Question 1

$$\lim_{n\to\infty} \frac{3.2^{n+1} - 4.5^{n+1}}{5.2^{n} + 7.5^{n}} =$$

a) $\frac{-20}{7}$

b) 0

c) $\frac{3}{5}$

d) $-\frac{4}{5}$

1. $$= \lim_{n\to\infty} \frac{3.2^n.2 - 4.5^n.5}{5.2^n + 7.5^n}$$

$$= \lim_{n\to\infty} \frac{5^n\left(6.\left(\frac{2}{5}\right)^n - 20\right)}{5^n\left(7 + 5(2/5)^n\right)} = \frac{0-20}{7+0} = -\frac{20}{7}$$

$$a)\frac{-20}{7}$$

Question 2

The function

$$f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$$

is not defined at x = 0. The value which should be assigned to f at x = 0 so that it is continuous at x = 0 is

a) loga + logb
b) 0
c) a – b
d) a + b

2. Since f(x) is continuous at x = 0

$$\therefore f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\log(1+ax) - \log(1-bx)}{x}$$

$$= \lim_{x\to 0} \frac{\frac{1}{1+ax}(a) + \left(\frac{1}{1-bx}\right)(-b)}{1} \quad by \;\; L.H.Rule$$

$$= \frac{\frac{a}{1+0} + \frac{b}{1-0}}{1} = a+b$$

$$\therefore f(0) = a+b$$

d) a + b

Question 3

If the function

$$f(x) = \begin{cases} \dfrac{1-\cos x}{x^2} & for\ x \neq 0 \\ k & x = 0 \end{cases}$$

is continuous at x = 0 then the value of k is

a)1 b)0

c)1/2 d)-1

3. Since f(x) is continuous at x = 0

$$\therefore f(0)=\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{1-\cos x}{x^2}\ \left(\frac{0}{0}\right)$$

$$K=\lim_{x\to 0}\frac{0-(-\sin x)}{2x}=\lim_{x\to 0}\frac{1}{2}.\frac{\sin x}{x}=\frac{1}{2}$$

c) 1/2

Question 4

$$\lim_{x \to 0} \frac{1 - \cos nx}{1 - \cos mx} =$$

a) $\frac{m}{n}$ b) $\frac{n}{m}$

c) $\frac{m^2}{n^2}$ d) $\frac{n^2}{m^2}$

4. $$\lim_{x\to 0}\frac{1-\cos nx}{1-\cos mx}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{0-(-\sin nx)n}{0-(-\sin mx)m}\ \textit{by LH rule}$$

$$=\lim_{x\to 0}\frac{n\sin nx}{m\sin mx}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{n.\cos nx\, n}{m.\cos mx.m}$$

$$=\frac{n^2.\cos 0}{m^2\cos 0}=\frac{n^2}{m^2}$$

d) $\dfrac{n^2}{m^2}$

Question 5

$$\lim_{n\to\infty}\left(1-\frac{1}{x}+\frac{1}{x^2}-\ldots\ldots\ldots to.n.terms\right)=$$

a) $\frac{1}{1-x}$ b) $\frac{1}{1+x}$

c) $\frac{x}{1+x}$ d) 0

5. $$= \lim_{n \to \infty} \left(1 + \left(-\frac{1}{x} \right) + \left(-\frac{1}{x} \right)^2 + \ldots . to\ n\ terms \right)$$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\left(-\frac{1}{x} \right)} = \frac{1}{1+\frac{1}{x}} = \frac{1}{\frac{x+1}{x}} = \frac{x}{1+x}$$

C) $\frac{x}{1+x}$

Question 6

$$\lim_{x \to 0} (1 + 3\tan x)^{3 \cot x} =$$

a) $e9$ b) $e3$

c) e d) 0

6. $\lim_{x\to 0}(1+3\tan x)^{\frac{3}{\tan x}} = \left((1+3\tan x)^{\frac{1}{3\tan x}}\right)^{9} = e^{9}$

a) e

Question 7

$$\lim_{x \to 0} \frac{10^x - 5^x - 2^x + 1}{x^2} =$$

a)log10

b)log5.log2

c)log25

d) 0

7. $$= \lim_{x \to 0} \frac{(5 \times 2)^x - 5^x - 2^x + 1}{x \; x} = \lim_{x \to 0} \frac{5^x.2^x - 5^x - 1(2^x - 1)}{x \quad x}$$

$$= \lim_{x \to 0} \frac{5^x(2^x - 1) - 1(2^x - 1)}{x \quad x} = \lim_{x \to 0} \frac{(5^x - 1)(2^x - 1)}{x \quad x}$$

