

If $\frac{d}{dx} [f(x) + C] = g(x) + 0$

then $\int g(x) dx = f(x) + C$

Thus the process of finding the function $f(x)$ whose differential coefficient w.r.t. ‘ x ’ is $g(x)$ is called the **integration** of $g(x)$ w.r.t. ‘ x ’

$$\int \sin ax dx = -\frac{\cos ax}{a} + C$$

$$\int \sec^2 7x dx = \frac{\tan 7x}{7} + C$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} + C$$

$$\int \frac{1}{x} dx = \log x + C$$

$$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$$

$$\int \sqrt{1 - \sin 2x} dx =$$

$$\int \sqrt{1 - \sin 2x} dx =$$

1) $\sin x - \cos x$

1

2) $\frac{1}{2}(\sin x + \cos x)^{3/2}$

3) $\sin x + \cos x$

4) $\sqrt{\sin x + \cos x}$

Ans:(3)

$$\begin{aligned}& \int \sqrt{1 - \sin 2x} \, dx \\& \int \sqrt{\cos^2 x + \sin^2 x - 2 \sin x \cos x} dx \\& = \int \sqrt{(\cos x - \sin x)^2} dx \\& = \int (\cos x - \sin x) dx \\& = \sin x + \cos x\end{aligned}$$

$$\int \frac{e^x}{3 \sinh x + 3 \cosh x} dx =$$

$$\int \frac{e^x}{3 \sinh x + 3 \cosh x} dx =$$

1) $\frac{x}{3}$

2) $\frac{x}{9}$

3) $\frac{x}{6}$

4) x

Ans (1)

$$3 \sinh x + 3 \cosh x =$$

$$3\left(\frac{e^x - e^{-x}}{2}\right) + 3\left(\frac{e^x + e^{-x}}{2}\right) = 3e^x$$

$$\int \frac{e^x}{3 \sinh x + 3 \cosh x} dx = \int \frac{e^x}{3e^x} dx =$$

$$\int \frac{1}{3} dx = \frac{x}{3}$$

$$\int \frac{\tan x}{\sec x + \tan x} dx =$$

$$\int \frac{\tan x}{\sec x + \tan x} dx =$$

- 1)** $\frac{1}{2}\log(\sec x + \tan x)$ **2)** $2\log(\sec \frac{x}{2})$
3) $\sec x - \tan x + x$ **4)** $\sec x + \tan x$

Ans (3)

$$\begin{aligned}\int \frac{\tan x}{\sec x + \tan x} dx &= \int \frac{\tan x(\sec x - \tan x)}{(\sec^2 x - \tan^2 x)} dx \\ \sec^2 x - \tan^2 x &= 1 \\ &= \int [\sec x \tan x - \tan^2 x] dx \\ &= \int [\sec x \tan x - (\sec^2 x - 1)] dx \\ &= \sec x - \tan x + x\end{aligned}$$

$$\frac{d[x^{n+1}]}{dx} = (n+1)x^n, \therefore \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$$

$$\text{Ex: } \int x^{100} dx = \frac{x^{101}}{101} + C, \quad \int x^{-17} dx = \frac{x^{-16}}{-16} + C,$$

$$\int x^{\frac{-1}{3}} dx = \frac{3x^{\frac{2}{3}}}{2} + C.$$

$$\therefore \int t^n dt = \frac{t^{n+1}}{n+1} + C, n \neq -1$$

$$\int \sin^n x \cos x dx = \frac{\sin^{n+1} x}{n+1} + C, n \neq -1$$

$$\int \frac{x^7 + x^5 + 1}{x^2 + 1} dx =$$

$$\int \frac{x^7 + x^5 + 1}{x^2 + 1} dx =$$

1) $\frac{x^6}{6} + \tan^{-1} x$

2) $\frac{x^6}{3!} + \log(x + 1)$

3) $\frac{x^6}{3!} + \frac{x^3}{3} + \tan^{-1} x$

4) $\frac{x^6}{3} + \tan^{-1} x$

Ans(1)

$$\begin{aligned} \int \frac{x^7 + x^5 + 1}{x^2 + 1} dx &= \int \frac{x^5(x^2 + 1) + 1}{x^2 + 1} dx \\ &= \int x^5 dx + \int \frac{1}{x^2 + 1} dx = \frac{x^6}{6} + \tan^{-1} x \end{aligned}$$

