

LIMITS, CONTINUITY
AND
DIFFERENTIATION

Question 1

$$\lim_{n \rightarrow \infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^n + 7.5^n} =$$

a) $\frac{-20}{7}$

b) 0

c) $\frac{3}{5}$

d) $-\frac{4}{5}$

Question 2

The function

$$f(x) = \frac{\log(1 + ax) - \log(1 - bx)}{x}$$

is not defined at $x = 0$. The value which should be assigned to f at $x = 0$ so that it is continuous at $x = 0$ is

- a) $\log a + \log b$
- b) 0
- c) $a - b$
- d) $a + b$

Question 3

If the function

$$f(x) = \begin{cases} \frac{1 - \cos x}{x^2} & \text{for } x \neq 0 \\ k & \end{cases}$$

is continuous at $x = 0$ then the value of k is

a) 1

b) 0

c) 1/2

d) -1

Question 4

$$\lim_{x \rightarrow 0} \frac{1 - \cos nx}{1 - \cos mx} =$$

a) $\frac{m}{n}$

b) $\frac{n}{m}$

c) $\frac{m^2}{n^2}$

d) $\frac{n^2}{m^2}$

Question 5

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{x} + \frac{1}{x^2} - \dots \dots \dots \text{to } n \text{ terms} \right) =$$

a) $\frac{1}{1-x}$

b) $\frac{1}{1+x}$

c) $\frac{x}{1+x}$

d) 0

Question 6

$$\lim_{x \rightarrow 0} (1 + 3 \tan x)^{3 \cot x} =$$

a) e^9

b) e^3

c) e

d) 0

Question 7

$$\lim_{x \rightarrow 0} \frac{10^x - 5^x - 2^x + 1}{x^2} =$$

a) log10

b) log5.log2

c) log25

d) 0

Question 8

If $f:\mathbb{R}\rightarrow\mathbb{R}$ is continuous such that $f(x+y) = f(x) + f(y) \forall x, y \in \mathbb{R}$, & $f(1) = 2$ then $f(100) =$

- a) 0 b) 100
c) 200 d) 400

Question 9

$$\lim_{x \rightarrow 0} \frac{\sin nx \{(a - n) nx - \tan x\}}{x^2} = 0$$

where n is a non zero positive integer,
then a is equal to

a) $\frac{n+1}{2}$

b) n^2+1

c) $\frac{1}{n+1}$

d) $n + \frac{1}{n}$

Question 10

$$\lim_{x \rightarrow T \arctan^{-1}(3)} \frac{T \tan^2 x - 2T \tan x - 3}{T \tan^2 x - 4T \tan x + 3} =$$

a) 1

b) 0

c) $\frac{1}{2}$

d) 2

a) 1

b) 0

c) $\frac{1}{2}$

d) 2

Question 11

$$\lim_{x \rightarrow 0} \frac{(1+x)\sin(2+x) - \sin 2}{x}$$

a) $2\cos 2$

b) $2 \cos 2 + \sin 2$

c) $\cos 2 + \sin 2$

d) 3

Question 12

Let

$$f(x) = \begin{cases} \frac{\sin \pi x}{5x} & \text{if } x \neq 0 \\ K & \text{if } x = 0 \end{cases}$$

if $f(x)$ is continuous at $x=0$, the value of k is

a) 1

b) 0

c) $\frac{\pi}{5}$

d) $\frac{5}{\pi}$

Question 13

If $f(x) = \cos(\log x)$ then $\left[f\left(\frac{x}{y}\right) + f(xy) \right]$ has the value

a) $f(x+y)$

b) $f(x) = f(y)$

c) $f(x)f(y)$

d) $2f(x)f(y)$

Question 14

$$\lim_{x \rightarrow 0} (1 - ax)^{\frac{1}{x}} =$$

a) e^{-a}

b) e

c) e^a

d) 1

Question 15

$$\lim_{\theta \rightarrow 0} \frac{\sin 3\theta \cdot \tan 4\theta}{\theta \cdot \sin 5\theta} =$$

