Answer

1) (c) $x^2 + y^2 + 2x - 4y - \frac{1}{3} = 0$

Here g = 1, f = -2, c = $-\frac{1}{3}$ $\therefore$ Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{1 + 4 + \frac{1}{3}} = \sqrt{\frac{16}{3}} = \frac{4}{\sqrt{3}}$

2) (b) Length of x- intercept = $2\sqrt{g^2 - c} = 2\sqrt{36 - 11} = 2\sqrt{25} = 10$

3) (d) Given lines form a triangle.

$\therefore$ Number of circles touching all the lines = 4

4) (b) $S_1 - S_2 = 0 \Rightarrow 3x + 5x + 4y - 5y - 5 + 6 = 0 \Rightarrow 8x - y + 1 = 0$

5. (c) $PT = \sqrt{1 + 9 - 2 + 12 - 11} = \sqrt{9} = 3$

6. (b) Centre = (4, 1)

Radius = $\perp^{lr}$ distance from (4, 1) to $3x + 4y - 1 = 0$

$= \left|\frac{12 + 4 - 1}{\sqrt{9 + 16}}\right| = \left|\frac{15}{5}\right| = 3$

Equation is $(x - 4)^2 + (y - 1)^2 = 9 \Rightarrow x^2 + y^2 - 8x - 2y + 8 = 0$

7. (a) Centre = (a, a), radius = a

Equation is $(x - a)^2 + (y - a)^2 = a^2$

i, e $x^2 + y^2 - 2ax - 2ay + a^2 = 0$

Substituting. (2, 1) we get $4 + 1 - 4a - 2a + a^2 = 0$

i, e. $a^2 - 6a + 5 = 0 \Rightarrow a = 1$, or $a = 5$

Equations are $(x - 1)^2 + (y - 1)^2 = 1$ or $(x - 5)^2 + (y - 5)^2 = 25$

i, e. $x^2 + y^2 - 2x - 2y + 1 = 0$ or $x^2 + y^2 - 10x - 10y + 25 = 0$

8. (b) (2, -3) satisfies both line and the circle

9. (a) Length = $\sqrt{c_1 - c_2} = \sqrt{4 - (-12)} = \sqrt{16} = 4$

10. (c) $\sqrt{4 + 9 + 12 + 6k - 6} = 7$

Squaring, $19 + 6k = 49 \Rightarrow 6k = 30 \Rightarrow k = 5$

11. (b) Common tangent is radical axis of two circles

$\therefore S_1 - S_2 = 0 \Rightarrow 2x + 2x + 4 + 4 = 0$

$4x + 8 = 0$

$\Rightarrow x + 2 = 0$

12. (d) $c_1 = (1, 3)$, $r_1 = r$, $c_2 = (4, -1)$, $r_2 = \sqrt{16 + 1 - 8} = \sqrt{9} = 3$

$r_1 - r_2 < c_1 c_2 < r_1 + r_2$

But $c_1 c_2 = \sqrt{9 + 16} = 5$

$\Rightarrow r - 3 < 5 < 3 + r \Rightarrow r - 3 < 5$ and $r + 3 > 5$

$\Rightarrow r < 8$ and $r > 2$

$\therefore 2 < r < 8$

13. (c) Solving $2x - 3y = 5 \times 4$

$3x - 4y = 7 \times 3$

$8x - 12y = 20$

$9x - 12y = 21$

$-x = -1 \Rightarrow x = 1,\ y = -1$

Centre = (1, -1)

Area = 154 $\Rightarrow \pi r^2 = 154$

$\frac{22}{7} \cdot \pi = 154 \qquad \Rightarrow r^2 = \frac{154 \times 7}{22} = 49$

