

INTEGRATIONS AND APPLICATIONS OF DEFINITE INTEGRALS

1. $\int \frac{x \sin x^2 e^{\sec x^2}}{\cos^2 x^2} dx =$
- a) $2 e^{\sec x^2} + c$ b) $\frac{1}{2} e^{\sec x} + c$ c) $e^{\sec x^2} + c$ d) $\frac{1}{2} \sec x^2 + c$
2. $\int \frac{dx}{3 \cos x - 4 \sin x + 5} =$
- a) $\tan^{-1} \left(\frac{1}{2} \tan \frac{x}{2} \right) + c$
 b) $\frac{1}{\tan \frac{x}{2} - 2} + c$
 c) $\frac{1}{2 - \tan \frac{x}{2}} + c$
 d) $\frac{1}{2 - \tan x} + c$
3. $\int \cos \sqrt{x} dx =$
- a) $2 [\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$
 b) $[\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$
 c) $2 [\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$
 d) $2 [\sqrt{x} \sin \sqrt{x} + \sin \sqrt{x}] + c$
4. $\int \frac{\sin x}{\sin^2 x + 4 \cos^2 x} dx =$
- a) $\cos x + c$
 b) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x}{\sqrt{3}} \right) + c$
 c) $\frac{1}{\sqrt{3}} \cot^{-1} \left(\frac{\sec x}{\sqrt{3}} \right) + c$
5. $\int \frac{e^x dx}{(1+e^x)(2+e^x)} =$
- a) $\log(2+e^x)(1+e^x) + c$
 b) $\log \left(\frac{2+e^x}{1+e^x} \right) + c$
 c) $\log \left(\frac{1+e^x}{2+e^x} \right) + c$
 d) $\log(1+e^x)^2$
6. $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$
- a) $\frac{1}{\sqrt{2}} \sin^{-1}(\sin x + \cos x) + c$
 b) $\sqrt{2} \sin^{-1}(\sin x + \cos x) + c$
 c) $\sqrt{2} \cos^{-1}(\cos x + \sin x) + c$
 d) $\sqrt{2} \sin^{-1}(\sin x - \cos x) + c$
7. $\int \frac{x^3 dx}{(1+x^2)^2} =$
- a) $-\frac{1}{1+x^2} + c$
 b) $\log \sqrt{1+x^2} + \frac{1}{2}(1+x^2) + c$
 c) $\log(1+x^2) + c$
 d) $-\frac{1}{1+x} + c$
8. $\int \tan x \sqrt{\sec x} dx =$
- a) $2\sqrt{\sec x}$
 b) $\frac{1}{2\sqrt{\sec x}}$
 c) $\sqrt{\cos x}$
 d) $\frac{2}{5}(\cos x)^{5/2}$
9. $\int \frac{x+4}{(x+5)^2} e^x dx$
- a) $\frac{e^x}{x+4}$
 b) $\frac{e^x}{x+5}$
 c) $\frac{e^x}{(x+5)^2}$
 d) $\frac{e^x}{(x+4)^2}$
10. $\int \frac{dx}{4x^2+12x+45} dx =$
- a) $\frac{1}{12} \tan^{-1} \left(\frac{2x+3}{6} \right)$
 b) $\frac{1}{3} \tan^{-1}(2x+3)$