$$= \log 5.\log 2$$

b)log5.log2

Question 8

If $f:R \rightarrow R$ is continuous such that $f(x+y) = f(x) + f(y) \ \forall \ x, y \in R$, & $f(1) = 2$ then $f(100) =$

a) 0 b) 100

c) 200 d) 400

8. $f(x + y) = f(x) + f(y)$

put $y = 1,\ x = 1 \qquad f(1 + 1) = f(x) + f(1)$

$f(2) = 2f(1)$

$x=2\ y = 1,\quad f(3) = f(2) + f(1) = 2(f(1) + f(1) = 3f(1)$

similarly $f(4) = 4\ f(1)$

$\therefore f(100) = 100\ f(1) = 100 \times 2$

$= 200$

c) 200

Question 9

$$\lim_{x \to 0} \frac{\sin nx\{(a-n)\,nx - Tanx\}}{x^2} = 0$$

where n is a non zero positive integer, then a is equal to

a) $\frac{n+1}{2}$

b) n^2+1

c) $\frac{1}{n+1}$

d) $n+\frac{1}{n}$

9. $\lim_{x\to 0} \frac{\sin nx}{x}\left\{\frac{(a-n)nx-\tan x}{x}\right\}=0$

$\lim_{x\to 0} \frac{\sin nx}{nx}.n\left\{\frac{(a-n)nx}{x}-\frac{\tan x}{x}\right\}=0$

$1\ n\{(a-n)n-1\}=0$

$an-n^2-1=\frac{0}{n}$

$an=n^2+1\ \therefore a=\frac{n^2+1}{n}$

$=n+\frac{1}{n}$

d) $n+\frac{1}{n}$

Question 10

$$\lim_{x \to T\,\mathrm{an}^{-1}(3)} \frac{T\,\mathrm{an}^2\,x - 2T\,\mathrm{an}\,x - 3}{T\,\mathrm{an}^2\,x - 4T\,\mathrm{an}\,x + 3} =$$

a) 1 b)0 c) $\frac{1}{2}$ d)2

a) 1 b)0

c) $\frac{1}{2}$ d)2

10. $$\frac{0}{0} = \lim_{x \to \tan^{-1} 3} \frac{2\tan x \sec^2 x - 2\sec^2 x - 0}{2\tan x \sec^2 x - 4\sec^2 x} \quad \textit{by LH Rule}$$

$$= \lim_{x \to \tan^{-1} 3} \frac{2\sec^2 x(\tan x - 1)}{2\sec^2 x(\tan x - 2)} = \frac{3-1}{3-2} = \frac{2}{1} = 2$$

d)2

Question 11

$$\lim_{x \to 0} \frac{(1+x)\sin(2+x) - \sin 2}{x}$$

a) 2cos 2

b)2 cos2+sin2

c) Cos2+sin2

d)3

11. $\frac{0}{0} = \frac{(1+x)\cos(2+x)+\sin(2+x)(0+1)-0}{1}$ *by LH Rule*

$$= (1+0)\cos(2+0)+\sin(2+0)$$

$$= \cos 2 + \sin 2$$

c) cos 2 + sin 2

Question 12

Let $$f(x)=\begin{cases}\dfrac{\sin \pi x}{5x} & if\ x \neq 0 \\ K & if\ x=0\end{cases}$$

if f(x) is continuous at x=0, the value of k is

a)1

b)0

c) $\dfrac{\pi}{5}$

d) $\dfrac{5}{\pi}$

12. Sinx f(x) is continuous $\therefore$ $f(0) = \lim_{x \to 0} f(x)$

$$K = \lim_{x \to 0} \frac{\sin \pi x}{5x} = \frac{\sin \pi x}{\pi x} . \frac{\pi}{5} = 1 = \frac{\pi}{5} = \frac{\pi}{5}$$

$$K = \frac{\pi}{5}$$

c) $\frac{\pi}{5}$

Question 13

If f(x)=cos(log x)then $\left[f\left(\frac{x}{y}\right) + f(xy) \right]$ has the value

a)f(x+y)

b)f(x)=f(y)

c)f(x)f(y)

d)2f(x)f(y)

13. $f(x/y) + f(x\,y)$ $= \cos(\log (x/y) + \cos (\log xy)$

$=\cos(\log x - \log y) + \cos (\log x + \log y)$

$= 2 \cos (\log x) \cos (\log y)$

$=2f(x)f(y)$

d)2f(x)f(y)