$$\int \frac{\sin^n x}{\cos^{n+2} x} dx = \int \tan^n x \sec^2 x dx$$

$$= \frac{\tan^{n+1} x}{n+1} + C$$

$$\int \frac{\cos^n x}{\sin^{n+2} x} dx = \int \cot^n x \sec^2 x dx$$

$$= - \frac{\cot^{n+1} x}{n+1} + C$$

$$\int \frac{1 - \cos^2 x}{\cos^4 x} dx =$$

$$\int \frac{1 - \cos^2 x}{\cos^4 x} dx =$$

- 1)** $-\frac{\cot^3 x}{3} + C$ **2)** $\frac{\tan^3 x}{3} + C$
3) $\cot^3 x + C$ **4)** $\tan^3 x + C$

Ans (2)

$$\begin{aligned}& \int \frac{1 - \cos^2 x}{\cos^4 x} dx \\&= \int \frac{\sin^2 x}{\cos^4 x} dx \\&= \int \tan^2 x \sec^2 x dx \\&= \frac{\tan^3 x}{3} + c \text{ (Put } \tan x = t)\end{aligned}$$

$$\int (ax + b)^n dx =$$

$$\frac{1}{a} \frac{(ax + b)^{n+1}}{n+1} + C,$$

$$n \neq -1$$

$$\int (x^2 + 2x + 1) dx = \int (x+1)^2 dx = \frac{(x+1)^3}{3}$$

$$\int (x^3 + 3x^2 + 3x + 1) dx = \int (x+1)^3 dx = \frac{(x+1)^4}{4} + C$$

$$\int (1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7) dx =$$

$$\int (1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7) dx =$$

1) $\frac{(1+x)^6}{6}$

2) $\frac{(1+x)^7}{7}$

3) $\frac{(1+x)^8}{2^4}$

4) $\frac{(1+x)^8}{2^3}$

Ans: (4)

$$\int (1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7) dx$$
$$= \int \frac{(1+x)^7}{7} dx = \frac{(1+x)^8}{2^3}$$

$$\int \frac{1 + \cos x}{x^2 + 2x \sin x + \sin^2 x} dx =$$

$$\int \frac{1 + \cos x}{x^2 + 2x \sin x + \sin^2 x} dx =$$

1) $-\frac{1}{x + \sin x}$

2) $-\frac{1}{3(x + \sin x)^3}$

3) $\log(x + \sin x)$ **4)** $\log \cos x$

$$\text{Ans. (1)} \int \frac{1 + \cos x}{x^2 + 2x \sin x + \sin^2 x} dx$$
$$= \int \frac{1 + \cos x}{(x + \sin x)^2} dx$$

let $t = x + \sin x$ then dt

$$= (1 + \cos x)dx$$

the integral becomes

$$\int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{x + \sin x}$$

Integrals of the type

$$\int \left(\frac{a\cos x + b\sin x}{c\cos x + d\sin x} \right) dx$$

Express numerator

$$a\cos x + b\sin x = \ell (\text{Dr}) + m \frac{d}{dx} (\text{Dr})$$

$$\ell = \frac{ac + bd}{c^2 + d^2} \quad \text{and} \quad m = \frac{ad - bc}{c^2 + d^2}$$

$$\int \left(\frac{a\cos x + b\sin x}{c\cos x + d\sin x} \right) dx = \ell x + m \log(\text{Dr})$$

$$\int \frac{3 \cos x + 2 \sin x}{2 \cos x + \sin x} dx =$$

$$\int \frac{3 \cos x + 2 \sin x}{2 \cos x + \sin x} dx =$$

- 1)** $\frac{8}{5} \log(2 \cos x + \sin x) - \frac{1}{5} x + C$
- 2)** $\frac{8}{5} \log(2 \cos x + \sin x) + \frac{1}{5} x + C$
- 3)** $-\frac{1}{5} \log(2 \cos x + \sin x) - \frac{8}{5} x + C$
- 4)** $-\frac{1}{5} \log(2 \cos x + \sin x) + \frac{8}{5} x + C$

Ans: (4)

