INTEGRATIONS AND APPLICATIONS OF DEFINITE INTEGRALS

1. (d)

Put $\sec x^2 = t$

$2x \sec x^2 \tan x^2\, dx = dt$

$$\int \frac{x \sin x^2 e^{\sec x^2}}{\cos^2 x^2}\, dx = \frac{1}{2} \int e^t\, dt = \frac{1}{2} e^t + c = \frac{1}{2} e^{\sec x^2} + C$$

2. (c)

Put $\tan \frac{x}{2} = t$

Then $\cos x = \frac{1-t^2}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$ $\quad dx = \frac{2dt}{1+t^2}$

$$\therefore \int \frac{dx}{3\cos x - 4\sin x + 5} = \int \frac{\frac{2dt}{1+t^2}}{3\left(\frac{1-t^2}{1+t^2}\right) - 4 \times \frac{2t}{1+t^2} + 5}$$

$$= \int \frac{2dt}{3(1-t^2) - 8t + 5(1+t^2)} = \int \frac{dt}{t^2 - 4t + 2} = \int \frac{dt}{(t-2)^2} = -\frac{1}{t-2} + C = -\frac{1}{\tan\frac{x}{2} - 2} + C$$

$$= \frac{1}{2 - \tan\frac{x}{2}} + c$$

3. (a)

Put $x = t^2 \Rightarrow dx = 2t\, dt$

$$\int \cos\sqrt{x}\, dx = \int \cos t .\, 2t\, dt = 2 \int t \cos t\, dt$$

$$= 2[t \sin t - \int \sin t\, dt] = 2\,[t \sin t + \cos t] + c$$

$$= 2\,[\sqrt{x} \sin\sqrt{x} + \cos\sqrt{x}\,] + c$$

4. (b)

$$\int \frac{\sin x\, dx}{\sin^2 x + 4\cos^2 x} = \int \frac{\sin x\, dx}{1 + 3\cos^2 x}$$

Put $\sqrt{3} \cos x = t \Rightarrow -\sqrt{3} \sin x\, dx = dt$

$$= \int \frac{-\frac{1}{\sqrt{3}} dt}{1+t^2} = \frac{1}{\sqrt{3}} \cot^{-1} t + c = \frac{1}{\sqrt{3}} \cot^{-1}(\sqrt{3} \cos x) + c = \frac{1}{\sqrt{3}} \tan^{-1} \frac{\sec x}{\sqrt{3}} + c$$

5. (c)

Put $e^x = t$, then $e^x\, dx = dt$

$$\int \frac{e^x\, dx}{(1+e^x)(2+e^x)} = \int \frac{dt}{(1+t)(2+t)} = \int \left(\frac{1}{1+t} - \frac{1}{2+t}\right) dt$$

$$= \log(1+t) - \log(2+t) + c = \log \frac{1+t}{2+t} + c = \log \frac{1+e^x}{2+e^x} + c$$

6. (d)

$$\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx = \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\, dx = \sqrt{2} \int \frac{(\sin x + \cos x)}{\sqrt{1 - (\sin x - \cos x)^2}}\, dx$$

Put $\sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

$$\sqrt{2} \int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} \sin^{-1} t + c = \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$$

7. (b)

Put $1 + x^2 = t$, then $2x\, dx = dt$

$$\int \frac{x^3}{(1+x^2)^2}dx = \int \frac{x^2\ x\ dx}{(1+x^2)^2} = \int \frac{(t-1)}{t^2}\ \frac{dt}{2}$$

$$= \frac{1}{2}\int \left(\frac{1}{t}-\frac{1}{t^2}\right)dt = \frac{1}{2}\left[\log\ t+\frac{1}{t}\right] + c = \frac{1}{2}\ \log\ (1+x^2) + \frac{1}{2\,(1+x^2)} + c$$

8. (a)

$$\int \tan x\ \sqrt{\sec x}\ dx = \int \frac{\tan x\ \sec\ x}{\sqrt{\sec\ x}}dx = 2\sqrt{\sec x} + c$$

9. (b)

$$\int e^x\ \frac{x+4}{(x+5)^2}dx = \int e^x\ \frac{x+5-1}{(x+5)^2}dx = \int e^x\left(\frac{1}{x+5}-\frac{1}{(x+5)^2}\right)dx$$

$$= \int e^x\ (f\ (x) + f^{\prime}\ (x)) = e^x\ f\ (x) = \frac{e^x}{x+5}$$

10. (a)

$$\int\frac{dx}{4x^2+12x+45} = \frac{1}{4}\int \frac{dx}{x^2+3x+\frac{45}{4}} = \frac{1}{4}\int\frac{dx}{\left(x+\frac{3}{2}\right)^2+\frac{45}{4}-\frac{9}{4}}$$

$$= \frac{1}{4}\int\frac{dx}{\left(x+\frac{3}{2}\right)+3^2} = \frac{1}{4}\times\frac{1}{3}\ \tan^{-1}\left(\frac{x+\frac{3}{2}}{3}\right) = \frac{1}{12}\tan^{-1}\left(\frac{2x+3}{6}\right) + c$$

11. (d)

$$\int e^{2\log\tan\sin x}\cos x\ dx = \int e^{\log\tan^2\sin x}\cos x\ dx \qquad [\because e^{\log t} = t]$$

$$= \int \tan^2\ (\sin x)\ \cos x\ dx = \int (\sec^2\ (\sin x) - 1).\cos x\ dx = \int (\sec^2\ t - 1)\ dt$$

