$$In\ \ an\ \ abelian\ \ group\ \left(a\ b\ c^{2}\ b^{-1}\ a^{-1}\right)^{-1}$$

$$= a^{-1} b^{-1} \left(c^{2}\right)^{-1} \left(b^{-1}\right)^{-1} \left(a^{-1}\right)^{-1} = a^{-1}\ b^{-1}\ c^{-2}\, b\ a$$

$$= \left(a^{-1}\, a\right)\left(b^{-1}\ b\right) c^{-2} = e\ e\, c^{-2} = c^{-2}$$

Ans is (1).

20) Which one of the following is a semi group under division.

1) Q 2) Z 3) C 4) {1, -1}

Sol: In 1) ,2), 3) If we take 0 in denominator we get infinity .therefore closure law fails

$\div$	1	-1
1	1	-1
-1	-1	1

In 4) From the table we can observe that Closure and Associative laws are satisfied

$$\therefore \{1, -1\} \text{ is a semi group}$$

Ans is (4)

21) Which of the following is false?

1) Every group is a monoid
2) Every monoid is a semi group
3) Every semi group is a monoid
4) Every monoid need not be a group

Sol:In 3) N is a semi group under + but it is not a monoid because under + , 0 is an identity and 0 $\notin$ N

For 4) Z is a monoid but it not a group under x

Every semi group need not be monoid

Ans is (3)

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22) On the set Q if a * b = a – b + 1 , $\forall\ a, b \in Q$

which one of the axiom is satisfied under *.

1) Identity axiom 2) Inverse axiom

3) Closure law 4) Commutative law.

Sol: For 1) Identity element does not exist.

If a * e = a = e * a then e identity

i.e. a * e = a & a = e * a

$a - e + 1 = a$	$a = e - a + 1$
$-e = -1$	$2a = e + 1$
$e = 1$	$e = 2a - 1$

Here e values are different .
Therefore Identity element does not exist.

2) Identity element does not exist implies inverse axiom is not satisfied.

3) Here if $a, b \in Q$ then, $a - b + 1 \in Q$ $\quad \forall\ a, b \in Q$

$\therefore$ **Ans is (3)**

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23) Which one of the following is not a group .

1) {0, 1, 2} + mod 3
2) {1, 2, 3, 4, 5} x mod 6
3) {0} + mod 3
4) {1, 2, 3, 4,} x mod 5

Sol; Construct the table for all the options then

$$In(2)\, 2 \times 3 = 6 = 0 \qquad O \notin set.$$

$$\therefore\ closure\ law\ fails$$

$$\therefore\ It\ is\ not\ a\ group$$

Ans is (2)

24) {-1, 0, 1} is not a multiplicative group because of failure of

1) Closure law

2) Associate law

3) Identity axiom

4) Inverse axiom

$\times$	-1	0	1
-1	1	0	-1
0	0	0	0
1	-1	0	1

$$\textit{Identity element is } 1$$

$$0^{-1} \textit{ does not exist}$$

$$\therefore \textit{Inverse axiom is not satisfied}$$

Ans is (4)

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25) For the square matrices of same order which of the following is true.

1) Matrix addition is not a group.
2) Matrix Multiplication is not a group
3) Matrix subtraction is a group.
4) Matrix division is a group

Ans. is (2)

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(26) The matrices $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

under $\times$ form

1) Infinite Semi Group

2) Infinite Group

3) Finite abelian Group

4) Infinite abelian Group.

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$$Let \quad A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad D = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\times$	A	B	C	D
A	A	B	C	D
B	B	A	D	C
C	C	D	A	B
D	D	C	B	A

Ans is (3).

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27) $\text{Let } G = \left\{ \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \theta \in R \right\}$ *is a group under X.*

The incorrect answer is

1) G is an infinite group

2) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ *is inverse of itself*

3) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ *is an element of G*

4) $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ *is an element of G*

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In (1), we know that set is a group.

In (2), If we find inverse of matrix we get it self.

In (4), when $\theta = 300^0$ we get this matrix.

In (3), $for\ \ any\ \theta,\ \cos\theta\ \&\ \sin\theta\ cannot$

$be\ equal\ \ to\ 1.$

$\therefore (3)\ is\ incorrect\ \ statement$

$$\therefore Ans\ \ is\,(3)$$

28) Which one is false in a group G

1)Identity is unique

2) Inverse is unique

3) Inverse of inverse of an element is itself

4) Inverse of an element is itself.

Sol; Inverse of an element need not be itself in a group

Ans is (4)

29) Which of the following is false?