$$c) \frac{1}{2} \tan^{-1} \left(\frac{2x+3}{6} \right)$$

$$d) \frac{1}{6} \tan^{-1} \left(\frac{2x+3}{6} \right)$$

$$11. \int e^{2 \log \tan \sin x} \cos x \, dx =$$

a) $\sin x + \tan \sin x$ b) $e^{2 \log \tan \cos x}$

c) $e^{\sin x}$

d) $\tan \sin x - \sin x$

$$12. \int e^x (\sin^2 x + \sin 2x) \, dx$$

a) $e^x \sin x$

b) $e^x \sin^2 x$

c) $e^x \sin 2x$

d) $e^x \sin x \cos x$

$$13. \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \, dx =$$

a) $\sec x - \operatorname{cosec} x$
c) $\cot x - \tan x$

b) $\sec x + \operatorname{cosec} x$
d) $\tan x - \cot x$

$$14. \int \sqrt{1 - \sin 2x} \, dx$$

a) $\sin x - \cos x$

b) $\sqrt{\sin x + \cos x}$

c) $\frac{1}{2}(\sin x + \cos x)^{3/2}$

d) $\sin x + \cos x$

$$15. \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) \, dx$$

a) $2e^x \tan \frac{x}{2}$

b) $e^x \tan \frac{x}{2}$

c) $e^x \tan x$

d) $\frac{e^x}{2} \tan \frac{x}{2}$

$$16. \int \frac{1 - \cos^2 x}{\cos^4 x} \, dx =$$

a) $-\frac{\cot^3 x}{3} + c$

b) $\cot^3 x + c$

c) $\frac{\tan^3 x}{3} + c$

d) $\tan^3 x + c$

$$17. \int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} \, dx =$$

a) $4 - \pi$

b) $2 + \pi$

c) $\pi - 4$

d) $1 + \pi$

18. The value of $\int_{\frac{1}{e}}^{\tan x} \frac{t \, dt}{1+t^2} + \int_{\frac{1}{e}}^{\cot x} \frac{dt}{t(1+t^2)}$ is

a) 1

b) $2e$

c) $\frac{2}{e}$

d) $\frac{e}{2}$

$$19. \int_1^{e^{37}} \frac{\pi \sin(\pi \log_e x)}{x} \, dx =$$

a) -2

b) 2

c) $2/\pi$

d) 2π

$$20. \int \frac{e^x(1+x)dx}{\sin^2(xe^x)} =$$

a) $\tan(e^x + x) + C$

b) $\cos^2(xe^x) + C$

c) $-\cot(xe^x) + C$

d) $-\cot \frac{x}{2} + C$

$$21. \int (1 + \cos x + \cos^2 x + \dots + \infty) \, dx =$$

a) $\tan \frac{x}{2} + C$

b) $\cot \frac{x}{2} + C$

c) $-\cot \frac{x}{2} + C$

d) $-\tan \frac{x}{2} + C$

$$22. \int \frac{\sec x \tan x}{10 + \tan^2 x} \, dx =$$

a) $\frac{1}{3} \tan^{-1} \left(\frac{\sec x}{3} \right) + C$

b) $\log(9 + \sec^2 x) + C$

c) $\tan^{-1}(\sec x) + C$

d) $\tan^{-1} \left(\frac{\sec x}{3} \right) + C$

$$23. \int \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} =$$

a) $\sin^{-1} \left(\frac{a}{b} \tan x \right) + C$

b) $\tan^{-1} \left(\frac{b}{a} \tan x \right) + C$

c) $\frac{1}{ab} \tan^{-1} \left(\frac{a}{b} \tan x \right) + C$

d) $\frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \tan x \right) + C$

$$24. \int \frac{x^2 + 1}{x^4 + 1} \, dx =$$

a) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 + 1}{\sqrt{2}x} \right) + C$