$= \tan t - t = \tan \sin x - \sin x + c$

12. (b)

$$\int e^x\ (\sin^2 x + 2\ \sin x \cos x)\ dx = \int e^x\ (f\ (x) + f^{\prime}(x)) = e^x\ f(x) = e^x\ \sin^2 x$$

13. (a)

$$\int \frac{\sin^3 x}{\sin^2 x\cos^2 x} + \frac{\cos^3 x}{\sin^2 x\cos^2 x} = \int \frac{\sin x}{\cos^2 x} + \int \frac{\cos x}{\sin^2 x}$$

$= \int (\sec x \tan x + \text{cosec } x \cot x)dx = \sec x - \text{cosec } x + c$

14. (d)

$$\int \sqrt{1-\sin 2x} = \int \sqrt{\cos^2 x+\sin^2 x-2\sin x\cos x}$$

$$= \int \sqrt{(\cos x-\sin x)^2}dx = \int \cos x - \sin x\ dx = \sin x + \cos x$$

15. (b)

$$\int e^x\left(\frac{1+\sin\ x}{1+\cos\ x}\right)dx = \int e^x\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx = \int e^x\left(\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right)dx$$

$$= e^x\ \tan\ \frac{x}{2} + c$$

This is in the form $\int e^x\ (f\ (x) + f^{\prime}\ (x))\ dx$ ∴ Answer is $e^x\ f(x)$ i.e. $e^x\ \tan x/2$

16. (d)

$$\int \frac{1-\cos^2 x}{\cos^4 x}dx = \int \frac{\sin^2 x}{\cos^4 x}\ dx = \int \tan^2 x\ \sec^2 x\ dx = \frac{\tan^3 x}{3} + c \text{ (Put } \tan x = t)$$

17. (a)

$$\int_0^{\log 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}\ dx$$

Put $e^x - 1 = t^2$

$e^x\,dx = 2t\,dt$

when $x = \log 5$, $t = 2$

$x = 0$, $t = 0$

$$\int_0^{\log 5} \frac{e^x\sqrt{e^x - 1}}{e^x + 3}\,dx = \int_0^2 \frac{2t^2}{4+t^2}\,dt = 2\int_0^2 \frac{t^2+4-4}{4+t^2}\,dt = 2\int_0^2 \left[1 - \frac{4}{4+t^2}\right] dt$$

$$= 2\left[t - 2\tan^{-1}\frac{t}{2}\right]_0^2 = 2\left[2 - 2\frac{\pi}{4}\right] = 2\left[\frac{4-\pi}{2}\right] = 4 - \pi$$

18. (a)

$$\int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$$

Put $t = \frac{1}{u} \Rightarrow dt = -\frac{1}{u^2}\,du$

When $t = \cot x$, $u = \tan x$ and $t = \frac{1}{e}$, $u = e$

$$= \int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{e}^{\tan x} \frac{-\frac{1}{u^2}\,du}{\frac{1}{u}\left(1+\frac{1}{u^2}\right)} = \int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} - \int_{e}^{\tan x} \frac{u}{1+u^2}\,du$$

$$= \int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{\tan x}^{e} \frac{t}{1+t^2}\,dt = \int_{1/e}^{e} \frac{t}{1+t^2}\,dt$$

$$= \frac{1}{2}\log\left(1+t^2\right)\Big|_{1/e}^{e} = \frac{1}{2}\left[\log(1+e^2) - \log\left(\frac{e^2+1}{e^2}\right)\right]$$

$$= \frac{1}{2}\log\frac{(1+e^2)}{\frac{(e^2+1)}{e^2}} = \frac{1}{2}\log e^2 = \log_e e = 1$$

19. (b)

Put $\pi \log_e x = t$, then $\frac{\pi}{x}\,dx = dt$

When $x = e^{37}$, $t = \pi \log_e e^{37} = 37\pi$

$X = 1$, $t = \pi \log_e 1 = 0$

$$\int_1^{e^{37}} \frac{\pi \sin(\pi \log_e x)}{x}\,dx = \int_0^{37\pi} \sin t\,dt = -\cos t\Big|_0^{37\pi}$$

$= -[\cos 37\pi - \cos 0] = -[-1 - 1] = 2$

20. (c)

Put $xe^x = t$, then $(1 + x)\,e^x\,dx = dt$

$$\therefore \int \frac{e^x(1+x)}{\sin^2(xe^x)}\,dx = \int \frac{dt}{\sin^2 t} = \int \operatorname{cosec}^2 t\,dt = -\cot t + c = -\cot(xe^x) + c$$

21. (c)

$$\int (1 + \cos x + \cos^2 x + \text{......... to } \infty) = \int \frac{1}{1-\cos x}\,dx \quad (\because \text{ it is a G. P})$$

$$= \int \frac{1}{2\sin^2\frac{x}{2}}\,dx = \int \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\,dx = -\cot\frac{x}{2} + c$$

22. (a)

$$\int \frac{\sec x \tan x}{10 + \tan^2 x}\,dx = \int \frac{\sec x \tan x\,dx}{9 + \sec^2 x} = \int \frac{dt}{3^2 + t^2} \quad (\text{Put } \sec x = t,\ \sec x \tan x\,dx = dt)$$

$$= \frac{1}{3}\tan^{-1}\frac{t}{3} + c = \frac{1}{3}\tan^{-1}\left(\frac{\sec x}{3}\right) + c$$

