

INTEGRATIONS AND APPLICATIONS OF DEFINITE INTEGRALS

1. $\int \frac{x \sin x^2 e^{\sec x^2}}{\cos^2 x^2} dx =$
 a) $2 e^{\sec x^2} + c$ b) $\frac{1}{2} e^{\sec x} + c$ c) $e^{\sec x^2} + c$ d) $\frac{1}{2} \sec x^2 + c$

2. $\int \frac{dx}{3 \cos x - 4 \sin x + 5} =$
 a) $\tan^{-1} \left(\frac{1}{2} \tan \frac{x}{2} \right) + c$ b) $\frac{1}{\tan \frac{x}{2} - 2} + c$
 c) $\frac{1}{2 - \tan \frac{x}{2}} + c$ d) $\frac{1}{2 - \tan x} + c$

3. $\int \cos \sqrt{x} dx =$
 a) $2 [\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$ b) $[\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$
 c) $2 [\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$ d) $2 [\sqrt{x} \sin \sqrt{x} + \sin \sqrt{x}] + c$

4. $\int \frac{\sin x}{\sin^2 x + 4 \cos^2 x} dx =$
 a) $\cos x + c$ b) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\sec x}{\sqrt{3}} \right) + c$
 c) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x}{3} \right) + c$ d) $\frac{1}{\sqrt{3}} \cot^{-1} \left(\frac{\sec x}{\sqrt{3}} \right) + c$

5. $\int \frac{e^x dx}{(1 + e^x)(2 + e^x)} =$
 a) $\log (2 + e^x) (1 + e^x) + c$ b) $\log \left(\frac{2 + e^x}{1 + e^x} \right) + c$
 c) $\log \left(\frac{1 + e^x}{2 + e^x} \right) + c$ d) $\log (1 + e^x)^2$

6. $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$
 a) $\frac{1}{\sqrt{2}} \sin^{-1} (\sin x + \cos x) + c$ b) $\sqrt{2} \sin^{-1} (\sin x + \cos x) + c$
 c) $\sqrt{2} \cos^{-1} (\cos x + \sin x) + c$ d) $\sqrt{2} \sin^{-1} (\sin x - \cos x) + c$

7. $\int \frac{x^3 dx}{(1 + x^2)^2} =$
 a) $-\frac{1}{1 + x^2} + c$ b) $\log \sqrt{1 + x^2} + \frac{1}{2} (1 + x^2) + c$
 c) $\log (1 + x^2) + c$ d) $-\frac{1}{1 + x} + c$

8. $\int \tan x \sqrt{\sec x} dx =$
 a) $2\sqrt{\sec x}$ b) $\frac{1}{2\sqrt{\sec x}}$ c) $\sqrt{\cos x}$ d) $\frac{2}{5} (\cos x)^{5/2}$

9. $\int \frac{x+4}{(x+5)^2} e^x dx$
 a) $\frac{e^x}{x+4}$ b) $\frac{e^x}{x+5}$ c) $\frac{e^x}{(x+5)^2}$ d) $\frac{e^x}{(x+4)^2}$

10. $\int \frac{dx}{4x^2 + 12x + 45} dx =$
 a) $\frac{1}{12} \tan^{-1} \left(\frac{2x+3}{6} \right)$ b) $\frac{1}{3} \tan^{-1} (2x + 3)$