c) $\frac{1}{2} \log \left(\frac{x+1}{x-1} \right) + C$

b) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + C$

d) $\tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + C$

25. $\int \sqrt{1 + \sin \left(\frac{x}{3} \right)} dx =$

a) $6 \left(\cos \frac{x}{6} - \sin \frac{x}{6} \right) + C$

c) $\frac{1}{6} \left(\cos \frac{x}{6} - \sin \frac{x}{6} \right) + C$

b) $6 \left(\sin \frac{x}{6} - \cos \frac{x}{6} \right) + C$

d) $\left(\sin \frac{x}{6} - \cos \frac{x}{6} \right) + C$

26. $\int \sec^2 x \csc^2 x dx =$

a) $\tan x - \cot x + C$

c) $\cot x - \tan x + C$

b) $\tan x + \cot x + C$

d) $\tan x \cot x + C$

27. $\int \csc^4 x dx =$

a) $\frac{1}{3} \cot^3 x + C$

c) $-\cot x - \frac{1}{3} \cot^3 x + C$

b) $\cot x + \frac{1}{3} \cot^3 x + C$

d) $-\cot x + \frac{1}{3} \cot^3 x + C$

28. $\int \sqrt{\frac{1-x}{1+x}} dx =$

a) $\sin^{-1} x + \sqrt{1-x^2} + C$

c) $\sin^{-1} x + \sqrt{x^2 - 1} + C$

b) $\cos^{-1} x + \sqrt{1-x^2} + C$

d) $\log \sqrt{1+x} + C$

29. $\int \sinh(\log x) dx =$

a) $\cosh \log x$

c) $x \cosh \log x$

b) $\frac{1}{x} \cosh \log x$

d) $\frac{x^2}{4} - \log \sqrt{x}$

30. $\int \frac{dx}{(2x-1)\sqrt{2x-2}} =$

a) $\tan^{-1}(2x-2)$

c) $3/2 \log(2x-3)$

b) $\tan^{-1} \sqrt{2x-2}$

d) none

31. $\int x^x \log(ex) dx =$

a) $x^x + c$

b) $x^x \log x + c$

c) $\frac{1}{2} x^{2x} + c$

d) $x^x \log(ex) + c$

32. $\int \frac{\cos x}{\cos \left(x - \frac{\pi}{4} \right)} dx =$

a) $\frac{1}{\sqrt{2}} [x + \log(\sin x + \cos x)]$

b) $\sqrt{2} [x + \log(\sin x + \cos x)]$

c) $\frac{1}{\sqrt{2}} x + \log(\sin x + \cos x)$

d) $\frac{1}{\sqrt{2}} x + \log(\sin x - \cos x)$

33. $\int \frac{x-1}{(x+1)^3} e^x dx =$

a) $e^x \frac{1}{(x+1)^2}$

b) $-e^x \frac{1}{(1+x)^2}$

c) $e^x \frac{1}{(1+x)^3}$

d) $e^x \frac{1}{(x+1)^3}$

34. If $\int \frac{\sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx = Ax + B \log(3 \sin x + 4 \cos x) + C$, then $A+B=$