Let Numerator,

$$3\cos x + 2\sin x = l(Dr) + m\left(\frac{d(Dr)}{dx}\right)$$

$$l = \frac{ac + bd}{c^2 + d^2} = \frac{6 + 2}{5};$$

$$m = \frac{ad - bc}{c^2 + d^2} = \frac{3 - 4}{5} = \frac{-1}{5}$$

$$\int \frac{3\cos x + 2\sin x}{2\cos x + \sin x} dx = lx + m\log(Dr)$$

$$= \frac{8}{5}x - \frac{1}{5}\log(2\cos x + \sin x)$$

$$\int \frac{5 \cot x - 2}{2 \cot x + 3} dx =$$

$$\int \frac{5 \cot x - 2}{2 \cot x + 3} dx =$$

1) $\frac{4}{13} \log(2 \cot x + 3) + \frac{19}{13} x + C$

2) $\frac{19}{13} \log(2 \cos x + 3 \sin x) + \frac{4}{13} x + C$

3) $\frac{11}{13} \log(2 \cot x + 3) + \frac{4}{13} x + C$

4) $\frac{19}{13} \log(2 \cot x + 3) + \frac{4}{13} x + C$

Ans: (2)

$$\int \frac{5 \cot x - 2}{2 \cot x + 3} dx = \int \frac{5 \cos x - 2 \sin x}{2 \cos x + 3 \sin x} dx$$
$$= l x + m \log(2 \cos x + 3 \sin x)$$

$$l = \frac{ac + bd}{c^2 + d^2} = \frac{4}{13}$$

$$m = \frac{ad - bc}{c^2 + d^2} = \frac{19}{13}$$

solution is

$$= \frac{19}{13} \log(2 \cos x + 3 \sin x) + \frac{4}{13} x + C$$

$$\int e^x [f(x) + f'(x)] dx = e^x f(x) + c$$

Examples:

$$1) \int e^x (\sin^2 x + \sin 2x) dx = e^x \sin^2 x + c$$

$$2) \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right) dx = e^x \sin^{-1} x + c$$

$$\int e^x (\tan x + \tan^2 x - 2011) dx =$$

$$\int e^x (\tan x + \tan^2 x - 2011) dx =$$

1) $-2011 - e^x \tan x + C$

2) $e^x (\tan x - 2012) + C$

3) $-e^x (2012 + \tan x) + C$

4) $2011 - e^x \cot x + C$

Ans:(2)

$$\begin{aligned}\int e^x [f(x) + f'(x)] dx &= e^x f(x) + C \\ \int e^x (-2012 + \tan x + 1 + \tan^2 x) dx \\ &= \int e^x (-2012 + \tan x + \sec^2 x) \\ &= e^x (-2012 + \tan x) + C\end{aligned}$$

$$\int e^x \left(1 + x + \frac{1}{x} - \frac{1}{x^2}\right) dx =$$

$$\int e^x \left(1 + x + \frac{1}{x} - \frac{1}{x^2}\right) dx =$$

1) $e^x \left(x + \frac{1}{x}\right)$

2) $\frac{e^x}{x}$

3) $e^x \left(\frac{1}{x} + \frac{1}{x^2}\right)$

4) $e^x \left(x + \frac{1}{x^2}\right)$

Ans: (1)

$$\int e^x \left(1 + x + \frac{1}{x} - \frac{1}{x^2}\right) dx$$

$$= \int e^x \left[\left(x + \frac{1}{x}\right) + \left(1 - \frac{1}{x^2}\right)\right] dx$$

$$= e^x \left(x + \frac{1}{x}\right) + C$$

$$\int e^x \left[\frac{x-5}{(x-4)^2} \right] dx =$$

$$\int e^x \left[\frac{x-5}{(x-4)^2} \right] dx =$$

1) $e^x \left(\frac{1}{x-5} \right)$

2) $e^x \left(\frac{1}{x-3} \right)$

3) $e^x \left(\frac{1}{x-6} \right)$

4) $e^x \left(\frac{1}{x-4} \right)$

$$\begin{aligned}
 \text{Ans:(d)} \quad & \int e^x \left[\frac{x+k}{(x+k+1)^2} \right] dx \\
 = & \int e^x \left[\frac{x+k+1-1}{(x+k+1)^2} \right] dx \\
 = & \int e^x \left[\frac{1}{(x+k+1)} - \frac{1}{(x+k+1)^2} \right] dx \\
 = & \int e^x [f(x) + f'(x)] dx \quad f(x) = \frac{1}{x+k+1} \\
 \therefore \int e^x \left[\frac{x+k}{(x+k+1)^2} \right] dx &= \frac{e^x}{x+k+1} + C \\
 \int e^x \left[\frac{x-5}{(x-4)^2} \right] dx &= e^x \left(\frac{1}{x-4} \right) \quad \text{here } k = -5
 \end{aligned}$$