- a) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2+1}{\sqrt{2x}} \right) + C$ b) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{2x}} \right) + C$
- c) $\frac{1}{2} \log \left(\frac{x+1}{x-1} \right) + C$ d) $\tan^{-1} \left(\frac{x^2-1}{\sqrt{2x}} \right) + C$
25. $\int \sqrt{1 + \sin\left(\frac{x}{3}\right)} dx =$
- a) $6 \left(\cos\frac{x}{6} - \sin\frac{x}{6} \right) + C$ b) $6 \left(\sin\frac{x}{6} - \cos\frac{x}{6} \right) + C$
- c) $\frac{1}{6} \left(\cos\frac{x}{6} - \sin\frac{x}{6} \right) + C$ d) $\left(\sin\frac{x}{6} - \cos\frac{x}{6} \right) + C$
26. $\int \sec^2 x \operatorname{cosec}^2 x dx =$
- a) $\tan x - \cot x + C$ b) $\tan x + \cot x + C$
- c) $\cot x - \tan x + C$ d) $\tan x \cot x + C$
27. $\int \operatorname{cosec}^4 x dx =$
- a) $\frac{1}{3} \cot^3 x + C$ b) $\cot x + \frac{1}{3} \cot^3 x + C$
- c) $-\cot x - \frac{1}{3} \cot^3 x + C$ d) $-\cot x + \frac{1}{3} \cot^3 x + C$
28. $\int \sqrt{\frac{1-x}{1+x}} dx =$
- a) $\sin^{-1} x + \sqrt{1-x^2} + C$ b) $\cos^{-1} x + \sqrt{1-x^2} + C$
- c) $\sin^{-1} x + \sqrt{x^2-1} + C$ d) $\log \sqrt{1+x} + C$
29. $\int \sinh(\log x) dx =$
- a) $\cosh \log x$ b) $\frac{1}{x} \cosh \log x$
- c) $x \cosh \log x$ d) $\frac{x^2}{4} - \log \sqrt{x}$
30. $\int \frac{dx}{(2x-1)\sqrt{2x-2}} =$
- a) $\tan^{-1}(2x-2)$ b) $\tan^{-1} \sqrt{2x-2}$
- c) $\frac{3}{2} \log(2x-3)$ d) none
31. $\int x^x \log(ex) dx =$
- a) $x^x + c$ b) $x^x \log x + c$ c) $\frac{1}{2} x^{2x} + c$ d) $x^x \log(ex) + c$
32. $\int \frac{\cos x}{\cos\left(x - \frac{\pi}{4}\right)} dx =$
- a) $\frac{1}{\sqrt{2}} [x + \log(\sin x + \cos x)]$ b) $\sqrt{2} [x + \log(\sin x + \cos x)]$
- c) $\frac{1}{\sqrt{2}} x + \log(\sin x + \cos x)$ d) $\frac{1}{\sqrt{2}} x + \log(\sin x - \cos x)$
33. $\int \frac{x-1}{(x+1)^3} e^x dx =$
- a) $e^x \frac{1}{(x+1)^2}$ b) $-e^x \frac{1}{(1+x)^2}$ c) $e^x \frac{1}{(1+x)^3}$ d) $e^x \frac{1}{(x+1)^3}$
34. If $\int \frac{\sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx = Ax + B \log(3 \sin x + 4 \cos x) + c$, then $A + B =$
- a) $\frac{4}{5}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) 2
35. If $I_n = \int_0^{\pi/4} \cot^n x dx$, then $I_n + I_{n-2} =$
- a) $\frac{1}{n}$ b) $\frac{1}{n-1}$ c) $\frac{1}{n-2}$ d) $\frac{1}{1-n}$
36. $\int_0^{\pi} \frac{dx}{\cos^2 x + 3 \sin^2 x} =$

- a) $\pi/\sqrt{2}$ b) π c) $\pi/\sqrt{3}$ d) $\frac{\pi}{2}$
37. $\int_0^2 \sqrt{\frac{2+x}{2-x}} dx =$
 a) $\frac{\pi}{2} + 1$ b) $\pi + 1$
 c) $\pi - 2$ d) $\pi + 2$
38. $\int_0^1 \sin^2 (\cos^{-1} x) dx$
 a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) 2 d) $\frac{\pi}{6}$
39. $\int_0^\infty \frac{dx}{(x^2 + a^2)(x^2 + b^2)} =$
 a) $\frac{\pi}{2ab(a+b)}$ b) $\frac{-\pi}{2ab(a+b)}$ c) $\frac{\pi}{ab(a+b)}$ d) $\frac{\pi ab}{a+b}$
40. $\int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{3 + \sin 2x}} dx =$
 a) $\pi/2$ b) $\pi/4$ c) $\pi/8$ d) π
41. $\int_0^{\pi/6} \sin 12x \cos 6x dx =$
 a) $2/9$ b) $1/9$ c) $1/18$ d) $5/18$
42. $\int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x} =$
 a) $\frac{1}{2} \log 2$ b) $2 \log 2$ c) $\log 2$ d) 7
43. $\int_0^1 \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx =$
 a) -1 b) 1 c) 0 d) $\pi/2 - 1$
44. $\int_0^1 \cos^{-1} \frac{2x}{1+x^2} dx =$
 a) $\pi - \log 2$ b) $-\log 2$ c) $\pi + \log 2$ d) $\log 2$
45. $\int_0^{2\pi} |x \sin x| dx =$
 a) 4π b) 3π c) 2π d) π
46. $\int_0^1 \frac{x^3}{1+x^8} dx =$
 a) $\frac{3\pi}{16}$ b) $-\frac{\pi}{16}$ c) $\frac{\pi}{16}$ d) $\frac{\pi}{4}$
47. $\int_1^4 e^{\sqrt{x}} dx =$
 a) $-2e^2$ b) $3e^2$ c) e d) $2e^2$
48. $\int_0^{\pi/2} |\sin x - \cos x| dx =$
 a) 0 b) $\sqrt{2} - 1$ c) $4\sqrt{2} + 2$ d) $2\sqrt{2} - 2$
49. $\int \frac{1}{(e^x - 1)^2} dx =$
 a) $-\frac{1}{e^x - 1}$ b) $-\log(e^x - 1) - \frac{1}{e^x - 1}$
 c) $x + \log(e^x - 1) - \frac{1}{e^x - 1}$ d) $x - \log(e^x - 1) - \frac{1}{e^x - 1}$
50. If $U_n = \int_1^e (\log x)^n dx$, $U_n + nU_{n-1} =$
 a) e^n b) e c) $e - 1$ d) 1