a) 1

b) $\frac{5}{12}$

c) 3

d) $\frac{12}{5}$

Question 16

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots \dots \dots \textit{to } n \textit{ terms} \right) =$$

a) $\frac{1}{2}$

b) 0

c) ∞

d) 1

Question 17

$$\lim_{x \rightarrow 2} \frac{2x^2 - 5x + 2}{x^2 - 3x + 2} =$$

a) 1

b) 2

c) 3

d) 8

Question 18

$$\lim_{x \rightarrow 0} \frac{\tan mx}{\tan nx} =$$

a) $\frac{m}{n}$

b) $\frac{m^2}{n^2}$

c) $\frac{n}{m}$

d) $\frac{n^2}{m^2}$

Question 19

$$\lim_{x \rightarrow \infty} \frac{(2+x)^{40} (4+x)^5}{(2-x)^{45}} \text{ is}$$

a) -1

b) 1

c) 16

d) 32

Question 20

20. If the three functions $f(x)$, $g(x)$ and $h(x)$ are such that $h(x) = f(x).g(x)$ and $f'(x).g'(x) = c$, where c is a constant, then

$$\frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2c}{f(x).g(x)} \text{ is equal to}$$

a) $\frac{h''(x)}{h(x)}$

b) $\frac{h(x)}{h'(x)}$

c) $h'(x).h''(x)$

d) $\frac{h(x)}{h''(x)}$

Question 21

21. The derivative of $e^{ax} \cos bx$ with respect to x is

$re^{ax} \cos\left(bx + \tan^{-1} \frac{b}{a}\right)$. When $a > 0$, $b > 0$, the value of r is

a) ab

b) $a + b$

c) $\sqrt{a^2 + b^2}$

d) $\frac{1}{\sqrt{ab}}$

Question 22

If $f(x) = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \dots + x^n$ then $f^{(1)}(1) =$

a) $\underline{n(n-1)}2^{n-2}$

b) $n(n-1)2^n$

c) $\underline{n(n-1)}2^{n-1}$

d) $(n-1)2^{n-1}$

Question 23

3. If $f(x) = \frac{g(x) + g(-x)}{2} + \frac{2}{[h(x) + h(-x)]^{-1}}$ where g & h are differentiable functions then $f'(0) =$

a) $3/2$

b) 0

c) 1

d) $1/2$

If $y = \cos^2\left(\frac{3x}{2}\right) - \sin^2\left(\frac{3x}{2}\right)$ then $\frac{d^2y}{dx^2}$ is —

a) $-3\sqrt{1-y^2}$

b) $9y$

c) $-9y$

d) $3\sqrt{1-y^2}$

Question 25

If $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots \infty$ with $|x| > 1$ then $\frac{dy}{dx} =$

a) $x^2 y^2$

b) $\frac{x^2}{y^2}$

c) $\frac{-y^2}{x^2}$

d) $\frac{y^2}{x^2}$

Question 26

If $\sin(x + y) + \cos(x + y) = \log(x + y)$ then $\frac{d^2 y}{dx^2} =$

a) 0

b) $-y/x$

c) 1

d) -1

Question 27

If $y = \sqrt{\tan x + \sqrt{\tan x + \dots \infty}}$ then $\frac{dy}{dx} =$

a) $\frac{\sec^2 x}{2y - 1}$

b) $\frac{\sec^2 x}{1 - 2y}$

c) $\frac{-\sec^2 x}{2y + 1}$

d) -1

Question 28

If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$ then $\frac{dy}{dx} + \frac{x^n}{n!} =$

a) x

b) $\frac{1}{x}$

c) y

d) $\frac{1}{y}$

Question 29

If $\sin y = \sqrt{x}$ then $\frac{dy}{dx} =$

a) $\frac{1}{2\sqrt{1-x^2}}$

b) $\frac{1}{2\sqrt{x-x^2}}$

c) $\frac{-1}{2\sqrt{x^2-x}}$

d) 2

Question 30

If $y = \sec^{-1}\left(\frac{\sqrt{x} + 10}{\sqrt{x} - 10}\right) + \sin^{-1}\left(\frac{\sqrt{x} - 10}{\sqrt{x} + 10}\right)$ then $\frac{dy}{dx} =$