$$\frac{1}{2} \int_0^1 (\sin^{-1} x + \cos^{-1} x) dx =$$

$$\int_0^{\frac{1}{2}} \left(\sin^{-1} x + \cos^{-1} x \right) dx =$$

1) $\frac{\pi}{2}$ 2) $\frac{\pi^2}{4}$

3) $\frac{\pi}{4}$ 4) $\frac{\pi^2}{8}$

Ans: (3)

$$\int_0^{\frac{1}{2}} (\sin^{-1} x + \cos^{-1} x) dx$$

$$= \int_0^{\frac{1}{2}} \left(\frac{\pi}{2} - \frac{x}{\sqrt{1-x^2}} \right) dx = \frac{\pi}{2} [x]_0^{\frac{1}{2}}$$

$$= \frac{\pi}{4}$$

General Properties of Definite Integrals

$$\text{I} \int_a^b f(x)dx = \int_a^b f(t)dt.$$

$$\text{II} \int_a^b f(x)dx = - \int_b^a f(x)dx.$$

$$\text{III} \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

where $a \leq c \leq b$

$$\int_0^4 |x - 1| dx =$$

$$\int_0^4 |x - 1| dx =$$

- 1) 4 2) 9/2 3) 5 4) 3

Ans:(3)

$$\int_0^4 |x - 1| dx =$$

$$\int_0^1 -(x - 1) dx + \int_1^4 (x - 1) dx$$

$$\left[\frac{-x^2}{2} + x \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^4$$

Evaluating we get I = 5

$$\text{IV} \quad \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

In Perticular

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{1 + e^{\sin x}} \right) dx =$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{1 + e^{\sin x}} \right) dx =$$

- 1) π 2) $\frac{\pi}{2}$ 3) $\frac{3\pi}{2}$ 4) $\frac{2\pi}{3}$

Ans: (2)

$$\text{Let } f(x) = \frac{1}{1+e^{\sin x}} \quad a = -\frac{\pi}{2} \quad b = \frac{\pi}{2}$$

$$f(a+b-x) = \frac{1}{1+e^{-\sin x}} = \frac{1}{1+\frac{1}{e^{\sin x}}} = \frac{e^{\sin x}}{e^{\sin x} + 1}$$

$$\text{If } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{1+e^{\sin x}} \right) dx \quad \text{then} \quad I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{e^{\sin x}}{1+e^{\sin x}} \right) dx$$

adding both

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+e^{\sin x}}{1+e^{\sin x}} \right) dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 dx = x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

- If $a + b = \frac{\pi}{2}$
- $\int_a^b \sin^2 x dx = \frac{b-a}{2}$
- $\int_a^b \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{b-a}{2}$
- $\int_a^b \frac{\tan^n x}{1 + \tan^n x} dx = \frac{b-a}{2}$
- $\int_a^b \frac{1}{1 + \tan^n x} dx = \frac{b-a}{2}$

$$\int_0^{\frac{\pi}{2}} \frac{7^{\frac{1}{3} \cos x} dx}{7^{\frac{1}{3} \sin x} + 7^{\frac{1}{3} \cos x}} =$$

$$\int_0^{\frac{\pi}{2}} \frac{7^{\frac{1}{3} \cos x} dx}{7^{\frac{1}{3} \sin x} + 7^{\frac{1}{3} \cos x}} =$$

1) π

2) 0

3) $\frac{\pi}{4}$

4) 1

Ans: (3)Standard problem

$$\int_0^{\frac{\pi}{2}} \frac{7^{n \cos x} dx}{7^{n \sin x} + 7^{n \cos x}} = \frac{\pi}{4}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \tan^3 x} dx =$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \tan^3 x} dx =$$