1) In a group of even order, there exists an element other than identity which is its own inverse.

2) In an abelian group, every element is its own inverse.

3) In an abelian group G $(a\,b)^{-1} = a^{-1}\,b^{-1}\,\forall\, a, b \in G$

4) In an abelian group G $(a\,b)^{2} = a^{2}\,b^{2}\,\forall\, a, b \in G$

$Ans is (2)$

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30) Which of the following is a semi group under subtraction.

1) {0}   2) Z   3) Q_0 4) R+

Sol:

$-$	0
0	0

In 2),3),4) Associative law fails

In 1) $(0 - 0) - 0 = 0 - 0 = 0$
similarly $0 - (0 - 0) = 0 - 0 = 0$
A.L. Satisfies

Ans is (1)

31) In group (G, *) for some element $a \in G$

if $a^2 = e$ then

1) $a = \sqrt{e}$ 2) a = e 3) $a^{-1} = a$ 4) a = 0

Sol: Given $a^2 = e \Rightarrow a * a = e$

$a^{-1} * (a * a) = a^{-1} * e \Rightarrow e * a = a^{-1}$

therefore $a = a^{-1}$

Ans is (3).

32) Which one of the following is not true in the group G $\forall\, a, b \in G$

1) $ab = ac \Rightarrow b = c$

2) $ab = cb \Rightarrow a = c$

3) $(ab)^{-1} = b^{-1}a^{-1}$

4) $(ab)^{-1} = a^{-1}b^{-1}$

Ans is (4)

33) Which of the following is true in the group G.

1) if $a^{-1} = a$ then G is abelian

2) $(a\ b)^{-2} = a^{-2}\ b^{-2}\ \forall\ a, b \in G$

3) if $a^2=a$ then $a = 0$ or $a=1$

4) if $a^2 = e^2$ then $a^{-1} = a$

$In\ 1)\ \forall\ a \in G\ is\ missed$

$\therefore\ This\ is\ not\ possible$

Ans is (4).

34) Which of the following is a subgroup of additive group of integers modulo 4.

1) {0, 1} + mod 4 2) {0, 2} + mod 4

3) {0, 2, 3} + mod4 4) {0, 4} + mod4

1)

+	0	1
0	0	1
1	1	2

2)

+	0	2
0	0	2
2	2	0

3)

	0	2	3
0	0	2	3
2	2	0	1
3	3	1	2

4)

+	0	3
0	0	3
3	3	2

Ans is (2)

35) Which one of the following is a subgroup of multiplicative group of set of non-zero reals R_0.

1) Q_0 2) Q_1 3) Z_0 4) Z

In 1) $Q_0 \subset R_0 \quad \therefore$ This is a subgroup

For 2) Q^1 *is a subset of* R_0 *but identity* $1 \notin Q^1$

$\therefore Q^1$ *is not a sub group*

For 3) $Z_0 \subset R_0$ *But inverse of* 2 *is* $1/2$ & $1/2 \notin Z_0$

$\therefore$ *inverse axiom is not satisfied.*

For 4) $Z \not\subset R_0$

$\therefore Z$ *is not a sub group*

Ans is (1)

36) Which of the following is a sub group of $Z_{11} - \{0\}$ under X mod 11.

1) {1, 3,5, 7}   2) {1, 7}   3) {1, 10}   4) {1, 5, 7}

Sol: In 1), 2) & 4) closure law is not satisfied

$\times_{11}$	1	10
1	1	10
10	10	1

$\therefore$ *Closure law satisfies*

Ans. is (3)

37) If H is a subset of group G then H is a subgroup of G if the following axioms are satisfied.

1) Closure and associative
2) Closure and Identity
3) Identity & Closure
3) Closure and inverse

Ans is (4)

38) If H and is subset of group G then H is a subgroup $\forall\, a, b \in H$ if

1) $ab \in H$

2) $a^{-1} b^{-1} \in H$

3) $ab^{-1} \in H$

4) $a \in H$

Ans. is (3)

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39) If H and K are subgroups of group G then which one is a subgroup of G

$1)\ H \cup K$ $\quad 2)\ H - K$

$3)\ H + K$ $\quad 3)\ H \cap K$

Ans. is 4)

40) The number of subgroups of the group

{ 1, 5, 7, 11} under X mod 12 is

1) 2 2) 4 3) 5 4) 1

$\times_{12}$	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

$$1^{-1}=1, 5^{-1}=5, 7^{-1}=7, 11^{-1}=11$$

$$\therefore (1, 5), (1, 7), (1, 11)\ are\ subgroup$$

$$with\ \ this\ \ \{1\}\ and\ set\ G\ itself\ are$$

$$sub\ groups\ \therefore 3+2=5$$

Ans is (3)

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41) Set of complex numbers Z with $|Z| = 1 \; under \; \times \; is$

1) an abelian group
2) Not a Group
3) Not a Monoid
4) Not a Semigroup

Ans. is (1)

42) Which of the following is a subgroup of additive group of integers

1) N 2) Z^+ 3) Set of even integers 4) $\bar{Z}$

N, Z^+, $\bar{Z}$ are subsets of Z but they are not groups under +.

