UNIT – II
MECHANICS

2.1 MOTION IN ONE DIMENSION

Mechanics is the oldest and the most fundamental of all branches of physics. Fundamentally, mechanics is the study of the state of rest as well as the state of motion of objects under the action of a force or forces.

The study of mechanics is broadly classified into 1) Dynamics and 2) Statics.

Dynamics deals with bodies in motion.

Statics deals with bodies at rest under the action of a system of forces.

Dynamics is again divided into kinematics and kinetics.

Kinematics deals with the description of motion without reference to the cause of motion. (Or How of motion?).

Kinetics deals with what moves and what causes it to move (i.e. it deals with the mass of the body in motion and force that causes motion). (Or Why of motion?).

Motion:

Two passengers sitting in a moving train are at rest relative to each other, but they are in motion relative to a stationary observer outside, in other words, rest and motion are relative. Thus we need a reference to describe rest and motion. The reference we use is called 'Frame of reference'

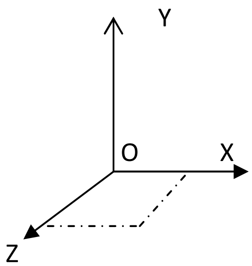
Frame of reference:

Frame of reference is the reference in which an observer sits and makes the observation. There are two types.

1) Inertial frame of reference.
2) Non-inertial frame of reference.

A frame of reference which is either at rest or moving with constant velocity is known as inertial frame of reference.

A frame of reference moving with some acceleration is known as non-inertial frame of reference.



To make observation we attach the frame of reference. Certain co-ordinate system consists of three mutually perpendicular axis x, y & z meeting at a common point 'O', the origin. When a particle moves in space, its position at any instant of time can be described with the help of its three positions coordinates **(x, y & z)** which change with time during the motion of particle.

Motion and Rest:

A body is said to be in motion if its position changes relative to other bodies regarded as fixed with the passage of time.

A body is said to be at rest when it occupies a same position relative to other body and does not change its position with the passage of time.

Motion is of two kinds, i.e,

1) Translatory motion. 2) Rotatory motion.

A body is said to have translatory motion when it moves such that all particles of the body perform the same motion, i.e., have equal velocities at any instant and describe similar paths or trajectory.

Ex: An arrow shot from a bow, a stone dropped from a height, a train moving on its rails, etc.

The motion of a body is said to be rotatory if it moves such that a particle or a line of particles in it remains fixed relative to the surroundings.

Ex: The wheel of a cycle in motion has rotatory motion. It has rotational motion about its axis.

The main characteristics of motion of a particle are its trajectory, co-ordinates, displacement, velocity, acceleration and path length.

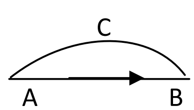
Motion in one dimension:

The motion of a particle is said to be in one dimension if only one out of three co-ordinates specifying the position of the particle change with time.
Ex: Car moving along a straight road.

Distance and Displacement:

Distance is the total length of the path described by a particle in a given interval of time or Actual distance traveled by a particle in given interval of time.

Displacement is a vector joining the initial position of the particle to its final position in a given interval of time.



Consider a body moving from A to B as shown in figure. The actual length of curved path ACB is the distance traversed by the body is given in time interval. The line segment AB gives the magnitude of displacement of body in the given time interval.
of displacement is directed from A to B through the straight line AB and the magnitude of displacement is AB.

Important Features:

a. Distance is a scalar quantity and displacement is a vector quantity.
b. Path length or distance is a positive scalar quantity which does not decrease with time and can never be zero for a moving body. Displacement of a body can be zero.
c. Magnitude of displacement can never be greater than distance.
d. When a body returns to its initial position, its displacement is zero but distance or path length is non-zero.
e. When body moves along a straight line, displacement is equal to distance.

Speed and Velocity:

Speed of a moving particle is defined as the distance covered per unit time.

$$\textbf{Speed} = \frac{\textbf{distance covered}}{\textbf{Time taken}}$$

It is a scalar having unit **ms^{-1}** and is always positive.

Speed is uniform if moving object covers equal distance in equal intervals of time.
For a non-uniform motion, we define average speed as the ratio of the total distance traveled to the total time taken.

$$\text{Average speed} = \frac{\text{total distance traveled}}{\text{total time taken.}}$$

Velocity

Velocity of a body is defined as the rate of its displacement.

$$\textbf{Velocity} = \frac{\textbf{displacement}}{\textbf{time.}}$$

Velocity is a vector whose SI unit is **ms^{-1}.** Velocity is said to be uniform if both its magnitude and direction remain unchanged otherwise it is non-uniform.

Average velocity is defined as the ratio of total displacement and total time.

$$\textbf{Average velocity} = \frac{\textbf{total displacement}}{\textbf{time taken}}$$

Velocity of a body at any instant of time is called its instantaneous velocity.