- 1) $\pi/2$ 2) $\pi/12$ 3) 0 4) $\pi/4$

Ans: (2)

$$\text{Let } I = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \tan^3 x} dx$$

$$\text{Here } a = \frac{\pi}{6}, \quad b = \frac{\pi}{3}, \quad a + b = \frac{\pi}{2}$$

$$I = \frac{b - a}{2} = \frac{\pi}{12}$$

$$\int_2^{13} \frac{\sqrt{x}}{\sqrt{15-x} + \sqrt{x}} dx =$$

$$\int_2^{13} \frac{\sqrt{x}}{\sqrt{15-x} + \sqrt{x}} dx =$$

- 1) $\frac{11}{2}$ 2) $\frac{1}{2}$ 3) 1 4) 2

$$\text{Ans:(1)} I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx \quad \dots \quad (1)$$

$$\begin{aligned} \text{Then } I &= \int_2^3 \frac{\sqrt{5-x}}{\sqrt{5-(5-x)} + \sqrt{5-x}} dx \\ &= \int_2^3 \frac{\sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} dx \quad \dots \quad (2) \end{aligned}$$

$$\begin{aligned} \text{Adding: } 2I &= \int_2^3 dx = x \Big|_2^3 = 3 - 2 = 1 \\ \therefore I &= \frac{1}{2} \end{aligned}$$

$$\int_0^1 x(1-x)^{16} dx =$$

$$\int_0^1 x(1-x)^{16} dx =$$

1) $1/301$

3) $2/301$

2) $1/272$

4) $1/306$

Ans: (4)

$$\begin{aligned} I &= \int_0^1 (1-x)x^{16} dx \\ &= \int_0^1 (x^{16} - x^{17}) dx \\ &= \frac{1}{17} - \frac{1}{18} = \frac{1}{306} \end{aligned}$$

$$\int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3 \sin x}{4 + 3 \cos x} \right) dx =$$

$$\int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3 \sin x}{4 + 3 \cos x} \right) dx =$$

- 1) $4/3$ 2) 2 3) 0 4) 1

Ans: (3)

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx \quad \dots \dots \dots (1)$$

Then

$$I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin \left(\frac{\pi}{2} - x \right)}{4+3\cos \left(\frac{\pi}{2} - x \right)} \right) dx$$

$$I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\cos x}{4+3\sin x} \right) dx \quad \dots \dots \dots (2)$$

Adding (1) and (2):

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \left[\log \left(\frac{4+3\sin x}{4+3\cos x} \right) + \log \left(\frac{4+3\cos x}{4+3\sin x} \right) \right] dx \\ &= \int_0^{\frac{\pi}{2}} \log 1 \, dx = \int_0^{\frac{\pi}{2}} 0 \, dx = 0 \therefore I = 0 \end{aligned}$$

$$\text{If } \int_0^\infty \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)(x^2 + c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$$

$$\text{then the value of } \int_0^\infty \frac{dx}{(x^2 + 4)(x^2 + 9)} =$$

If $\int_0^\infty \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)(x^2 + c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$

then the value of $\int_0^\infty \frac{dx}{(x^2 + 4)(x^2 + 9)} =$

1) $\pi/60$

2) $\pi/20$

3) $\pi/40$

4) $\pi/80$

Ans: (1)

Put $a = 2$, $b = 3$ and $c = 0$

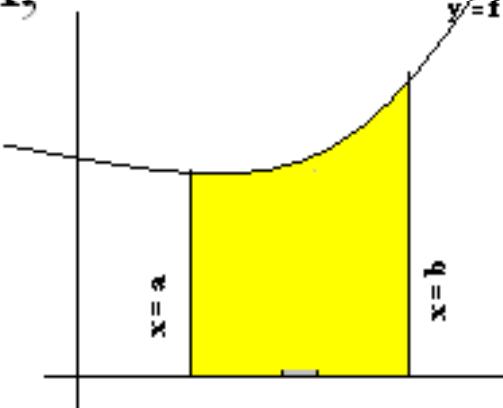
$$\text{Then } \int_0^{\infty} \frac{x^2 dx}{(x^2 + 4)(x^2 + 9)(x^2 + 0)} =$$

$$\frac{\pi}{2(2+3)(3+0)(0+2)}$$

$$\text{i.e., } \int_0^{\infty} \frac{dx}{(x^2 + 4)(x^2 + 9)} = \frac{\pi}{60}$$

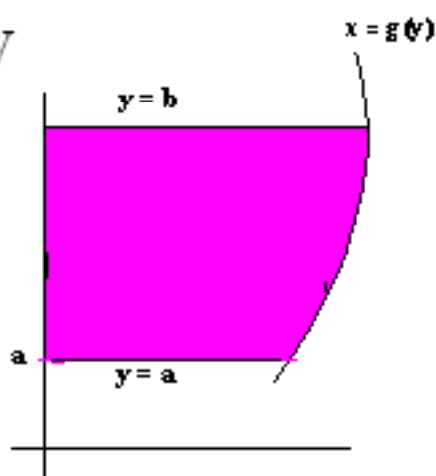
Definite integral as AREA

1. Area of the region bounded by $y=f(x)$,
x – axis and the ordinates $x=a$ and $x=b$
is $A = \int_a^b y dx = \int_a^b f(x)dx$,