Question 14

$$\lim_{x \to 0} (1 - ax)^{\frac{1}{x}} =$$

a) e^{-a}

b) e

c) e^{a}

d) 1

14. $= \lim_{n \to \infty} \left[(1 - ax)^{\frac{1}{-ax}} \right]^{-a} = e^{-a}$

a) e^{-a}

Question 15

$$\lim_{\theta \to 0} \frac{\sin 3\theta \cdot \tan 4\theta}{\theta \cdot \sin 5\theta} =$$

a) 1

b) $\frac{5}{12}$

c) 3

d) $\frac{12}{5}$

15. $\lim_{\theta \to 0} \frac{\sin 3\theta}{3\theta} 3 \frac{\tan 4\theta}{4\theta} 4\theta . \frac{5\theta}{\sin 5\theta} . \frac{1}{5\theta} = \frac{3 \times 4}{5} = \frac{12}{5}$

d) $\frac{12}{5}$

Question 16

$$\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\ldots\ldots\ldots to.n.terms\right)=$$

a) $\frac{1}{2}$ b)0

c)∞ d) 1

16. $$= \lim_{n\to\infty}\left(\frac{1+2+3+.....+n}{n^2}\right)$$

$$= \lim_{n\to\infty}\frac{n(n+1)}{2n^2} = \lim_{n\to\infty}\frac{n(n)\left(1+\frac{1}{n}\right)}{2n^2} = \frac{1}{2}$$

a) $\frac{1}{2}$

Question 17

$$\lim_{x \to 2} \frac{2x^2 - 5x + 2}{x^2 - 3x + 2} =$$

a)1

b)2

c)3

d)8

17. $= \lim_{x \to 2} \frac{4x-5}{2x-3} (by\ LH\ Rule)$

$= \frac{8-5}{4-3} = 3$

c)3

Question 18

$$\lim_{x \to 0} \frac{\tan mx}{\tan nx} =$$

a) $\frac{m}{n}$

b) $\frac{m^2}{n^2}$

c) $\frac{n}{m}$

d) $\frac{n^2}{m^2}$

18. $= \lim_{x \to 0} \frac{\sec^2 mx.m}{\sec^2 nx.n} \left(LH\ Rule\right)$

$= \frac{m}{n}$

a) $\frac{m}{n}$

Question 19

$$\lim_{x\to\infty}\frac{(2+x)^{40}(4+x)^{5}}{(2-x)^{45}}$$ is

a)-1 b)1

c)16 d)32

$$19.\ =\lim_{x\to\infty}\frac{x^{40}\left(\frac{2}{x}+1\right)^{40}\ x^{5}\left(\frac{4}{x}+1\right)^{5}}{x^{45}\left(\frac{2}{x}-1\right)^{45}}$$

$$=\frac{(1+0)(0+1)}{(0-1)^{45}}=-1$$

a)-1

Question 20

20. If the three functions f(x), g(x) and h(x) are such that h(x) = f(x).g(x) and $f^1(x).g^1(x) = c$, where c is a constant, then

$$\frac{f''(x)}{f(x)} + \frac{g''(x)}{g(x)} + \frac{2c}{f(x).g(x)}$$ is equal to

a) $\frac{h^{11}(x)}{h(x)}$

b) $\frac{h(x)}{h'(x)}$

c) h'(x).h''(x)

d) $\frac{h(x)}{h''(x)}$

20. $h(x) = f(x)\, g(x)$

$h^1(x) = f(x)\, g^1(x) + g(x)\, f^{1(x)}$

$h^{11}(x) = f(x)\, g^{11}(x) + g^1(x)\, f^1(x) + g(x)\, f^{11}(x) + f^1(x)\, g^1(x)$

$= f^{11}(x)\, g(x) + g^{11}(x) f(x) + 2f^1(x) g^1(x)$

$$\frac{h^{||}(x)}{h(x)} = \frac{f^{11}(x)}{f(x)} + \frac{g^{11}(x)}{g(x)} + \frac{2C}{f(x)g(x)}$$

a) $\dfrac{h^{11}(x)}{h(x)}$

Question 21

21. The derivative of $e^{ax}\cos bx$ with respect to x is

$re^{ax}\cos\left(bx+Tan^{-1}\frac{b}{a}\right)$. When a > 0, b > 0, the value of r is

a) ab

b) a + b

c) $\sqrt{a^2+b^2}$

d) $\frac{1}{\sqrt{ab}}$

21. $y = e^{ax} \cos bx$

$$\frac{dy}{dx} = e^{ax}(-\sin bx)b + \cos bx\, e^{ax} a$$

$$= e^{ax}(a\cos bx - b\sin bx)$$

$$= e^{ax}(r\cos\theta\cos bx - r\sin\theta\sin bx)$$

$$= r\, e^{ax}(\cos(bx+\theta)$$

$$= r\ e^{ax}(\cos(bx+\tan^{-1} b/a))$$

put $a = r\cos\theta$

$b = r\sin\theta$

$r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}(b/a)$

c) $\sqrt{a^2 + b^2}$

Question 22

$$\text{If } f(x)=1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+.....+x^n \text{ then } f^{11}(1)=$$