a) 4/5

b) 2/5

c) 1/5

d) 2

35. If $I_n = \int_0^{\pi/4} \cot^n x dx$, then $I_n + I_{n-2} =$

a) $\frac{1}{n}$

b) $\frac{1}{n-1}$

c) $\frac{1}{n-2}$

d) $\frac{1}{1-n}$

36. $\int_0^{\pi} \frac{dx}{\cos^2 x + 3 \sin^2 x} =$

- a) $\pi/\sqrt{2}$ b) π c) $\pi/\sqrt{3}$ d) $\frac{\pi}{2}$
37. $\int_0^2 \sqrt{\frac{2+x}{2-x}} dx =$
 a) $\frac{\pi}{2} + 1$ b) $\pi + 1$
 c) $\pi - 2$ d) $\pi + 2$
38. $\int_0^1 \sin^2(\cos^{-1}x) dx$
 a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) 2 d) $\frac{\pi}{6}$
39. $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)} =$
 a) $\frac{\pi}{2ab(a+b)}$ b) $\frac{-\pi}{2ab(a+b)}$ c) $\frac{\pi}{ab(a+b)}$ d) $\frac{\pi ab}{a+b}$
40. $\int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{3 + \sin 2x}} dx =$
 a) $\pi/2$ b) $\pi/4$ c) $\pi/8$ d) π
41. $\int_0^{\pi/6} \sin 12x \cos 6x dx =$
 a) $2/9$ b) $1/9$ c) $1/18$ d) $5/18$
42. $\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} =$
 a) $1/2 \log 2$ b) $2 \log 2$ c) $\log 2$ d) 7
43. $\int_0^1 \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx =$
 a) -1 b) 1 c) 0 d) $\pi/2 - 1$
44. $\int_0^1 \cos^{-1} \frac{2x}{1+x^2} dx =$
 a) $\pi - \log 2$ b) $-\log 2$ c) $\pi + \log 2$ d) $\log 2$
45. $\int_0^{2\pi} |x \sin x| dx =$
 a) 4π b) 3π c) 2π d) π
46. $\int_0^1 \frac{x^3}{1+x^8} dx =$
 a) $\frac{3\pi}{16}$ b) $-\frac{\pi}{16}$ c) $\frac{\pi}{16}$ d) $\frac{\pi}{4}$
47. $\int_1^4 e^{\sqrt{x}} dx =$
 a) $-2e^2$ b) $3e^2$ c) e d) $2e^2$
48. $\int_0^{\pi/2} |\sin x - \cos x| dx =$
 a) 0 b) $\sqrt{2} - 1$ c) $4\sqrt{2} + 2$ d) $2\sqrt{2} - 2$
49. $\int \frac{1}{(e^x - 1)^2} dx =$
 a) $-\frac{1}{e^x - 1}$ b) $-\log(e^x - 1) - \frac{1}{e^x - 1}$
 c) $x + \log(e^x - 1) - \frac{1}{e^x - 1}$ d) $x - \log(e^x - 1) - \frac{1}{e^x - 1}$
50. If $U_n = \int_1^e (\log x)^n dx$, $U_n + nU_{n-1} =$
 a) e^n b) e c) $e - 1$ d) 1

51. If $\int_{-1}^4 f(x) dx = 4$ and $\int_2^4 [3 - f(x)] dx = 7$, then the value of $\int_2^{-1} f(x) dx =$
- a) 2 b) -3 c) -5 d) 1
52. $\int_0^4 \frac{(4-x)^4}{x^4 + (4-x)^4} dx =$
- a) $\frac{-3}{2}$ b) $\frac{2}{3}$ c) 4 d) 2
53. $\int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx =$
- a) $\pi/6$ b) $\pi/4$ c) $\pi/2$ d) 0
54. $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}} =$
- a) π b) $\frac{\pi}{2}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{4}$
55. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \tan x} dx =$
- a) $\pi/2$ b) $\pi/12$ c) 0 d) $\pi/4$
56. The value of the integral $\int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx$ is
- a) $\frac{a-b}{2}$ b) $b-a$ c) $\frac{b-a}{2}$ d) $\frac{a+b}{2}$
57. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right) d\theta =$
- a) 0 b) 1 c) 2 d) 7
58. $\int_1^3 (x-1)(x-2)(x-3) dx =$
- a) 0 b) 1 c) 2 d) 3
59. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin |x| dx =$
- a) 0 b) 1 c) -1 d) 2
60. If $I_1 = \int_0^{\frac{\pi}{2}} \log \sin x dx$ and $I_2 = \int_0^{\frac{\pi}{2}} \log \cos x dx$, then
- a) $I_1 = 2I_2$ b) $2I_1 = I_2$ c) $I_1 + I_2 = 0$ d) $I_1 - I_2 = 0$
61. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx =$
- a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{8} \log 2$ d) $\frac{3\pi}{8} \log 2$
62. $\int_0^{\infty} \frac{1}{x^4 + 2x^2 + 1} dx =$
- a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\tan^{-1} 2$ d) $\tan^{-1} 4$
63. $\int_0^1 \frac{x dx}{(x+1)^4} =$
- a) 1/2 b) 4 c) 0 d) $\frac{1}{12}$
64. The area bounded by $y = x^2 - 3x + 2$ and x-axis is

a) $\frac{1}{3}$ sq. units b) $\frac{1}{6}$ sq. units c) $\frac{2}{3}$ sq. units d) $\frac{1}{12}$ sq. units

65. Area bounded by the curve $y^2 = 16x$ and the line $y = mx$ is $\frac{2}{3}$ then m is equal to
 a) 3 b) 4 c) 1 d) 2

66. Area enclosed by $y = \sin x$ and x axis between $x = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ is
 a) 0 b) 1 c) 2 d) none