$\therefore (x - 1)^2 + (y + 1)^2 = 49$

$X^2 + y^2 - 2x + 2y - 47 = 0$

14. (d) Solving $x + y = 6$

$x + 2y = 4$

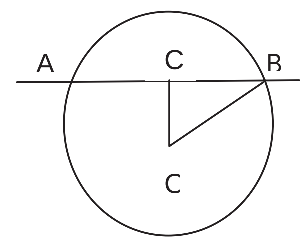
$y = -2, \quad x = 8$

Centre = (8, -2), Eqn is $(x - 8)^2 + (y + 2)^2 = r^2$

Substituting. (6, 2), $(6 - 8)^2 + (2 + 2)^2 = r^2$

$r^2 = 4 + 16 = 20 \qquad \Rightarrow r^2 = 20 \qquad r = \sqrt{20}$

15. (c) $r = \sqrt{2}$



$\overline{OC} = \perp^{lr}$ distance from (0, 0) to $2x - y + 1 = 0$

$= \frac{1}{\sqrt{5}}$

$\therefore \overline{AB} = 2\,\overline{BC} = 2\sqrt{\overline{OB}^2 - \overline{OC}^2} = 2\sqrt{2 - \frac{1}{5}} = 2.\frac{3}{\sqrt{5}} = \frac{6}{\sqrt{5}}$

16. (a) $x^2 + y^2 = 7$ .... (1)

$y = 3x + K$ .... (2)

Condition is $c^2 = a^2 (m^2 + 1)$

$K^2 = 7 (9 + 1) = 70$

$K = \sqrt{70}$

17. (b) Centre = (-4 , 3)

Radius = $\perp^{lr}$ distance from (-4, 3) to $5x - y - 3 = 0 = \left|\frac{-20 - 3 - 3}{\sqrt{25 + 1}}\right| = \sqrt{26}$

Eqn is $(x + 4)^2 + (y - 3)^2 = 26$

$x^2 + y^2 + 8x - 6y - 1 = 0$

18. (b) $x^2 + y^2 + x + 3y - \frac{k}{3} = 0$ ....(1)

$x^2 + y^2 - x + \frac{y}{2} + \frac{k}{4} = 0$ ....(2)

$2g_1g_2 + 2f_1 f_2 = C_1 + C_2$

$$2\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) + 2\left(\frac{3}{2}\right)\frac{1}{4} = \frac{-k}{3} + \frac{k}{4} \Rightarrow -\frac{1}{2} + \frac{3}{4} = \frac{-4k + 3k}{12} \Rightarrow \frac{-2 + 3}{4} = \frac{-k}{2} \Rightarrow \frac{1}{4} = \frac{-k}{12} \Rightarrow k = \frac{-12}{4} = -3$$

19. (b)radius = y cordinate of centre

$\sqrt{g^2 + f^2 - c} = |-f| \Rightarrow g^2 + f^2 - c^2 = f^2 \Rightarrow c = g^2$

c = $\left(\frac{P}{2}\right)^2 = \frac{P^2}{4} \Rightarrow P^2 = 4c$

20. (d) g = - 2sin θ, f = -2cos θ, c = -12

$r = \sqrt{g^2 + f^2 - c^2} = \sqrt{4\cos^2\theta + 4\sin^2\theta + 12} = \sqrt{4+12} = \sqrt{16} = 4$

∴ Circumference = 2πr = 8π

21. (a)Equation is $x^2 + y^2 - ax - by = 0$

Here a = 5, b=6 ∴ $x^2 + y^2 - 5x - 6y = 0$

22. (b)$x^2 + y^2 - x + 2y + \frac{18}{7} = 0$ .... (1)

$x^2 + y^2 - \frac{7x}{4} + 2y + 5 = 0$ .... (2)

Radical axis is (1) – (2)

$-x + \frac{7x}{4} + 2y - 2y + \frac{18}{7} - 5 = 0$

$\frac{-4x+7x}{4} + \frac{18-35}{7} = 0$

$\frac{3x}{4} - \frac{17}{7} = 0$

$\frac{21x-68}{28} = 0 \quad \Rightarrow 21x - 68 = 0$

23. (d) The family of circles passing through the origin is $x^2 + y^2 + 2gx + 2fy = 0$