a) $n(n-1)2^{n-2}$

b) $n(n-1)2^{n}$

c) $n(n-1)2^{n-1}$

d) $(n-1)2^{n-1}$

22. $f(x)=1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+\ldots+x^n$

$f(x) = (1 + x)^n$

$f^1(x) = n(1+x)^{n-1}$

$f^{11}(x) = n(n-1)\ (1+x)^{n-2}$

$f^{11}(1) = n(n-1)\ (1+1)^{n-2} = 2^{n-2}n(n-1)$

a) $n(n-1)2^{n-2}$

Question 23

3. If $f(x)=\frac{g(x)+g(-x)}{2}+\frac{2}{\left[h(x)+h(-x)\right]^{-1}}$ where g & h are differentiable functions then $f^{1}(0)=$

a) 3/2 b) 0

c) 1 d) ½

23. $f(x)=\dfrac{g(x)+g(-x)}{2}+2\left[h(x)+h(-x)\right]$

$$f^1(x)=\frac{g^1(x)+g^1(-x)(-1)}{2}+2\left[h^1(x)+h^1(-x)(-1)\right]$$

$$f^1(0)=\frac{g^1(0)-g^1(-0)}{2}+2\left[h^1(0)-h^1(-0)\right]$$

$$=0+0=0$$

b) 0

Question 24

If $y = \cos^2\left(\frac{3x}{2}\right) - \sin^2\left(\frac{3x}{2}\right)$ then $\frac{d^2y}{dx^2}$ is __

a) $-3\sqrt{1-y^2}$

b) 9y

c) -9y

d) $3\sqrt{1-y^2}$

24. $y = \cos 2\left(\frac{3x}{2}\right) = \cos 3x$

$$\frac{dy}{dx} = -\sin 3x.3$$

$$\frac{d^2y}{dx^2} = -3\cos 3x.3 = -9y$$

c)-9y

Question 25

If $y = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots\infty$ with $|x| > 1$ then $\frac{dy}{dx} =$

a) x^2y^2

b) $\frac{x^2}{y^2}$

c) $\frac{-y^2}{x^2}$

d) $\frac{y^2}{x^2}$

25. $y = 1 + \frac{1}{x} + \frac{1}{x^2} + ...\infty \; is \; in \; G.P$

$$= S\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{x}} = \frac{x}{x-1} \qquad \therefore \frac{y}{x} = \frac{1}{x-1}$$

$$\frac{dy}{dx} = \frac{(x-1)-x}{(x-1)^2} = -\left(\frac{1}{x-1}\right)^2 = -\frac{y^2}{x^2}$$

c) $\dfrac{-y^2}{x^2}$

Question 26

If $\sin(x + y) + \cos(x + y) = \log(x + y)$ then $\frac{d^2y}{dx^2} =$

a)0 b)-y/x

c)1 d)-1

26. $\cos(x+y)\left(1+\frac{dy}{dx}\right)-\sin(x+y)\left(1+\frac{dy}{dx}\right)-\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)=0$

$\left(1+\frac{dy}{dx}\right)\left(\cos(x+y)-\sin(x+y)-\frac{1}{x+y}\right)=0$

$1+\frac{dy}{dx}=0$

a) 0

Question 27

If $y = \sqrt{\tan x + \sqrt{\tan x + \cdots \infty}}$ *then* $\frac{dy}{dx} =$

a) $\frac{\sec^2 x}{2y-1}$

b) $\frac{\sec^2 x}{1-2y}$

c) $\frac{-\sec^2 x}{2y+1}$

d) -1

27. $\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

a) $\frac{\sec^2 x}{2y-1}$

Question 28

If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots + \frac{x^n}{n!}$ then $\frac{dy}{dx} + \frac{x^n}{n!} =$

a) x

b) $\frac{1}{x}$

c) y

d) $\frac{1}{y}$

28. $\frac{dy}{dx} = 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \ldots\ldots + \frac{nx^{n-1}}{n!}$

$\frac{dy}{dx} + \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \ldots\ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}$