23. (d)

$$\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \int \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x} = \frac{1}{b}\int \frac{dt}{a^2+t^2} \quad [\text{Put } b\tan x = t,\ b\sec^2 x = dt]$$

$$= \frac{1}{b} \times \frac{1}{a} \tan^{-1}\left(\frac{t}{a}\right) + c = \frac{1}{ab} \tan^{-1}\left(\frac{b \tan x}{a}\right) + c$$

24. (b)

$$\int \frac{x^2\left(1+\frac{1}{x^2}\right)}{x^2\left(x^2+\frac{1}{x^2}\right)} dx = \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left[\left(x-\frac{1}{x}\right)^2+2\right]}$$

Put $x - \frac{1}{x} = t$, then $\left(1+\frac{1}{x^2}\right) dx = dt$

$$= \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}\,x}\right) + c$$

25. (b)

$$\int \sqrt{1+\sin\left(\frac{x}{3}\right)} dx = \int \sqrt{\sin^2\frac{x}{6}+\cos^2\frac{x}{6}+2\sin\frac{x}{6}\cos\frac{x}{6}} dx = \int \sqrt{\left(\sin\frac{x}{6}+\cos\frac{x}{6}\right)^2} dx$$

$$= -6\cos\frac{x}{6} + 6\sin\frac{x}{6} + c = 6\left(\sin\frac{x}{6}-\cos\frac{x}{6}\right) + c$$

26. (a)

$$\int \sec^2 x \operatorname{cosec}^2 x \, dx = \int \frac{1}{\cos^2 x \sin^2 x} dx = \int \frac{\sin^2 x+\cos^2 x}{\cos^2 x \sin^2 x} dx = \int \left(\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}\right) dx$$

$$= \int (\sec^2 x + \operatorname{cosec}^2 x)\, dx = \tan x - \cot x + c$$

27. (c)

$$\int \operatorname{cosec}^4 x \, dx = \int (1 + \cot^2 x) \operatorname{cosec}^2 x \, dx$$

put $\cot x = t$

$-\operatorname{cosec}^2 x \, dx = dt$

$$= \int (1 + t^2)(-dt) = -\left(t+\frac{t^3}{3}\right) + c = -\cot x - \frac{\cot^3 x}{3} + c$$

28. (a)

$$\int \sqrt{\frac{1-x}{1+x}} dx = \int \frac{\sqrt{1-x}}{\sqrt{1+x}} \times \frac{\sqrt{1-x}}{\sqrt{1-x}} dx$$

$$= \int \frac{1-x}{\sqrt{1-x^2}} dx = \int \left[\frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}\right] dx = \sin^{-1} x + \sqrt{1-x^2} + c$$

29. (d)

$$\int \sinh \log x \, dx = \int \frac{e^{\log x} - e^{-\log x}}{2} dx = \int \frac{x - 1/x}{2} dx = \tfrac{1}{2}\left[\frac{x^2}{2} - \log x\right] = \frac{x^2}{4} - \log\sqrt{x} + c$$

30. (b)

Take $\sqrt{2x-2} = t$, $2x - 1 = 2x - 2 + 1 = t^2 + 1$

$$\frac{2}{2\sqrt{2x-2}} dx = dt \qquad dx = t\,dt$$

$$\therefore \int \frac{dx}{(2x-1)\sqrt{2x-2}} = \int \frac{t\,dt}{(t^2+1)t} = \tan^{-1} t = \tan^{-1}\sqrt{2x-2} + c$$

31. (a)

$$\int x^x \log_e ex \, dx = \int x^x (\log_e e + \log_e x)\, dx = \int x^x (1 + \log_e x)\, dx = x^x + c$$

32. (a)

$$\int \frac{\cos x}{\cos\left(x-\frac{\pi}{4}\right)} dx = \int \frac{\cos x \, dx}{\frac{1}{\sqrt{2}}(\cos x+\sin x)}$$

$$= \frac{\sqrt{2}}{2} \int \frac{\cos x - \sin x + \cos x + \sin x}{(\cos x + \sin x)} dx = \frac{1}{\sqrt{2}} \int \left[\frac{\cos x - \sin x}{\cos x + \sin x} + 1\right] dx$$

$$= \frac{1}{\sqrt{2}} [\log(\cos x + \sin x) + x] + c$$

33. (a)

$$\frac{x-1}{(x+1)^3} = \frac{(x+1)-2}{(x+1)^3} = \frac{1}{(1+x)^2} - \frac{2}{(x+1)^3} = f(x) + f^{l}(x)$$

$$I = \int e^x [f(x) + f^{l}(x)] dx = e^x f(x) = e^x \frac{1}{(1+x)^2}$$

34. (a)

$$N_r = l(D_r) + m\frac{d}{dx}(D_r)$$

Sin x + 3 cos x = l (3 sin x + 4 cos x) + m (3 cos x – 4 sin x)

= (3l – 4m) sin x+ (4l + 3m) cos x

$\Rightarrow$ 3l – 4m = 1

4l + 3m = 3

$\therefore$ 9l – 12m = 3

16l + 12m = 12

25 l = 15

$\Rightarrow l = \frac{3}{5}$

$\therefore m = \frac{1}{5}$

$$\therefore \int \frac{\sin x + 3\cos x}{3\sin x + 4\cos x} dx = \frac{3}{5} x + \frac{1}{5} \log(3\sin x + 4\cos x) \quad \therefore A + B = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