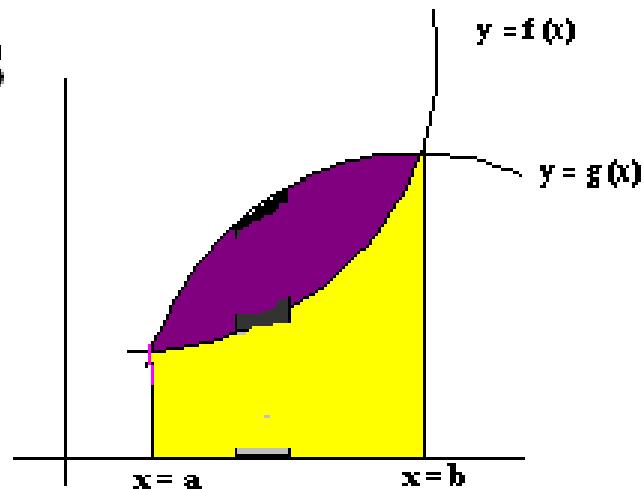
2. Area of the region bounded by $x=g(y)$,
y – axis and the lines $y = a$ and $y = b$ is

$$A = \int_a^b x \, dy = \int_a^b g(y) \, dy$$



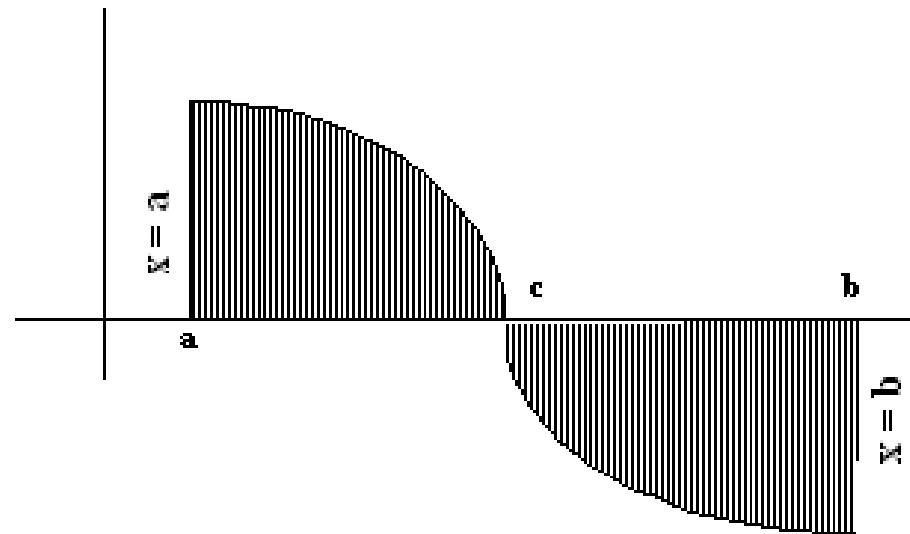
3. If two curves $y=f(x)$ and $y = g(x)$ intersect at $x = a$ and $x = b$ then area between the curves

$$A = \left| \int_a^b [f(x) - g(x)] dx \right|$$



4. If $f(x) \geq 0$ for $a \leq x \leq c$ and $f(x) \leq 0$, $c \leq x \leq b$, then

$$\int_a^b f(x) dx = \left| \int_a^c f(x) dx \right| + \left| \int_c^b f(x) dx \right|$$



The area bounded by $y = x^2 - 3x + 2$ and
x – axis is

The area bounded by $y = x^2 - 3x + 2$ and
x – axis is

- | | |
|----------------------------|----------------------------|
| 1) $\frac{1}{3}$ sq. units | 2) $\frac{1}{6}$ sq. units |
| 3) $\frac{2}{3}$ sq. units | 4) 1 sq. units |

Ans: (2)