$= y$

c) y

Question 29

If $\sin y = \sqrt{x}$ *then* $\dfrac{dy}{dx} =$

a) $\dfrac{1}{2\sqrt{1-x^2}}$

b) $\dfrac{1}{2\sqrt{x-x^2}}$

c) $\dfrac{-1}{2\sqrt{x^2-x}}$

d) 2

29. $y = \sin^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

b) $$\frac{1}{2\sqrt{x-x^2}}$$

Question 30

If $y = \sec^{-1}\left(\frac{\sqrt{x}+10}{\sqrt{x}-10}\right) + \sin^{-1}\left(\frac{\sqrt{x}-10}{\sqrt{x}+10}\right)$ then $\frac{dy}{dx} =$

a)0

b) $\frac{1}{2\left(\sqrt{x}-10\right)}$

c) $\frac{-1}{2\sqrt{x}}$

d)None

30. $y = \cos^{-1}\left(\frac{\sqrt{x}-10}{\sqrt{x}+10}\right) + \sin^{-1}\left(\frac{\sqrt{x}-10}{\sqrt{x}+10}\right) = \frac{\pi}{2}$

$\therefore \frac{dy}{dx} = 0$

a)0

Question 31

If $y = b\left[\tan^{-1}\frac{y}{x} + \tan^{-1}\frac{x}{y}\right]$ *then* $\frac{dy}{dx} =$

a)1 b)0

c)-2 d) 1

31. $y = b\left(\cot^{-1}\frac{x}{y} + \tan^{-1}\frac{x}{y}\right) = b\frac{\pi}{2}$

$$\frac{dy}{dx} = 0$$

b) 0

Question 32

If $x^x = y^y$ then $\frac{dy}{dx} =$

a) $\frac{-y}{x}$

b) $\frac{-x}{y}$

c) $1+\log\left(\frac{x}{y}\right)$

d) $\frac{1+\log x}{1+\log y}$

32. $X^x = y^y$

$$x^x\left(1+\log x\right)=y^y\left(1+\log y\right)\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{x^x\left(1+\log x\right)}{y^y\left(1+\log y\right)}=\frac{x^x\left(1+\log x\right)}{x^x\left(1+\log y\right)}=\frac{1+\log x}{1+\log y}$$

d) $\dfrac{1+\log x}{1+\log y}$

Question 33

If $y = \sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$ then $\frac{dy}{dx} =$

a) $\frac{2}{\sqrt{1-x^2}}$

b) $\frac{a}{\sqrt{1-a^2}}$

c) $\frac{2a}{\sqrt{1-a^2x^2}}$

d) $\frac{2a^2}{1-a^2x^2}$

33. $y = \sin^{-1}\left(2(ax) - \sqrt{1-(ax)^2}\right)$

$y = 2\sin^{-1}(ax)$

$$\frac{dy}{dx} = 2\frac{1}{\sqrt{1-(ax)^2}}a = \frac{2a}{\sqrt{1-a^2x^2}}$$

c) $\dfrac{2a}{\sqrt{1-a^2x^2}}$

Question 34

If $x = a(\cos\theta + \theta\sin\theta)$ & $y = a(\sin\theta - \theta\cos\theta)$ then $a\theta\frac{d^2y}{dx^2} =$

a)$\cos\theta$ b)$\sec^2\theta$

c)$\sec^3\theta$ d)None

34. $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\left(\cos\theta - \left(\theta\left(-\sin\theta\right) + \cos\theta.1\right)\right)}{\left(-\sin\theta + \theta\cos\theta + \sin\theta\right)}$$

$$= \frac{a\left(\cos\theta + \theta\sin\theta - \cos\right)}{a\theta\ \cos\theta} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

$$\frac{d^2y}{dx^2} = \sec^2\theta\frac{d\theta}{dx} = \sec^2\theta\frac{1}{a\theta\cos\theta}$$

$$a\theta\ \frac{d^2y}{dx^2} = \sec^2\theta\ \sec\theta = \sec^3\theta$$

c) $\sec^3\theta$

Question 35

If $y = \tan^{-1}\left(\frac{a\sin x + b\cos x}{a\cos x - b\sin x}\right)$ then $\frac{dy}{dx} =$

a)-1 b)1

c)$x + \alpha$ d)0

35. $y = \tan^{-1}\left(\dfrac{a\cos x\left(\dfrac{\sin x}{\cos x}+\dfrac{b}{a}\right)}{a\cos x\left(1-\dfrac{b}{a}\dfrac{\sin x}{\cos x}\right)}\right) = \tan^{-1}\left(\dfrac{\tan x + b/a}{1-\tan x.b/a}\right)$

$= \tan^{-1}(\tan x) + \tan^{-1}(b/a)$

$= x + \tan^{-1}(b/a)$

$\dfrac{dy}{dx} = 1+0 = 1$

b) 1

Question 36

Let $f(x + y) = f(x)\, f(y)$ for all x & y. suppose $f(5) = 2$.