35. (d)

$$I_n + I_{n-2} = \int_0^{\frac{\pi}{4}} \cot^n x \, dx + \int_0^{\frac{\pi}{4}} \cot^{n-2} x \, dx = \int_0^{\frac{\pi}{4}} (\cot^n x + \cot^{n-2} x) \, dx$$

$$= \int_0^{\frac{\pi}{4}} \cot^{n-2} x (\cot^2 x + 1) \, dx = \int_0^{\frac{\pi}{4}} \cot^{n-2} x \, . \, \mathrm{cosec}^2 x \, dx$$

$$= \left[-\frac{\cot^{n-1} x}{n-1}\right]_0^{\frac{\pi}{4}} = -\left[\frac{1}{n-1} - 0\right] = \frac{1}{1-n}$$

36. (d)

$$\int_0^{\pi} \frac{dx}{\cos^2 x + 3\sin^2 x} = \int_0^{\pi} \frac{\sec^2 x}{1 + 3\tan^2 x}$$

$\frac{\sec^2 x}{1+3\tan^2 x}$ is discontinuous at $x = \frac{\pi}{2}$

37. (d)

$$\int_0^2 \sqrt{\frac{2+x}{2-x}} \, dx = \int_0^2 \frac{\sqrt{2+x}}{\sqrt{2-x}} \times \frac{\sqrt{2+x}}{\sqrt{2+x}} dx = \int_0^2 \frac{2+x}{\sqrt{4-x^2}} dx = \int_0^2 \left(\frac{2}{\sqrt{4-x^2}} + \frac{x}{\sqrt{4-x^2}}\right) dx$$

$$= \left[2\sin^{-1}\frac{x}{2} - \sqrt{4-x^2}\right]_0^2 = [2\sin^{-1} 1 - 0] - [0 - \sqrt{4}] = 2 \times \frac{\pi}{2} + 2 = \pi + 2$$

38. (b)

$$\int_0^1 \sin^2(\cos^{-1} x) \, dx = \int_0^1 1 - [\cos(\cos^{-1} x)]^2 \, dx = \int_0^1 1 - x^2 \, dx = x - \frac{x^3}{3}\Big|_0^1$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

39. (a)

$$\frac{1}{(x^2+a^2)(x^2+b^2)} = \frac{1}{b^2-a^2}\left[\frac{1}{x^3+a^2} - \frac{1}{x^2+b^3}\right]$$

$$\therefore I = \frac{1}{b^2-a^2}\left[\frac{1}{a}\tan^{-1}\frac{x}{a} - \frac{1}{b}\tan^{-1}\frac{x}{b}\right]\Bigg|_0^{\infty}$$

$$= \frac{1}{b^2-a^2}\left[\frac{1}{a}.\frac{\pi}{2} - \frac{1}{b}.\frac{\pi}{2}\right] = \frac{\pi}{2}\ \frac{1}{b^2-a^2}\ \frac{b-a}{ab} = \frac{\pi}{2ab(a+b)}$$

40. (b)

$(\sin x + \cos x)dx = d(\sin x - \cos x)$; $3 + \sin 2x = 4 - (\sin x - \cos x)^2$

$$\therefore I = \int_0^{\pi/4} \frac{d(\sin x - \cos x)}{\sqrt{4-(\sin x - \cos x)^2}} = \frac{1}{2}\sin^{-1}[(\sin x - \cos x)]\Big|_0^{\pi/4}$$

$$= \frac{1}{2}\ [\sin^{-1} 0 - \sin^{-1}(0-1)] = \frac{1}{2}\sin^{-1} 1 = \pi/4$$

41. (a)

$$I = \int_0^{\pi/6} \frac{1}{2}\ [\sin 18x + \sin 6x]dx = -\frac{1}{2}\left[\frac{1}{18}\cos 18x + \frac{1}{6}\cos 6x\right]\Bigg|_0^{\pi/6}$$

$$= -\frac{1}{2}\left[\left(-\frac{1}{18} - \frac{1}{6}\right)2\right] = \frac{1}{18} + \frac{1}{6} = \frac{4}{18} = \frac{2}{9}$$

42. (c)

Put $t = \tan x/2$; $1 + \sin x + \cos x = 1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = \frac{1+t^2+2t+1-t^2}{1+t^2} = \frac{2(1+t)}{1+t^2}$

$$\therefore I = \int_0^1 \frac{2dt}{2(1+t)} = \log(1+t)\Big|_0^1 = \log 2$$

43. (b)

Put $x = \sin\theta$

$dx = \cos\theta\, d\theta$

When $x = 0$, $\theta = \frac{\pi}{2}$

$x = 1$, $\theta = \frac{\pi}{2}$

$$\int_0^1 \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = \int_0^{\frac{\pi}{2}} \theta\sin\theta\, d\theta = [-\theta\cos\theta + \sin\theta]_0^{\pi/2} = 1$$

44. (d)

$$I = \int_0^1 \left[\frac{\pi}{2} - \sin^{-1}\frac{2x}{1+x^2}\right]dx = \frac{\pi}{2} - 2\int_0^1 \tan^{-1} x\, dx$$

$$= \frac{\pi}{2} - 2\left[x\tan^{-1}x - \frac{1}{2}\log(1+x^2)\right]\Bigg|_0^1 = \frac{\pi}{2} - 2\left[\left(\frac{\pi}{4} - \frac{1}{2}\log 2\right) - 0\right] = \log 2$$