It cuts $x^2 + y^2 + 4x - 6y - 13 = 0$ orthogonally

$\therefore 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

2g. (2) + 2f (-3) = 0 – 13

4g – 6f = -13 ⇒ 4g – 6f + 13 = 0

Loucs of (-g, -f) is -4x – 6(-y) + 13 = 0

-4x + 6y + 13 = 0 i, e. 4x – 6y -13 = 0

24. (b) $x^2 + y^2 - 3x - 4y + 5 = 0$ .... (1)

$x^2 + y^2 - \frac{7x}{3} + \frac{8y}{3} + \frac{11}{3} = 0$ ....(2)

Radical axis is (1) - (2)

$-3x + \frac{7x}{3} - 4y - \frac{8y}{3} + 5 - \frac{11}{3} = 0$

Multiply by 3, -9x + 7x – 12y – 8y + 15 – 11 = 0 ⇒-2x – 20y + 4 = 0

∴ slope = $\frac{-a}{b} = \frac{2}{-20} = -\frac{1}{10}$

25. (c) Locus is the radical axis of two circles i, e. 8x – 12y + 5 = 0

26. (c) centre = (2, 3) r = 2

Eqn is $(x-2)^2 + (y-3)^2 = 4$

$x^2 + y^2 - 4x - 6y + 9 = 0$

27. (d) 3x + 3y + 7 = 0 ⇒ $\frac{3x}{\sqrt{18}} + \frac{3y}{\sqrt{18}} + \frac{7}{\sqrt{18}} = 0 \Rightarrow x.\frac{1}{\sqrt{2}} + y.\frac{1}{\sqrt{2}} = \frac{-7}{3\sqrt{2}}$

Comparing with x cos $\propto$ + y sin $\propto$ = p, we get p = $\left|\frac{-7}{3\sqrt{2}}\right| = \frac{7}{3\sqrt{2}}$

28. (a) Solving 3x + 4y = 5

5x + 4y = 4

-x = 1

i, e. x= -1 any y = 2

Substituting in λx + 4y = 6, we get -λ + 8 = 6

-λ = -2 λ=2

29. (d) (slope of AB) x (slope of x – y = 0) = -1 $\Rightarrow \frac{k+3}{h-4} \times 1 = -1$

$\Rightarrow$k +3 = -h + 4

h + k= 1 .... (1)

mid point of AB = $\left(\frac{4+h}{2}, \frac{-3+k}{2}\right)$

Substituting in x – y = 0

$\frac{4+h}{2} - \left(\frac{-3+k}{2}\right) = 0$

4 + h + 3 – k = 0

h – k = -7 .... (2)

Solving (1) and (2)

2h = -6 $\Rightarrow$ h = -3 and k = 4 $\therefore$ B = (-3, 4)

30. (d) tan θ = $\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \tan 45^0 = \left|\frac{\frac{-3}{k} - \left(\frac{-2}{5}\right)}{1 + \left(\frac{-3}{k}\right)\left(\frac{-2}{5}\right)}\right|$

i, e. $1 = \left|\frac{\frac{-3}{k} + \frac{2}{5}}{1 + \frac{6}{5k}}\right| \Rightarrow \left|\frac{-15+2k}{5k+6}\right| = 1 \quad \therefore \frac{-15+2k}{5k+6} = \pm 1 \Rightarrow \frac{-15+2k}{5k+6} = 1$ and $\frac{-15+2k}{5k+6} = -1$

i, e. -15+2k = 5k + 6 $\Rightarrow$ 3k = -21 i, e. k = -7 | -15 + 2k = -5k - 6 $\Rightarrow$ 7k = 9 $\Rightarrow$ k $= \frac{9}{7}$