$f^{|}(0) = 3$ then $f^{1}(5)$ is

a)4 b)3

c)8 d)6

36. Put $y = 5$ in $f(x+y) = f(x)\, f(y)$

$f(x + 5) = f(x)\, f(5)$

$f(x+5) = f(x)\, 2$

$f^1(x+5) = 2f^1(x)$

put $x = 0$ $f^1(0+5) = 2f^1(0)$

$f^1(5) = 2 \times 3 = 6$

d) 6

Question 37

The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.to $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

a)2

b) $\frac{1}{2}$

c)-1

d)1

37. $y = \sin^{-1}\left(\frac{2x}{1+x}\right) and\ Z = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 90 - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$= 2\tan^{-1}x \qquad = 90 - 2\tan^{-1}x$

$$\frac{dy}{dz} = \frac{dy/dx}{dz/dx} = \frac{2/1+x^2}{0-2/1+x^2} = -1$$

c) -1

Question 38

If $2x^2 + 4xy + 3y^2 = 0$ then $\frac{d^2y}{dx^2} =$

a) 0

b) $\frac{1}{2}$

c) $\frac{1}{(2x + y)^2}$

d) $\frac{4}{3}$

38. It is a homogeneous eqn of degree 2 and the RHS = 0

$$\therefore \frac{d^2y}{dx^2} = 0$$

a) 0

Question 39

If $y = \tan^{-1}\left(\frac{x+a}{1-xa}\right)$ then $\frac{dy}{dx} =$

a) $\frac{1}{a^2+x^2}$

b) $\frac{1}{1+x^2}$

c) $\frac{y}{a^2+x^2}$

d) 0

39. $y = \tan^{-1} x + \tan^{-1} a$

$$\frac{dy}{dx} = \frac{1}{1+x^2} + 0 = \frac{1}{1+x^2}$$

b) $\dfrac{1}{1+x^2}$

Question 40

If $y = \tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log\left(ex^2\right)}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$ *then* $\frac{d^2y}{dx^2} =$

a) 0 b)-1

c)2 d)1

40. $y = \tan^{-1}\left[\frac{\log e - \log x^2}{\log e + \log x^2}\right] + \tan^{-1}\left[\frac{3 + 2\log x}{1 - 3.2\log x}\right]$

$= \tan^{-1}\left[\frac{1 - 2\log x}{1 + 1.2\log x}\right] + \tan^{-1}\left[\frac{3 + 2\log x}{1 - 3.2\log x}\right]$

$= \text{Tan}^{-1}1 - \tan^{-1}(2\log x) + \tan^{-1}3 + \tan^{-1}(2\log x)$

$= \tan^{-1}1 + \tan^{-1}3$

$\frac{dy}{dx} = 0$

a) 0

Question 41

If $y = \sin x^0$ then $\frac{dy}{dx} =$

a) $\cos x^\circ$

b) $\frac{\pi}{180}\cos x^\circ$

c) $\frac{180}{\pi}\cos x^\circ$

d) 1

41. $y = \sin \dfrac{\pi x}{180}$

$$\frac{dy}{dx} = \cos \frac{\pi x}{180} . \frac{\pi}{180} = \frac{\pi}{180} \cos x^0$$

b) $\dfrac{\pi}{180} \cos x^\circ$

Question 42

If $y = \text{Tan}^{-1}\left(\frac{2x}{1-x^2}\right)$ then $\frac{dy}{dx} =$

a) $-\frac{1}{x}$

b) $\sec^{-1}\left(\frac{2x}{1-x^2}\right)$

c) $\frac{1}{1+x^2}$

d) $\frac{2}{1+x^2}$

42. $y = 2\tan^{-1}x$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

d) $\dfrac{2}{1+x^2}$

Question 43

If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$ then $\frac{dy}{dx} =$

a)1 b)x

c)y d)∞

43. $\frac{dy}{dx} = 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \ldots\ldots = y$

c) y

Question 44

If $f(x) = \cos^{-1}\left[\frac{1-(\log x)^2}{1+(\log x)^2}\right]$ then $f'(e) =$