Put $y = 0$, $x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$

$$A = \left| \int_1^2 [x^2 - 3x + 2] dx \right|$$
$$= \left| \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_1^2 \right| = \frac{1}{6} \text{ sq. units}$$

Area enclosed by $y = \sin x$ and x axis

between $x = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ is

Area enclosed by $y = \sin x$ and x axis

between $x = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ is

- 1) 0
- 2) 1
- 3) 2
- 4) 4

Ans: (3)

Area enclosed by $y = \sin x$ and x axis

between $x = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ is

$$\begin{aligned} A &= 2 \int_0^{\pi/2} \sin x \, dx \\ &= 2 [-\cos x]_0^{\pi/2} = 2 \end{aligned}$$

Area of the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ is

Area of the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ is

- 1) 100π
- 2) 50π
- 3) 25π
- 4) 10π

Ans (4)

$$a = 5, b = 2$$

Area of the ellipse = $\pi ab =$

$$\pi (5)(2) = 10\pi$$

The area of the figure bounded by
 $y^2 = 8x$ and $y = 2x$ is

The area of the figure bounded by
 $y^2 = 8x$ and $y = 2x$ is

- 1) 1 2) $\frac{1}{4}$ 3) $\frac{4}{3}$ 4) 2

Ans: (3)

$$y^2 = 8x \text{ ----- (1)}$$

$a = 2$ [comparing with $y^2 = 4ax$]

$$y = 2x \text{ ----- (2)} \quad m = 2$$

[comparing with $y = mx$]

$$\text{Required area} = \frac{8a^2}{3m^3} = \frac{8 \cdot 2^2}{3 \cdot 2^3} = \frac{4}{3}$$

Area between the curve $y = x^2$
and $y = 2 - x^2$ is

Area between the curve $y = x^2$
and $y = 2 - x^2$ is

- 1) $\frac{2}{3}$
- 2) $\frac{8}{3}$
- 3) $\frac{1}{3}$
- 4) $\frac{4}{3}$

Ans: (2)