- a) $\frac{1}{3}$ sq. units b) $\frac{1}{6}$ sq. units c) $\frac{2}{3}$ sq. units d) $\frac{1}{12}$ sq. units
65. Area bounded by the curve $y^2 = 16x$ and the line $y = mx$ is $\frac{2}{3}$ then m is equal to
a) 3 b) 4 c) 1 d) 2
66. Area enclosed by $y = \sin x$ and x axis between $x = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ is
a) 0 b) 1 c) 2 d) none
67. The area between the parabolas $y^2 = 9x$ and $x^2 = 9y$ is _____ sq. units
a) 9 b) 18 c) 27 d) 7
68. Area of the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ is
a) 100π b) 50π c) 25π d) 10π
69. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x}{\sin x + \cos x} dx =$
a) $\pi/4$ b) $\pi/8$ c) $\pi/6$ d) $\pi/12$
70. $\int_{-\pi}^{3\pi} \log(\sec \theta - \tan \theta) d\theta =$
a) 1 b) 0 c) 2 d) 7
71. $\int_0^{\frac{\pi}{2}} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx =$
a) $(a + b)\pi$ b) $(a + b)\frac{\pi}{2}$ c) $(a + b)\frac{\pi}{4}$ d) $a + b$
72. $\int_0^{\pi} x \cos^2 x \sin x dx =$
a) $1/3$ b) $2/3$ c) $\pi/3$ d) 1
73. $\int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}} dx =$
a) 5 b) 10 c) $5/2$ d) 7
74. $\int_0^{\pi} \frac{dx}{1 + 2^{\cos x}} =$
a) π b) 0 c) $\pi/2$ d) 1
75. $\int_0^1 x(1-x)^n dx =$
a) $\frac{1}{n(n+1)}$ b) $\frac{1}{(n+1)(n+2)}$ c) $\frac{1}{n+1} + \frac{1}{n+2}$ d) $\frac{1}{n(n-1)}$
76. $\int_0^1 x^3(1-x^2)^5 dx =$
a) $1/30$ b) $1/42$ c) $1/84$ d) $10/231$
77. $\int_0^{\pi/2} \log \tan x dx =$
a) $\pi/2$ b) $\pi/4$ c) $\pi/8$ d) 0
78. $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^2 x} dx =$
a) 1 b) π c) $\pi/4$ d) $\pi/2$
79. $\int_0^{\frac{\pi}{2}} \frac{1}{1 + (\cot x)^{101}} dx =$
a) 0 b) $\pi/2$ c) $\pi/3$ d) $\pi/4$

80. The value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \left[\log \left(\frac{1-x}{1+x} \right) \right] dx$ is
- a) 0 b) $e^{1/2}$ c) 1 d) $2e^{1/2}$
81. The area of the region bounded by the curve $y^2 = 2y - x$ and the y - axis is
- a) $\frac{1}{3}$ sq. units b) $\frac{2}{3}$ sq. units c) $\frac{4}{3}$ sq. units d) $\frac{5}{3}$ sq. units
82. The area enclosed by the parabola $y^2 = 2x$ and the straight line $y = x$ is _____ sq. units
- a) $16/3$ b) $2/3$ c) $4/3$ d) $8/3$
83. The area of the figure bounded by $y^2 = 9x$ and $y = 3x$ is
- a) 1 b) $1/4$ c) $1/2$ d) 2
84. The area enclosed between the curves $y = \sin^2 x$ and $y = \cos^2 x$ in the interval $0 \leq x \leq \pi$ is
- a) 2 sq. units b) $\frac{1}{2}$ sq. units c) 1 sq. units d) 3 sq. units
85. Area bounded by the curve $y = \log_e x$, $x > 0$ and x axis
- a) 1 sq. units b) $\frac{1}{2}$ sq. units c) 2 sq. units d) 4 sq. units
86. Area bounded by the curve $y = x^3$, the x - axis and the ordinate $x = -2$ and $x = 1$ is
- a) 9 b) $\frac{15}{4}$ c) $\frac{17}{4}$ d) $\frac{1}{4}$
87. The area enclosed by the parabola $x^2 + 4y = 4$ and x - axis is _____ sq. units
- a) $16/3$ b) $8/3$ c) $4/3$ d) 2
88. The area bounded by the curve $y = \log x$, x - axis and ordinate $x = e$ is
- a) e b) 1 c) ∞ d) 2
89. $\int_0^1 \log \left(\sin \frac{\pi}{2} x \right) dx =$
- a) $-\log_e^3$ b) $-\log_e^2$ c) \log_e^7 d) \log_e^5
90. $\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx =$
- a) 0 b) 1 c) $\pi/4$ d) none of these