45. (c)

$$\int_0^{2\pi} |x\sin x|\, dx = \int_0^{\pi} |x\sin x|\, dx + \int_{\pi}^{2\pi} |x\sin x|\, dx$$

$$= \int_0^{\pi} x\sin x\, dx + \int_{\pi}^{2\pi} -(x\sin x)\, dx = [-x\cos x + \sin x]_0^{\pi} - [-x\cos x + \sin x]_{\pi}^{2\pi}$$

$$= [\pi - 0] - [-2\pi + \pi] = 2\pi$$

46. (c)

$$\int_0^1 \frac{x^3}{1+x^8}dx = \int_0^1 \frac{\frac{dt}{4}}{1+t^2}$$ [Put $x^4 = t$, $4x^3dx = dt$, $x = 0;\ t = 0,\ x = 1;\ t = 1$]

$$= \left[\frac{1}{4}\tan^{-1} t\right]_0^1 = \frac{1}{4} \times \frac{\pi}{4} = \frac{\pi}{16}$$

47. (d)

$$\int_1^4 e^{\sqrt{x}}\ dx = \int_1^2 e^t.2t\ dt$$ [Put $x = t^2$, $dx = 2t\ dt$, $x = 1,\ t = 1,\ x = 4,\ t = 2$]

$$= 2\ \left|te^t - e^t\right|_1^2 = 2\ \left[e^t(t-1)\right]_1^2 = 2\ [e^2 - 0] = 2e^2$$

48. (d)

If $0 \le x < \frac{\pi}{4}$, $\cos x > \sin x$ so that $\sin x - \cos x < 0$

$$\int_0^{\frac{\pi}{2}} |\sin x - \cos x|\ dx = \int_0^{\frac{\pi}{4}} |\sin x - \cos x|\ dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} |\sin x - \cos x|\ dx$$

$$= \int_0^{\frac{\pi}{4}} -(\sin x - \cos x)\ dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\ dx$$

$$= \left[\sin x + \cos x\right]_0^{\pi/4} + \left[-\cos x - \sin x\right]_{\pi/4}^{\pi/2} = \left[\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)-(1+0)\right] + \left[(0-1)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{2}{\sqrt{2}} - 1 - 1 + \frac{2}{\sqrt{2}} = 2\sqrt{2} - 2$$

49. (d)

$$\int \frac{1}{(e^x-1)^2}\ dx = \int \frac{(1-e^x)+e^x}{(e^x-1)^2}dx = \int \frac{-1}{e^x-1} + \frac{e^x}{(e^x-1)^2}dx$$

$$= \int \frac{(e^x-1)-e^x}{e^x-1} + \frac{e^x}{(e^x-1)^2}dx = \int 1 - \frac{e^x}{e^x-1} + \frac{e^x}{(e^x-1)^2}dx$$

$$= x - \log(e^x - 1) - \frac{1}{e^x-1}$$

50. (b)

$U_n = \int_1^e (\log x)^x\ dx$, apply integration by parts

$$= (\log x)^n . x\Big|_1^e - \int_1^e n\ (x \log x)^{n-1} \frac{1}{x}\ dx$$

$$= (e - 0) - n\int_1^e (\log x)^{n-1} = e - nU_{n-1}$$

$\therefore U_n + nU_{n-1} = e$

51. (c)

$$\int_2^4 [3 - f(x)]dx = 7 \Rightarrow 3x\Big|_2^4 - \int_2^4 f(x)\ dx = 7$$

$$\Rightarrow \int_2^4 f(x)\ dx = 6 - 7 = -1$$

Now $\int_{-1}^4 f(x)\ dx = \int_{-1}^2 f(x)\ dx + \int_2^4 f(x)\ dx$

$$4 = \int_{-1}^2 f(x)\ dx - 1 \Rightarrow \int_{-1}^2 f(x)\ dx = 5 \Rightarrow \int_2^{-1} f(x)\ dx = -5$$

52. (d)

Let $I = \int_0^4 \frac{(4-x)^4}{x^4+(4-x)^4}\ dx$ ---------- (1)

Then $I = \int_0^4 \frac{[4-(4-x)]^4}{[4-x]^4+[4-(4-x)]^4}dx = \int_0^4 \frac{x^4}{(4-x)^4+x^4}\ dx$ ----------- (2)

(1) + (2) gives

$$2I = \int_0^4 1\,dx = x\Big|_0^4 = 4 \Rightarrow I = 2$$

53. (d)

Let $I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x}\,dx$ ---------- (1)

Then $I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}\,dx$ ------ (2) [Applying property 5)

(1) + (2) gives $2I = \int_0^{\pi/2} 0\,dx = 0$

$\Rightarrow I = 0$

54. (d)

Put $x = a \sin\theta$, then $dx = a \cos\theta$

When $x = 0$, $\theta = 0$

$X = 1$, $\theta = \pi/2$

$$\therefore I = \int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}} = \int_0^{\pi/2} \frac{\cos\theta}{\cos\theta + \sin\theta}\,d\theta = \frac{\pi}{4}$$

55. (b)

Let $I = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \tan x}\,dx$ ------------- (1)

Then $I = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \tan\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}\,dx = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \tan\left(\frac{\pi}{2} - x\right)} = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \cot x}\,dx$

$= \int_{\pi/6}^{\pi/3} \frac{\tan x}{\tan x + 1}\,dx$ ------------ (2)

(1) + (2) gives $2I = \int_{\pi/6}^{\pi/3} 1\,dx = x\Big|_{\pi/6}^{\pi/3} = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$

56. (c)

