Question 1

If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then $x = ?$

a) 0

c) -2

b) -1

d) -3

Answer (b)

Solution:

$$\text{When } x = 0$$

$$[\tan^{-1}(0)]^2 + [\cot^{-1}(0)\}^2 = 0^2 + \left(\frac{\pi}{2}\right)^2$$

$$= \frac{\pi^2}{4} \neq \frac{5\pi^2}{8}$$

When $x = -1$

$$[\tan^{-1}(-1)]^2 + [\cot^{-1}(-1)\}^2$$

$$= \left(\frac{-\pi}{4}\right)^2 + \left(\frac{3\pi}{4}\right)^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} = \frac{10\pi^2}{16} = \frac{5\pi^2}{8}$$

MATHEMATICS

Question 2

The value of

$$\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right]=?$$

a) $\frac{2b}{a}$

b) $\frac{2a}{b}$

c) $\frac{a}{b}$

d) $\frac{b}{a}$

Answer (a)

Solution:

$$\text{Take } \theta = \frac{1}{2}\cos^{-1}\frac{a}{b} \Rightarrow \theta = \cos^{-1}\frac{a}{b} \Rightarrow$$

$$\cos 2\theta = \frac{a}{b}$$

$$= \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{1 + \tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{1 - \tan^2\theta}$$

$$= \frac{2(1 + \tan^2\theta)}{1 - \tan^2\theta} = 2\sec 2\theta = \frac{2b}{a}$$

Question 3

If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

and $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{3}$

then $(x, y) = ?$

a) $\left(1, \frac{1}{2}\right)$

c) $(-1, 1)$

b) $\left(\frac{1}{2}, 1\right)$

d) $(1, 1)$

Answer (b)

Solution:-

Given $\cos^{-1}x - \cos^{-1}y = \frac{\pi}{3}$

$$\left(\frac{\pi}{2} - \sin^{-1}x\right) - \left(\frac{\pi}{2} - \sin^{-1}y\right) = \frac{\pi}{3}$$

$$\sin^{-1}y - \sin^{-1}x = \frac{\pi}{3}$$

Also $\sin^{-1} y + \sin^{-1} x = \frac{2\pi}{3}$

Adding $2\sin^{-1} y = \pi \Rightarrow \sin^{-1} y = \frac{\pi}{2}$

$y = \sin\frac{\pi}{2} = 1$

when $y = 1$, $\sin^{-1} y + \sin^{-1} x = \frac{2\pi}{3}$

$$\sin^{-1} x = \frac{2\pi}{3} - \frac{\pi}{2}$$

$$\sin^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \sin\frac{\pi}{6} = {}^{1}/_{2}$$

Question 4

Let (x, y) be such that

$$\sin^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(2xy) = \frac{\pi}{2}$$

then (x, y) lies on

a) the circle $x^2 + y^2 = 1$

b) line $y = x$

c) $(x^2 - 1)(y^2 - 1) = 0$

d) $(4x^2 - 1)(y^2 - 1) = 0$

Answer (a)

Solution:- $\sin^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(2xy) = \frac{\pi}{2}$

$$\cos^{-1} y + \cos^{-1}(2xy) = \frac{\pi}{2} - \sin^{-1} x$$

$$= \cos^{-1} x$$

$$\therefore \quad \cos^{-1}(2xy) = \cos^{-1}x - \cos^{-1}y$$

$$\cos^{-1}(2xy) = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$2xy = xy + \sqrt{1-x^2}\sqrt{1-y^2}$$

$$\Rightarrow xy = \sqrt{1-x^2}\sqrt{1-y^2}$$

Squaring on both sides

$$x^2y^2 = (1 - x^2)(1 - y^2)$$

$$x^2 + y^2 = 1$$

Question 5

If $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\sqrt{\frac{3}{x}} - \frac{\pi}{6} = 0$ then $x =$

a) 2

b) 3

c) 4

d) 5

Answer (c)

Solution:- **Given equation**

$$= \sin^{-1}(1) - \frac{\pi}{6} = \sin^{-1}\sqrt{\frac{3}{x}}$$

$$\frac{\pi}{2} - \frac{\pi}{6} = \sin^{-1}\sqrt{\frac{3}{x}}$$

$$\sin\frac{\pi}{3} = \sqrt{\frac{3}{x}}$$

$$\frac{\sqrt{3}}{2} = \sqrt{\frac{3}{x}} \Rightarrow \frac{1}{2} = \frac{1}{\sqrt{x}} \Rightarrow x = 4$$

Question 6

$$\sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right\}\right]=?$$

a) $\frac{\pi}{4}$

c) 0

b) $\frac{\pi}{6}$

d) $\frac{\pi}{2}$

Answer (c)

Solution:

we have $\sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{4-2\sqrt{3}}{8}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$$\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} = \sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}} = 15°$$

$$\cos^{-1}\frac{\sqrt{12}}{4} = \cos^{-1}\frac{\sqrt{3}}{2} = 30°$$

$$\sec^{-1}\sqrt{2} = 45°$$

$\therefore$ **given expression is equal to**

$$\sin^{-1}[\cot(15° + 30° + 45°)]$$

$$= \sin^{-1}[\cot 90°]$$

$$= \sin^{-1} 0 = 0$$

Question 7

$The\ value\ of\ cosec^2(\cot^{-1} 3) - sec^2(\tan^{-1} 2) = ?$

$a)$ 10

$b)$ 7

$c)$ 2

$d)$ 5

Answer (d)

Solution:-

$$\textit{Given expression}$$

$$= [1 + cot^2(\cot^{-1} 3)] - [1 + tan^2(\tan^{-1} 2)]$$

$$= [1 + (3)^2] - [1 + (2)^2]$$

$$= 10 - 5 = 5$$

Question 8

$$If\ \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]=\alpha\ then\ x^2$$

$a)\ \sin\alpha$

$b)\ \cos 2\alpha$

$c)\ \cos\alpha$

$d)\ \sin 2\alpha$

Answer (d)

Solution:-

From the given relation, we have

$$\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]=\tan\alpha$$

$$\frac{2\sqrt{1+x^2}}{2\sqrt{1-x^2}}=\frac{1+\tan\propto}{1-\tan\propto}$$

$(by\ componendo\ and\ dividendo)$

$$\frac{1+x^2}{1-x^2} = \frac{(\cos\propto + \sin\propto)^2}{(\cos\propto - \sin\propto)^2}$$

$$\frac{1+x^2}{1-x^2} = \frac{1+\sin 2\propto}{1-\sin 2\propto}$$

$$\frac{2}{2x^2} = \frac{2}{2\sin 2 \propto}$$

$$\Rightarrow x^2 = \sin 2 \propto$$

(Again by componendo and dividendo)