a) does not exists

b) $\frac{2}{e}$

c) $\frac{1}{e}$

d) 1

44. = f(x) = $2\tan^{-1}(\log x)$

$$f^1(x) = \frac{2}{x\left(1+(\log x)^2\right)}$$

$$f^1(e) = \frac{2}{e\left(1+(\log e)^2\right)} = \frac{1}{e}$$

c) $\frac{1}{e}$

Question 45

If $y = e^{(5-x)}$ then $\frac{dy}{dx}$ at $x = o$ is

a)e log 5

b)e^5. log5

c)-e log5

d)e

45. $y = e^{(5^{-x})}$

$$\frac{dy}{dx} = e^{(5^{-x})}.5^{-x}.\log 5^{-1}$$

$$at\ x = 0$$

$$\frac{dy}{dx} = e.1.(-1)\log 5 = -e\log 5$$

c)-e log5

Question 46

If $f(x) = x\,|x|$ then it's derivative is

a)$2x$ b)$-2x$

c)$2\,|x|$ d)$2x^2$

46. $f^1(x) = x\dfrac{x}{|x|} + |x|$

$$= \frac{x^2}{|x|} + |x|$$

$$= \frac{|x|^2}{|x|} + |x| = |x| + |x| = 2|x|$$

c) $2\,|x|$

47. If $y = \tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right]$ *then* $\frac{dy}{dx} =$

a)0 b)cos 2x c)sin 2x d)-1

47. $y = \tan^{-1}\left[\dfrac{1-\tan x}{1+\tan x}\right] = \tan^{-1}\left[\dfrac{\tan 45 - \tan x}{1+\tan 45.\tan x}\right]$

$= \tan^{-1}\left(\tan\left(45-x\right)\right) = 45 - x$

$\dfrac{dy}{dx} = -1$

d) -1

48. Let $U = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ & $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ then $\frac{du}{dv} =$

a) $\frac{1}{2}$ b)x c) $\frac{1-x^2}{1+x^2}$ d)1

48. $U = 2\tan^{-1}x \qquad v = 2\tan^{-1}x$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{2/1+x^2}{2/1+x^2} = 1$$

d) 1

49. The derivative of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.t. $\cot^{-1}\left(\frac{1-3x^2}{3x-x^3}\right)$ is

a) $\frac{3}{2}$ b) 1 c) $\frac{1}{2}$ d) $\frac{2}{3}$

49. U = $2\tan^{-1}x$ $\qquad v = \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right) = 3\tan^{-1} x$

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{2/1+x^2}{3/1+x^2} = \frac{2}{3}$$

d) $\dfrac{2}{3}$

50. If $y = \sin^{-1}\left(\frac{5\sin x + 4\cos x}{\sqrt{41}}\right)$ then $\frac{dy}{dx} =$

a) ½ b) 0 c) 1 d) $\frac{1}{\sqrt{41}}$

50. $y = \sin^{-1}\left(\frac{5}{\sqrt{41}}\sin x + \frac{4}{\sqrt{41}}\cos x\right)$

$= \sin^{-1}\left(\sin x\cos\beta + \cos x\sin\beta\right) = \sin^{-1}\left(\sin\left(x+\beta\right)\right)$

$= x + \beta$

$\frac{dy}{dx} = 1$

c) 1

51. If $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right)+\tan^{-1}\left(\frac{2+3x}{3-2x}\right)$ then $\frac{dy}{dx}$

a) $\frac{1}{1+x^2}$ b) $\frac{5}{1+25x^2}$ c) 1 d) -1

51. $y = \tan^{-1}\left(\frac{5x - x}{1 + 5x.x}\right) + \tan^{-1}\left(\frac{3(2/3 + x)}{3(1 - 2/3\ \ x)}\right)$

$= \tan^{-1} 5x - \tan^{-1} x + \tan^{-1} \frac{2}{3} + \tan^{-1} x$

$\frac{dy}{dx} = \frac{5}{1 + 25x^2}$

b) $\frac{5}{1 + 25x^2}$

52. If $y = \tan^{-1}\left(\frac{3a^2x - x^3}{a(a^2 - 3x^2)}\right)$ then $\frac{dy}{dx}$ =

a) $\frac{3a^2}{a^2 + x^2}$ b) $\frac{3a}{a^2 + x^2}$ c) $\frac{a}{a^2 + x^2}$ d) $\frac{3}{a^2 + x^2}$

52. $y = \tan^{-1}\left(\dfrac{a^3\left(3\dfrac{x}{a} - \left(\dfrac{x}{a}\right)^3\right)}{a^3\left(1 - 3\left(\dfrac{x}{a}\right)^2\right)}\right) = 3\tan^{-1}\dfrac{x}{a}$

$$\frac{dy}{dx} = \frac{3}{1+\left(\dfrac{x}{a}\right)^2}\,\frac{1}{a} = \frac{3a}{a^2+x^2}$$

b) $\dfrac{3a}{a^2+x^2}$

53. If $y = \sin^{-1}\left[\sqrt{x-ax} - \sqrt{a-ax}\right] then \frac{dy}{dx} =$

a) $\frac{1}{2\sqrt{x}\sqrt{1-x}}$

b) $\frac{1}{\sin\sqrt{a-ax}}$

c) $\sin^{-1}\sqrt{x}\sin^{-1}\sqrt{a}$

d) 0

53. $y = \sin^{-1}\left(\sqrt{x(1-a)} - \sqrt{a(1-x)}\right)$

$= \sin^{-1}\left(\sqrt{x}\sqrt{1-\left(\sqrt{a}\right)^2} - \sqrt{a}\sqrt{1-\left(\sqrt{x}\right)^2}\right)$