a) 0

b) $\frac{1}{2(\sqrt{x} - 10)}$

c) $\frac{-1}{2\sqrt{x}}$

d) None

Question 31

$$\text{If } y = b \left[\tan^{-1} \frac{y}{x} + \tan^{-1} \frac{x}{y} \right] \text{ then } \frac{dy}{dx} =$$

a) 1

b) 0

c) -2

d) 1

Question 32

If $x^x = y^y$ then $\frac{dy}{dx} =$

a) $\frac{-y}{x}$

b) $\frac{-x}{y}$

c) $1 + \log\left(\frac{x}{y}\right)$

d) $\frac{1 + \log x}{1 + \log y}$

Question 33

If $y = \sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$ then $\frac{dy}{dx} =$

a) $\frac{2}{\sqrt{1-x^2}}$

b) $\frac{a}{\sqrt{1-a^2}}$

c) $\frac{2a}{\sqrt{1-a^2x^2}}$

d) $\frac{2a^2}{1-a^2x^2}$

Question 34

If $x = a(\cos\theta + \theta\sin\theta)$ & $y = a(\sin\theta - \theta\cos\theta)$ then $a\theta \frac{d^2y}{dx^2} =$

a) $\cos\theta$

b) $\sec^2\theta$

c) $\sec^3\theta$

d) None

Question 35

If $y = \tan^{-1}\left(\frac{a \sin x + b \cos x}{a \cos x - b \sin x}\right)$ then $\frac{dy}{dx} =$

a) -1

b) 1

c) $x + \alpha$

d) 0

Question 36

Let $f(x + y) = f(x) f(y)$ for all x & y . suppose $f(5) = 2$.

$f'(0) = 3$ then $f'(5)$ is

a)4

b)3

c)8

d)6

Question 37

The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.to $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

a) 2

b) $\frac{1}{2}$

c) -1

d) 1

Question 38

If $2x^2 + 4xy + 3y^2 = 0$ then $\frac{d^2y}{dx^2} =$

a) 0

b) $\frac{1}{2}$

c) $\frac{1}{(2x + y)^2}$

d) $\frac{4}{3}$

Question 39

If $y = \tan^{-1} \left(\frac{x + a}{1 - xa} \right)$ then $\frac{dy}{dx} =$

a) $\frac{1}{a^2 + x^2}$

b) $\frac{1}{1 + x^2}$

c) $\frac{y}{a^2 + x^2}$

d) 0

Question 40

$$\text{If } y = \tan^{-1} \left(\frac{\log \left(\frac{e}{x^2} \right)}{\log (ex^2)} \right) + \tan^{-1} \left(\frac{3 + 2 \log x}{1 - 6 \log x} \right) \text{ then } \frac{d^2 y}{dx^2} =$$

a) 0

b) -1

c) 2

d) 1

Question 41

If $y = \sin x^\circ$ then $\frac{dy}{dx} =$

a) $\cos x^\circ$

b) $\frac{\pi}{180} \cos x^\circ$

c) $\frac{180}{\pi} \cos x^\circ$

d) 1

Question 42

If $y = \text{Tan}^{-1}\left(\frac{2x}{1-x^2}\right)$ then $\frac{dy}{dx} =$

a) $-\frac{1}{x}$ b) $\sec^{-1}\left(\frac{2x}{1-x^2}\right)$

c) $\frac{1}{1+x^2}$ d) $\frac{2}{1+x^2}$

Question 43

If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ then $\frac{dy}{dx} =$

a) 1

b) x

c) y

d) ∞

Question 44

If $f(x) = \cos^{-1} \left[\frac{1 - (\log x)^2}{1 + (\log x)^2} \right]$ then $f'(e) =$