31. (a) $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{2\sqrt{4-1}}{1+1}\right| = \sqrt{3} \quad \Rightarrow \quad \theta = 60^0$

32. (a) Condition is abc + 2fgh – af^2 – bg^2 – ch^2 = 0

i. e. k(3) (8)+2 (7) (4) $\frac{7}{2}$-k(49)– 3(1b) -8.$\left(\frac{49}{4}\right)$ = 0 $\Rightarrow$ 24k + 196 – 49k – 48–98= 0 $\Rightarrow$25k = 50

$\Rightarrow$ k =2

33. (b) Let m and 3m be the slopes of the two lines then m + 3m = -$\frac{2h}{b}$ and m x 3m = $\frac{a}{b}$

4m = =-$\frac{2h}{b}$ and 3m^2 = $\frac{a}{b}$ $\Rightarrow$ m = -$\frac{h}{2b}$ *and* $3\left(\frac{h^2}{4b^2}\right) = \frac{a}{b}. \Rightarrow h^2 = \frac{4ab}{3}.$

34. (d) Product of perpendiculars from (x_1 , y_1) to ax^2 + 2hxy + by^2 = 0 is $\left|\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}}\right|$

Here (x_1 , y_1) = (1, 2) a = 1, h = 2, b = 1

$\therefore$ Product $= \left|\dfrac{1+2(2).2+4}{\sqrt{(1-1)^2+4.4}}\right| = \left|\dfrac{1+8+4}{\sqrt{16}}\right| = \dfrac{13}{4}$

35. (c) Required distance $= \left|2\sqrt{\dfrac{g^2-ac}{a(a+b)}}\right| = \left|2\sqrt{\dfrac{9-8}{1(1+1)}}\right| = \left|2\dfrac{1}{\sqrt{2}}\right| = \sqrt{2}$

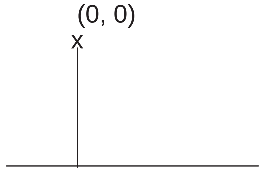
36. (b) Dist. $= \left|\dfrac{-20-110}{\sqrt{25+144}}\right| = \dfrac{130}{13} = 10$

37. (a) $x+2y+7+\lambda(2x-3y-9)=0 \Rightarrow (0,\ 0) \Rightarrow \lambda = \dfrac{7}{9}$

$$9x+18y+63+14x-21y-63=0$$

$$23x-3y=0$$

38. (c)

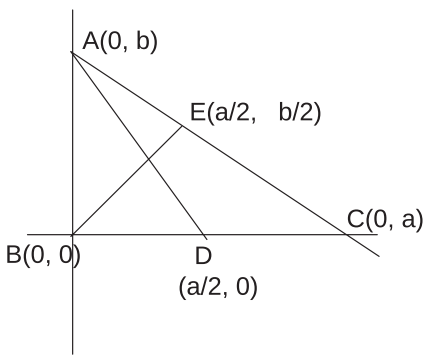


h + k = 2 ——— (1)

$\dfrac{k}{h}\times -1 = -1 \Rightarrow k = h$

$\therefore\ (h,\ k) = (1,\ 1)$

39. (b)



Slope of AD x Slope of BE = - 1

$$\frac{-b}{\frac{a}{2}} \times \frac{\frac{b}{2}}{\frac{a}{2}} = -1$$

$$\frac{-2b}{a} \times \frac{b}{a} = -1$$

$$2b^2 = a^2$$

$$a = \pm b\sqrt{2}$$

40. (a) The ratio in which line joining the points $(x_1,\ y_1)$ and $(x_2,\ y_2)$ is divided by the line

$$ax+by+c=0 \text{ is } -\left(\frac{ax_1+by_1+c}{ax_2+by_2+c}\right)$$

i, e. ratio $= -\left(\dfrac{2-3}{8-6}\right) = -\left(\dfrac{-1}{2}\right) = \dfrac{1}{2} = 1:2$