67. The area between the parabolas $y^2 = 9x$ and $x^2 = 9y$ is _____ sq. units
 a) 9 b) 18 c) 27 d) 7

68. Area of the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ is
 a) 100π b) 50π c) 25π d) 10π

69. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\sin x + \cos x} dx =$
 a) $\pi/4$ b) $\pi/8$ c) $\pi/6$ d) $\pi/12$

70. $\int_{-\pi}^{3\pi} \log(\sec \theta - \tan \theta) d\theta =$
 a) 1 b) 0 c) 2 d) 7

71. $\int_0^{\frac{\pi}{2}} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx =$
 a) $(a + b)\pi$ b) $(a + b)\frac{\pi}{2}$ c) $(a + b)\frac{\pi}{4}$ d) $a + b$

72. $\int_0^{\pi} x \cos^2 x \sin x dx =$
 a) $1/3$ b) $2/3$ c) $\pi/3$ d) 1

73. $\int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}} dx =$
 a) 5 b) 10 c) $5/2$ d) 7

74. $\int_0^{\pi} \frac{dx}{1 + 2^{\cos x}} =$
 a) π b) 0 c) $\pi/2$ d) 1

75. $\int_0^1 x(1-x)^n dx =$
 a) $\frac{1}{n(n+1)}$ b) $\frac{1}{(n+1)(n+2)}$ c) $\frac{1}{n+1} + \frac{1}{n+2}$ d) $\frac{1}{n(n-1)}$

76. $\int_0^1 x^3 (1-x^2)^5 dx =$
 a) $1/30$ b) $1/42$ c) $1/84$ d) $10/231$

77. $\int_0^{\pi/2} \log \tan x dx =$
 a) $\pi/2$ b) $\pi/4$ c) $\pi/8$ d) 0

78. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin^{\frac{3}{2}} x} dx =$
 a) 1 b) π c) $\pi/4$ d) $\pi/2$

79. $\int_0^{\frac{\pi}{2}} \frac{1}{1 + (\cot x)^{101}} dx =$
 a) 0 b) $\pi/2$ c) $\pi/3$ d) $\pi/4$

80. The value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \left[\log\left(\frac{1-x}{1+x}\right) \right] dx$ is

81. The area of the region bounded by the curve $y^2 = 2y - x$ and the y - axis is

- a) $\frac{1}{3}$ sq. units b) $\frac{2}{3}$ sq. units c) $\frac{4}{3}$ sq. units d) $\frac{5}{3}$ sq. units

82. The area enclosed by the parabola $y^2 = 2x$ and the straight line $y = x$ is _____ sq. units

- a) $16/3$ b) $2/3$ c) $4/3$ d) $8/3$

83. The area of the figure bounded by $y^2 = 9x$ and $y = 3x$ is

84. The area enclosed between the curves $y = \sin^2 x$ and $y = \cos^2 x$ in the interval $0 \leq x \leq \pi$ is

- a) 2 sq. units b) $\frac{1}{2}$ sq. units c) 1 sq. units d) 3 sq. units

85. Area bounded by the curve $y = \log_e x$, $x > 0$ and x axis

- a) 1 sq. units b) $\frac{1}{2}$ sq. units c) 2 sq. units d) 4 sq. units

86. Area bounded by the curve $y = x^3$, the x - axis and the ordinate $x = -2$ and $x = 1$ is

87. The area enclosed by the parabola $x^2 + 4y = 4$ and x - axis is _____ sq. units

- a) $16/3$ b) $8/3$ c) $4/3$ d) 2

88. The area bounded by the curve $y = \log x$, x -axis and ordinate $x = e$ is

$$89. \quad \int_0^1 \log \left(\sin \frac{\pi}{2} x \right) dx =$$

- a) $-\log_e^3$ b) $-\log_e^2$ c) \log_e^7 d) \log_e^5

$$90. \quad \int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx =$$