Let $I = \int_a^b \frac{f(x)\,dx}{f(x) + f(a + b - x)}$ --------- (1)

Then $I = \int_a^b \frac{f(a + b - x)\,dx}{f(a + b - x) + f(x)}$ ------- (2)

(1) + (2) gives $2I = \int_a^b dx = b - a \Rightarrow I = \frac{b - a}{2}$

57. (a)

$f(-\theta) = \log\left(\frac{2 - \sin\theta}{2 + \sin\theta}\right) = \log\left(\frac{2 + \sin\theta}{2 - \sin\theta}\right) = -f(\theta)$

$\therefore \log\left(\frac{2 - \sin\theta}{2 + \sin\theta}\right)$ is an odd function

Hence $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2 - \sin\theta}{2 + \sin\theta}\right) d\theta = 0$

58. (a)

Put $x - 2 = y$ then $dx = dy$

When $x = 1$, $y = -1$

$x = 3$, $y = 1$

$\therefore \int_1^3 (x - 1)(x - 2)(x - 3)\,dy = \int_{-1}^{1} (y + 1)y(y - 1)\,dy = \int_{-1}^{1} (y^3 - y)dy$

$= 0$ $\quad y^3 - y$ is odd

59. (d)

$\int_{-\pi/2}^{\pi/2} \sin|x|\,dx = \int_{-\pi/2}^{0} -\sin x\,dx + \int_0^{\pi/2} \sin x\,dx$

$= \cos x\Big|_{-\pi/2}^{0} + (-\cos x)\Big|_0^{\pi/2}$

$= \cos 0 - \cos\frac{\pi}{2} + \left(-\cos\frac{\pi}{2} + \cos 0\right) = 2$

60. (a)

f (x) = log sin x then f (π - x) = f (x)

$\therefore I_1 = \int_0^{\pi} \log \sin x \, dx = 2 \int_0^{\pi/2} \log \sin x \, dx = 2 \int_0^{\pi/2} \log \cos x \, dx = 2I_2$

61. (c)

Put x = tan θ, then dx = $\sec^2 d\theta$

When x = 0, θ = 0

X = 1, θ = $\frac{\pi}{4}$

$I = \int_0^1 \frac{\log(1+x)}{1+x^2} dx = \int_0^{\pi/4} \frac{\log(1+\tan\theta)\ \sec^2\theta\ d\theta}{(1+\tan^2\theta)}$

$= \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta = \frac{\pi}{8} \log 2$

62. (a)

Put x = tan θ, then dx = $\sec^2\theta\, d\theta$

When x = 0, θ = 0

X = ∞, θ = π/2

$I = \int_0^{\infty} \frac{dx}{x^4+2x^2+1} = \int_0^{\infty} \frac{dx}{(x^2+1)^2} = \int_0^{\pi/2} \frac{\sec^2\theta\ d\theta}{(\tan^2\theta+1)^2}$

$= \int_0^{\pi/2} \cos^2\theta\, d\theta = \int_0^{\pi/2} \frac{1+\cos 2\theta}{2} d\theta$

$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2} = \frac{1}{2}\left[\frac{\pi}{2}+0-0\right] = \frac{\pi}{4}$

63. (d)

Put x + 1 = t, then dx = dt

When x = 0, t = 1

X = 1, t = 2

$\therefore \int_0^1 \frac{x\ dx}{(x+1)^4} = \int_1^2 \frac{t-1}{t^4} dt = \int_1^2 \left(\frac{1}{t^3} - \frac{1}{t^4}\right) dt$

$= \left[-\frac{1}{2t^2} + \frac{1}{3t^3}\right]_1^2 = \left(-\frac{1}{8}+\frac{1}{24}\right) - \left(-\frac{1}{2}+\frac{1}{3}\right)$

$= \left(-\frac{1}{12}\right) - \left(\frac{-1}{6}\right) = -\frac{1}{12} + \frac{1}{6} = \frac{1}{12}$

64. (b)

Put y = 0, $x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$

$A = \left|\int_1^2 [x^2 - 3x + 2]\, dx\right| = \left|\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2\right| = \frac{1}{6}$ sq. units

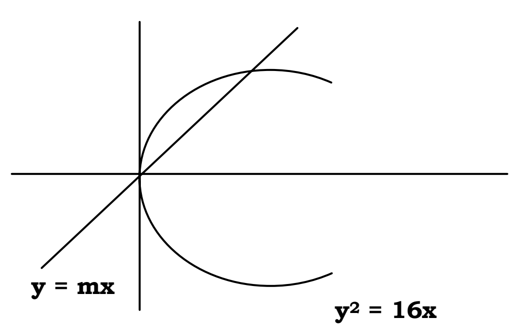
65. (b)

$y^2 = 16x$ --------- (1)

y = mx ----------- (2)

$m^2x^2 = 16x \Rightarrow x\,(m^2x - 16) = 0$

$\Rightarrow x = 0,\ x = \frac{16}{m^2}$

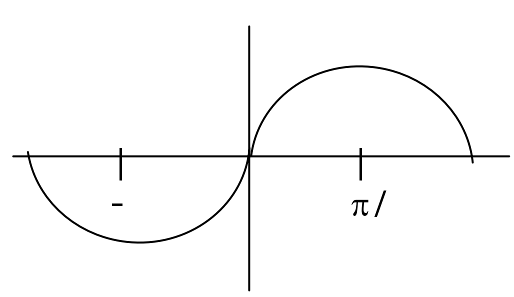


$A = \int_0^{16/m^2} (\sqrt{16}\, x - mx) dx = \frac{2}{3}$

$\Rightarrow \left[4 \times \frac{2}{3} x^{3/2} - \frac{mx^2}{2}\right]_0^{16/m^2} = \frac{2}{3}$