$= \sin^{-1}\sqrt{x} - \sin^{-1}\sqrt{a}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}}$

a) $\dfrac{1}{2\sqrt{x}\sqrt{1-x}}$

54. $y = (1 + x)(1 + x^2)(1 + x^4)\ldots.(1 + x^{2n})$ then $\frac{dy}{dx}$ *at* $x = 0$ *is*

a) 1 b) -1 c) 0 d) none

54. $y=\dfrac{(1-x)(1+x)(1+x^2)(1+x^4)....(1+x^{2n})}{(1-x)}$

$$y=\frac{(1-x^2)(1+x^2)(1+x^4)....(1+x^{2n})}{(1-x)}$$

$$y=\frac{(1-x^4)(1+x^4)....(1+x^{2n})}{(1-x)}$$

$$=\frac{\left(1-(x^{2n})^2\right)}{1-x}=\frac{1-x^{4n}}{1-x}$$

$$\frac{dy}{dx}=\frac{(1-x)(0-4nx^{4n-1})(-1)-(1-x^{4n})(0-1)}{(1-x)^2}$$

$at\ x=0$

$$\frac{dy}{dx}=\frac{0-(1-0)(-1)}{(1-0)}=1$$

a) 1

55. If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots\cdots$ then $\frac{dy}{dx}$ =

a) $-x$ b) x c) y d) $-y$

55. $$\frac{dy}{dx} = 0 - 1 + \frac{2x}{2!} - \frac{3x^2}{3!} + \ldots.$$

$$\frac{dy}{dx} = -\left(1 - \frac{2x}{2!} + \frac{3x^2}{3!} - \ldots.\right)$$

$$= -y$$

d) -y

56. If $y = \log |x| . x \neq 0$ then $\frac{dy}{dx}$ is

a) $\frac{1}{|x|}$ b) $\frac{1}{x}$ c) $\frac{1}{-x}$ d) 1

56. $\frac{dy}{dx} = \frac{1}{|x|} \cdot \frac{x}{|x|} = \frac{x}{|x|^2} = \frac{x}{x^2} = \frac{1}{x}$

b) $\frac{1}{x}$

57. If $x = a\,\mathrm{Cos}^3\theta$ and $y = a\,\mathrm{Sin}^3\theta$, then dy/dx =

a) $\sqrt[-3]{\frac{x}{y}}$ b) $\sqrt[-3]{\frac{y}{x}}$ c) $\sqrt[3]{\frac{y}{x}}$ d) $\sqrt[3]{\frac{x}{y}}$

57. $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = \frac{-\sin\theta}{\cos\theta} = -\sqrt[3]{\frac{a\sin^3\theta}{a\cos^3\theta}}$$

$$= -\sqrt[3]{\frac{y}{x}}$$

b) $-\sqrt[3]{\frac{y}{x}}$

58. If f(x) is an even function & $f^{1}(x)$ exists then $f^{1}(e) + f^{1}(-e)$ is

a) > 0 b) 0 c) ≥ 0 d) <0

58. $f(-x) = f(x)$

$f^1(-x)(-1) = f^1(x)$

$f^1(-e) = f^1(e) \quad \therefore f^1(e) + f^1(-e) = 0$

b) o

59. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ the $\frac{dy}{dx}=$

a) $\frac{x}{\sqrt{1-x^4}}$ b) $\frac{x}{\sqrt{1+x^4}}$ c) $\frac{x^2}{\sqrt{1+x^4}}$ d) $\frac{x^2}{\sqrt{1-x^4}}$

59. $y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x^2$

$$\frac{dy}{dx} = 0 - \frac{1}{2}\frac{(-1).2x}{\sqrt{1-x^4}} = \frac{x}{\sqrt{1-x^4}}$$

a) $\dfrac{x}{\sqrt{1-x^4}}$

60. If $x = \dfrac{1-t^2}{1+t^2} \quad y = \dfrac{2t}{1+t^2}, \quad then \quad \dfrac{dy}{dx} =$

a) $-\dfrac{x}{y}$ b) $\dfrac{x}{y}$ c) $-\dfrac{y}{x}$ d) $\dfrac{y}{x}$

60. Put t = tan θ

$\therefore x = \cos 2\theta$ and $y = \sin 2\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos 2\theta}{-2\sin 2\theta} = -\frac{y}{x}$$

c) $-\frac{y}{x}$