$y = x^2$ and $y = 2 - x^2$ meet

$$\text{if } x^2 = 2 - x^2 \Rightarrow x = \pm 1$$

$$\therefore \text{Area} = \int_{-1}^1 (x^2 - (2 - x^2)) dx$$

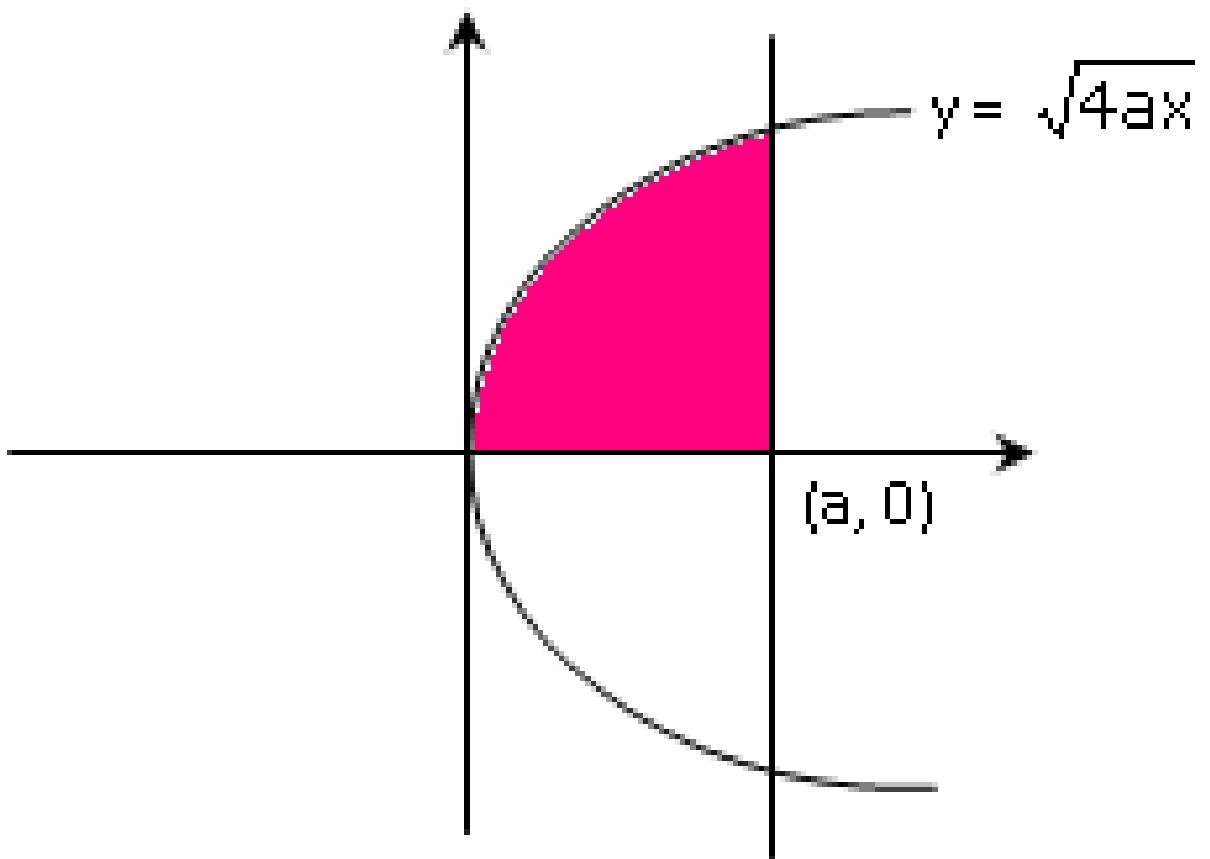
$$= \int_{-1}^1 (2x^2 - 2) dx$$

$$= \left| \frac{2x^3}{3} - 2x \right|_{-1}^1 = \left| \frac{4}{3} - 4 \right| = \frac{8}{3}$$

Area lying between the parabola
 $y^2 = 4ax$ and its latus rectum is

Area lying between the parabola
 $y^2 = 4ax$ and its latus rectum is

- 1) $\frac{8a^2}{3}$
- 2) $\frac{8a}{3}$
- 3) $\frac{4a}{3}$
- 4) $\frac{4a^2}{3}$



Ans: (1)

$$\text{Area} = 2 \int_0^a \sqrt{4ax} \, dx$$

$$= 2 \times 2\sqrt{a} = \int_0^a x^{1/2} \, dx = 4\sqrt{a} \left[\frac{x^{3/2}}{3/2} \right]_0^a$$

$$= \frac{8\sqrt{a}}{3} [a^{3/2} - 0] = \frac{8}{3} a^2$$

Area between $y^2 = 6x$ and $x^2 = 6y$ is

Area between $y^2 = 6x$ and $x^2 = 6y$ is

- 1) 36
- 2) 6
- 3) 2
- 4) 12

Ans: (4)

$$4a = 6 \Rightarrow a = \frac{3}{2}$$

$$\text{Area} = \frac{16a^2}{3} = \frac{16}{3} \times \frac{9}{4} = 12$$

$$\int a^x e^x dx =$$

$$\int a^x e^x dx =$$

$$1) \frac{e^x a^x}{\log a}$$

$$2) \frac{e^x a^x}{\log(ae)}$$

$$3) \frac{(ae)^x}{\log a}$$

$$4) (ae)^x$$

Ans: (2)

We have $\int k^x dx = \frac{k^x}{\log k}$

$$\int a^x e^x dx = \int (ae)^x$$

$$= \frac{(ae)^x}{\log(ae)} = \frac{e^x a^x}{\log(ae)}$$

$$\int \frac{\sin x}{\sin^2 x + 4\cos^2 x} dx =$$

$$\int \frac{\sin x}{\sin^2 x + 4\cos^2 x} dx =$$

1) $\cos x + C$

2) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\sec x}{\sqrt{3}} \right) + C$

3) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x}{3} \right) + C$

4) $\tan^{-1} \left(\frac{\sec x}{\sqrt{3}} \right) + C$

Ans:(2)

$$\int \frac{\sin x \ dx}{\sin^2 x + 4 \cos^2 x} = \int \frac{\sin x \ dx}{1 + 3 \cos^2 x}$$

Put $\sqrt{3} \cos x = t$

$$\Rightarrow -\sqrt{3} \sin x \ dx = dt$$

$$= \int \frac{-\frac{1}{\sqrt{3}} dt}{1+t^2} = \frac{1}{\sqrt{3}} \cot^{-1} t + C$$

$$= \frac{1}{\sqrt{3}} \cot^{-1} (\sqrt{3} \cos x) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sec x}{\sqrt{3}} + C$$

**THANK
YOU**

**THANK
YOU**