$\Rightarrow \frac{8 \times 64}{3m^3} - \frac{256}{2m^3} = \frac{2}{3} \Rightarrow m = 4$

66. (c)



$A = 2 \int_0^{\pi/2} \sin x \, dx = 2 \left[-\cos x\right]_0^{\pi/2} = 2$

67. (c)

$4a = 9 \Rightarrow a = \frac{9}{4}$

$\therefore A = \frac{16a^2}{3} = \frac{16}{3} \left(\frac{9}{4}\right)^2 = \frac{16 \times 9 \times 9}{3 \times 4 \times 4} = 27$

68. (d)

a = 5, b = 2

Area of the ellipse = $\pi ab = \pi (5)(2) = 10\pi$

69. (d)

Let $I = \int_{\pi/6}^{\pi/3} \frac{\sin x}{\sin x + \cos x} dx$ ----------- (1)

Then $I = \int_{\pi/6}^{\pi/3} \frac{\sin\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)}{\sin\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right) + \cos\left(\frac{\pi}{6} + \frac{\pi}{3} - x\right)} dx$

$= \int_{\pi/6}^{\pi/3} \frac{\cos x}{\cos x + \sin x} dx$ --------- (2)

(1) + (2) gives

$2I = \int_{\pi/6}^{\pi/3} dx = x \Big|_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$

70. (b)

Let $I = \int_{-\pi}^{3\pi} \log (\sec\theta - \tan\theta \, d\theta$ ---------- (1)

Then $I = \int_{-\pi}^{3\pi} \log (\sec (2\pi - \theta) - \tan (2\pi - \theta)] \, d\theta$

$= \int_{-\pi}^{3\pi} \log (\sec\theta + \tan\theta) \, d\theta$ ---------- (2)

Adding (1) and (2)

$2I = \int_{-\pi}^{3\pi} \log 1 \, d\theta = 0 \Rightarrow I = 0$

71. (c)

$I = \int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx$ ---------- (1)

Then $I = \int_0^{\pi/2} \frac{a \sin\left(\frac{\pi}{2} - x\right) + b \cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx = \int_0^{\pi/2} \frac{a \cos x + b \sin x}{\cos x + \sin x} dx$ -------- (2)

$2I = \int_0^{\pi/2} (a + b) \, dx = (a + b) \frac{\pi}{2} \therefore I = (a + b) \frac{\pi}{4}$

72. (c)

$I = \int_0^{\pi} x \cos^2 x \sin x \, dx = \int_0^{\pi} (\pi - x) \cos^2 (\pi - x) \sin (\pi - x) dx$

$= \int_0^{\pi} (\pi - x) \cos^2 x \sin x \, dx = \pi \int_0^{\pi} \cos^2 x \sin x \, dx - \int_0^{\pi} x \cos^2 x \sin x \, dx$

$I = \pi \left[-\frac{\cos^3 x}{3}\right]_0^{\pi} - I$

$2I = \pi \left[\frac{1}{3} + \frac{1}{3}\right] \Rightarrow I = \frac{\pi}{3}$

73. (a)

Let $I = \int_0^{10} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{10-x}} dx$ --------- (1)

Then $I = \int_0^{10} \frac{\sqrt{10-x}}{\sqrt{10-x} + \sqrt{x}} dx$ --------- (2)

$2I = \int_0^{10} dx = 10$

$I = 5$

74. (c)

Let $I = \int_0^{\pi} \frac{dx}{1 + 2^{\cos x}}$ --------- (1)

Let $I = \int_0^{\pi} \frac{dx}{1 + 2^{\cos(\pi - x)}} dx = \int_0^{\pi} \frac{dx}{1 + 2^{-\cos x}}$

$= \int_0^{\pi} \frac{2^{\cos x}}{2^{\cos x} + 1} dx$ -------- (2)

Then $2I = \int_0^{\pi} dx = \pi \Rightarrow I = \frac{\pi}{2\text{`}}$

75. (b)

Put $1 - x = t$, then $-dx = dt$

When $x = 0$, $t = 1$

$x = 1$, $t = 0$

$\therefore \int_0^1 x(1-x)^n dt = \int_0^1 (1-t)\, t^n (-dt) = \int_0^1 (t^n - t^{n+1}) dt$

$= \left[\frac{t^n}{n+1} - \frac{t^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$

76. (c)

Put $1 - x^2 = t$ then $-2x\, dx = dt$

When $x = 0$, $t = 1$

$x = 1$, $t = 0$

$\int_0^1 x^3 (1-x^2)^5 dx = \int_0^1 x^2 (1-x^2)^5 x dx = \int_1^0 (1-t) t^5 \left(-\frac{dt}{2}\right) = \frac{1}{2}\int_0^1 (t^5 - t^6) dt$

$= \frac{1}{2}\left[\frac{t^6}{6} - \frac{t^7}{7}\right]_0^1 dt = \frac{1}{2}\left[\frac{1}{6} - \frac{1}{7}\right] = \frac{1}{84}$

77. (d)

$I = \int_0^{\frac{\pi}{2}} \log \tan x dx$ ---------------(1)

Then $I = \int_0^{\frac{\pi}{2}} \log \tan\left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \log \cot x dx$ ----------------(2)