- a) does not exist b) $\frac{2}{e}$
c) $\frac{1}{e}$ d) 1

Question 45

If $y = e^{(5-x)}$ then $\frac{dy}{dx}$ at $x = 0$ is

a) $e \log 5$

b) $e^5 \cdot \log 5$

c) $-e \log 5$

d) e

Question 46

If $f(x) = x |x|$ then it's derivative is

a) 2x

b) -2x

c) 2 |x|

d) $2x^2$

Question 47

47. If $y = \tan^{-1} \left[\frac{\cos x - \sin x}{\cos x + \sin x} \right]$ then $\frac{dy}{dx} =$

a) 0

b) $\cos 2x$

c) $\sin 2x$

d) -1

Question 48

48. Let $U = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ & $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ then $\frac{du}{dv} =$

a) $\frac{1}{2}$

b) x

c) $\frac{1-x^2}{1+x^2}$

d) 1

Question 49

49. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.t. $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

a) $\frac{3}{2}$

b) 1

c) $\frac{1}{2}$

d) $\frac{2}{3}$

Question 50

50. If $y = \sin^{-1} \left(\frac{5 \sin x + 4 \cos x}{\sqrt{41}} \right)$ then $\frac{dy}{dx} =$

a) $\frac{1}{2}$

b) 0

c) 1

d) $\frac{1}{\sqrt{41}}$

Question 51

51. If $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right) + \tan^{-1}\left(\frac{2+3x}{3-2x}\right)$ then $\frac{dy}{dx}$

a) $\frac{1}{1+x^2}$

b) $\frac{5}{1+25x^2}$

c) 1

d) -1

Question 52

52. If $y = \tan^{-1} \left(\frac{3a^2x - x^3}{a(a^2 - 3x^2)} \right)$ then $\frac{dy}{dx} =$

a) $\frac{3a^2}{a^2 + x^2}$

b) $\frac{3a}{a^2 + x^2}$

c) $\frac{a}{a^2 + x^2}$

d) $\frac{3}{a^2 + x^2}$

Question 53

53. If $y = \sin^{-1} \left[\sqrt{x - ax} - \sqrt{a - ax} \right]$ then $\frac{dy}{dx} =$

a) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$

b) $\frac{1}{\sin \sqrt{a - ax}}$

c) $\sin^{-1} \sqrt{x} \sin^{-1} \sqrt{a}$

d) 0

Question 54

54. $y = (1 + x)(1 + x^2)(1 + x^4)\dots(1 + x^{2n})$ then $\frac{dy}{dx}$ at $x = 0$ is

a) 1

b) -1

c) 0

d) none

Question 55

55. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$ then $\frac{dy}{dx} =$

a) $-x$

b) x

c) y

d) $-y$

Question 56

56. If $y = \log |x|$, $x \neq 0$ then $\frac{dy}{dx}$ is

a) $\frac{1}{|x|}$

b) $\frac{1}{x}$

c) $\frac{1}{-x}$

d) 1

Question 57

57. If $x = a \cos^3\theta$ and $y = a \sin^3\theta$, then $\frac{dy}{dx} =$

a) $\sqrt[3]{\frac{x}{y}}$

b) $\sqrt[3]{\frac{y}{x}}$

c) $\sqrt[3]{\frac{y}{x}}$

d) $\sqrt[3]{\frac{x}{y}}$

Question 58

58. If $f(x)$ is an even function & $f^{-1}(x)$ exists then $f^{-1}(e) + f^{-1}(-e)$ is

a) > 0

b) 0

c) ≥ 0

d) < 0

Question 59

59. If $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$ the $\frac{dy}{dx} =$

a) $\frac{x}{\sqrt{1-x^4}}$

b) $\frac{x}{\sqrt{1+x^4}}$

c) $\frac{x^2}{\sqrt{1+x^4}}$

d) $\frac{x^2}{\sqrt{1-x^4}}$

Question 60

60. If $x = \frac{1-t^2}{1+t^2}$ $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx} =$

- a) $-\frac{x}{y}$ b) $\frac{x}{y}$ c) $-\frac{y}{x}$ d) $\frac{y}{x}$