$\therefore$ They are not subgroups.

Ans is (3)

MATHEMATICS

43) Which one of the following is a semi group under x.

1) Z-
2) R-
3) Q-
4) set of odd integers

Ans is (4)

44) The set of square matrices form an abelian group under

1) + 2) - 3) × 4) None

Square matrices are of order 2x2,3x3

then addition,subtraction,multiplication may not be possible.

Ans is (4)

45) On set of reals R, $if\ a * b = \sqrt{a^2 + b^2}$ then identity element is

1) 0 2) 1 3) -1 4) does not exist

Sol: Identity axiom is a * e =a

$$\sqrt{a^2+e^2} = a, \qquad a^2+e^2 = a^2, \qquad e^2 = 0 \quad \therefore e = 0$$

But 0 is not identity element when a = - 1

$$Because \quad -1 * e = -1 \Rightarrow \sqrt{1+e^2} = -1$$

$$But \ \sqrt{1+e^2} \neq -1 \ \therefore \ e \ cannot \ be \ identity$$

Ans is(4)

MATHEMATICS

46) In the group $(R_0, *)$ if $a * b = \frac{ab}{10} \ \forall \ a, b \in R_0$

and $2 * (5 * x) = 4$ $then \ x =$

1)10 2) 20 3) 30 4) 40

$Given \ 2*(5*x) = 4$ $(2*5)*x = 4 \ [\because A.L. \ is \ satisfied]$

$$\frac{2(5)}{10} * x = 4$$

$1 * x = 4$ $\frac{1(x)}{10} = 4$ $\therefore x = 40$

Ans is (4)

47) In the group $(Z, *)$ If $a * b = a + b + 8$,

$\forall\ a, b \in Z$ & $\left(2^{-1} * x\right) * 4 = 1$ then x =

1) -1 2) $3/2$ 3) $-11/2$ 4) $-2/11$

In this case given (2^{-1} * x)* 4 =1

We can eliminate 2^{-1} by inserting 2 from left side.

$$Consider\ 2 * \left(2^{-1} * x\right) * 4 = 2 * 1$$

$$\left(2 * 2^{-1}\right) * \left(x * 4\right) = 2 * 1 \Rightarrow e * \left(x * 4\right) = 2 * 1$$

$$\Rightarrow x + 4 + 8 = 2 + 1 + 8 \quad \therefore \quad x = -1$$

Ans is (1)

MATHEMATICS

48) In the set $\left\{\begin{pmatrix}1 & x\\0 & 1\end{pmatrix},\ x \in R_0\right\}$ the inverse of $\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}$ under $\times$

1) $\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}$ 2) $\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$ 3) $\begin{pmatrix}1 & 3\\2 & 3\end{pmatrix}$ 4) $\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$Sol:\ Let\ X = \begin{pmatrix}1 & x\\0 & 1\end{pmatrix},\quad X^{-1} = \begin{pmatrix}1 & y\\0 & 1\end{pmatrix}\ \&\ E = \begin{pmatrix}1 & e\\0 & 1\end{pmatrix}$

$From\ the\ identity\ axiom\quad X\,E = X$

$$\begin{pmatrix}1 & x\\0 & 1\end{pmatrix}\begin{pmatrix}1 & e\\0 & 1\end{pmatrix} = \begin{pmatrix}1 & x\\0 & 1\end{pmatrix}$$

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$$\begin{pmatrix} 1 & e+x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \qquad e + x = x \qquad \therefore e = 0$$

$$\therefore \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ is an identity} \qquad \text{From inverse axiom } X\,X^{-1} = E$$

$$\begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & y+x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$y + x = 0 \therefore y = -x$$

$$\text{Substituting} \quad \therefore X^{-1} = \begin{pmatrix} 1 & -x \\ 0 & 1 \end{pmatrix}$$

$$\text{put } x = 3 \quad \therefore \quad X^{-1} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$$

Ans is (1)