(1) + (2) gives $2I = \int_0^{\frac{\pi}{2}} 0 x dx = 0$

$I = 0$

78. (d)

$\int_0^{\frac{\pi}{2}} 1 dx = x\Big|_{\theta}^{\pi/2} = \frac{\pi}{2}$

79. (d)

$I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + (\cot x)^{101}} dx = \int_0^{\frac{\pi}{2}} \frac{\sin^{101} x}{\sin^{101} x + \cos^{101} x} dx = \frac{\pi}{4}$

80. (a)

cos x is even function and $\log\left(\frac{1-x}{1+x}\right)$ is odd function

$\therefore$ cos x $\log\left(\frac{1-x}{1+x}\right)$ is odd function

81. (c)

$y^2 = 2y - x$ meets the y – axis when x = 0

$\therefore y^2 = 2y \Rightarrow y(y-2) = 0 \Rightarrow y = 0,2$

$$\text{Area, } A = \int_0^2 x\,dy = \int_0^2 (2y - y^2)\,dy = \left[y^2 - \frac{y^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$$

82. (b)

$y^2 = 2x$ -------------- (1)

$y = x$ --------------(2)

$\Rightarrow x^2 = 2x \Rightarrow x(x-2) = 0 \Rightarrow x = 0,2$

$$\therefore \text{area} = A = \int_0^2 \left[\sqrt{2}\, x^{\frac{1}{2}} - x\right]dx = \left[\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^2 = \frac{8}{3} - 2 = \frac{2}{3}$$

83. (c)

$y^2 = 9x$ ------------ (1)

$y = 3x$ -------------(2)

$(3x)^2 = 9x \Rightarrow 9x(x-1) = 0$ x = 0,1

$$A = \int_0^1 \left[3\sqrt{x} - 3x\right]dx = \left[2x^{\frac{3}{2}} - \frac{3x^2}{2}\right]_0^1 = 2 - \frac{3}{2} = \frac{1}{2}$$

84. (c)

$\sin^2 x = \cos^2 x \Rightarrow \tan^2 x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$

$$A = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin^2 x - \cos^2 x)\,dx = -\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos 2x\,dx = -\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = -\frac{1}{2}\left[\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right] = 1\text{sq unit}$$

85. (a)

$\log_e x = 0 \Rightarrow x = 1$

$$A = \int_0^1 \log_e x\,dx = \left[x\log_e x - x\right]_0^1$$

$= 1$ sq unit

86. (c)

$f(x) = x^3$

f (-2) < 0 and f (1) >0

$\therefore$ f (x) crosses x axis in [-2 , 1]

$f(x) = 0 \Rightarrow x = 0$

$$A = \left|\int_{-2}^{0} x^3 dx\right| + \left|\int_0^1 x^3 dx\right| = \left|\left[\frac{x^4}{4}\right]_{-2}^0\right| + \left|\left[\frac{x^4}{4}\right]_0^1\right| = |-4| + \left|\frac{1}{4}\right| = \frac{17}{4}$$

87. (b)

$x^2 + 4y = 4 \Rightarrow y = \frac{1}{4}(4 - x^2)$

$$y = 0 \Rightarrow \frac{1}{4}\left(4 - x^2\right) = 0 \Rightarrow x = \pm 2$$

$$A = \int_{-2}^{2} y\,dx = \frac{1}{4}\int_{-2}^{2}\left(4 - x^2\right)dx$$

$$A = \frac{1}{4}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{1}{4}\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = \frac{1}{4}\left[16 - \frac{16}{3}\right] = \frac{1}{4}\left(\frac{32}{3}\right) = \frac{8}{3}$$

88. (b)

$\log_e x = 0 \Rightarrow x = 1$

$$A = \int_{1}^{e} \log x\,dx = \left[x\log x - x\right]_1^e$$

$= [e\log_e e - e] - [0 - 1]$

$= 1$

89. (b)

Put $\frac{\pi}{2}x = t$

$\frac{\pi}{2}dx = dt$

When $x = 0$, $t = 0$

$x = 1$, $t = \frac{\pi}{2}$

$$\int_0^1 \log \sin\frac{\pi}{2}x\,dx = \int_0^{\pi/2} \log \sin t.\frac{2}{\pi}dt = \frac{2}{\pi}\int_0^{\pi/2} \log \sin t\,dt = \frac{2}{\pi}\left(-\frac{\pi}{2}\log\frac{2}{e}\right) = -\log\frac{2}{e}$$

90. (c)

Put $\sin^2 x = t$ Then, $2 \sin x \cos x\,dx = dt$ i.e., $\sin 2x\,dx = dt$

when $x = 0$, $t = 0$

$$x = \frac{\pi}{4},\ t = \frac{1}{2} \quad \therefore \int_0^{\pi/4} \frac{\sin 2x}{\sin 4x + \cos 4x}dx = \int_0^{1/2} \frac{dt}{t^2 + (1-t)^2}$$

$$= \int_0^{1/2} \frac{dt}{2t^2 - 2t + 1} = \frac{1}{2}\int_0^{1} \frac{dt}{t^2 - t + \frac{1}{2}} = \frac{1}{2}\int_0^{1/2} \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \frac{1}{2} - \frac{1}{4}}$$

$$= \frac{1}{2}\int_0^{1/2} \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \frac{1}{4}} = \left[\tan^{-1}(2t-1)\right]_0^{1/2} = [\tan^{-1}0 - \tan^{-1}(-1)]$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$