Important Features

a. Speed is a scalar quantity. Velocity is a vector quantity.

b. Velocity can be negative, zero or positive, but speed is never negative.

c. If motion takes place in the same direction, then the average speed and average velocity are the same.

d. If a particle moves a distance at speed 'v_1' and comes back with speed 'v_2', then

$$V_{avg} = \frac{2\, v_1 v_2}{v_1 + v_2}$$

e. If a particle moves in two equal intervals of time at different speeds v_1 and v_2 respectively, then

$$V_{avg} = \frac{v_1 + v_2}{2}$$

Acceleration

Acceleration of a particle is defined as the rate of change of velocity.

$$\textbf{Acceleration} = \frac{\textbf{change in velocity}}{\textbf{time taken}}$$

It is a vector having unit **ms^{-2}.**

Acceleration may be uniform or variable.

Acceleration of a particle is said to be uniform, if the velocity of the particle changes by equal amount in equal intervals of time, however small the intervals may be. Otherwise acceleration is said to be variable.

Constant acceleration

Uniform acceleration having the same direction of velocity is called constant acceleration.

Ex: Acceleration due to gravity is 'g'.

Equations of motion of bodies with constant acceleration

The following symbols are used with reference to equations of motion.

u → initial velocity a → uniform acceleration v → final velocity

s → distance travelled in time 't' t → time interval for which motion is considered.

Velocity at time't'

By definition, acceleration 'a' is given by, $a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{(v - u)}{t}$

Therefore, $v = u + at$.

Distance‘s’ traveled in time‘t’

In the case of motion with variable velocity,

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time taken}} = \frac{s}{t}$$

As the acceleration is uniform,

$$\text{Average velocity} = \frac{(u + v)}{2} = \frac{s}{t}$$

Or $s = \frac{(u + v)}{2}$ **OR** $s = \frac{u + u + at}{2}$ (Because, $v = u + at$)

Hence, $\mathbf{s = ut + \frac{1}{2} at^2}$

Velocity acquired in traveling a distance‘s’:

$$\text{Average velocity} = \frac{\text{distance traveled}}{\text{time taken}}$$

$\frac{(u + v)}{2} = \frac{s}{t}$ Or $(u + v)\, t = 2s$

By definition, $a = \frac{(v - u)}{t}$ Or $t = \frac{(v - u)}{a}$

Therefore, $\frac{(u + v)(v - u)}{a} = 2s$

i.e. $v^2 - u^2 = 2as$

Hence, $\mathbf{v^2 = u^2 + 2as}$

Important Features of uniform motion

- Uniform motion is a straight line motion with constant velocity.
- In uniform motion displacement and distance are equal.
- No net force is required for an object to be in uniform motion.
- The velocity in uniform motion does not depend upon the time interval.
- The velocity in uniform motion is independent of choice of origin.

Uniform motion

A particle is said to be in uniform motion if it has equal displacements in equal intervals of time however small these intervals may be.

Distance S_n traveled during nth second of motion:

Distance traveled during nth second is equal to the difference between the distance traveled in ‘n’ seconds and (n-1) seconds.

$S_n = (un + \frac{1}{2} an^2) - [u (n-1) + \frac{1}{2} a (n-1)^2]$

$= u + an - \frac{1}{2} a$

Hence, $\mathbf{S_n = u + a (n - 1/2)}$.

Acceleration in terms of distance traveled in two consecutive equal intervals of time

If s_1 and s_2 are the distances traveled in two consecutive intervals of time ‘t’ each ‘a’ the constant acceleration and ‘u’ the initial velocity for s_1, then

$S_1 = ut + \frac{1}{2} at^2$ and

$s_1 + s_2 = u (2t) + \frac{1}{2} a (2t)^2 = 2\, ut + 2\, at^2$

Therefore, $(s_1 + s_2 - 2s_1) = at^2$

i.e. $s_2 - s_1 = at^2$

Hence, $\mathbf{a = \frac{(s_2 - s_1)}{t^2}}$

Motion under gravity from a height:

Any body dropped, moves towards the earth with nearly constant acceleration. Its motion is an example of one dimensional motion under gravity. This acceleration is the same for all bodies which are dropped and independent of their size, mass or composition and is called "acceleration due to gravity".

It is always directed towards the center of the earth and is denoted by 'g'.

The equation of motion for a freely falling body are obtained by replacing 'a' by 'g'

i.e. for a body falling freely under gravity,

1. $\mathbf{V = u + gt}$
2. $\mathbf{S = ut + ½\, gt^2}$
3. $\mathbf{V^2 = u^2 + 2gh}$
4. $\mathbf{S_{nt} = u + ½\, g\, (2n–1)}$

For a body rising vertically against gravity,

1. $\mathbf{V = u - gt}$
2. $\mathbf{S = ut – ½\, gt^2}$
3. $\mathbf{V^2 = u^2 – 2gh}$
4. $\mathbf{S_n = u – ½\, g\, (2n-1)}$

Graphs:

Graph is the pictorial representation of variation of one quantity with respect to another.

Graphs are extensively used in science. There are two main reasons for plotting experimental data on graphs, they are,

a) To communicate the result in an easily understandable manner.
b) To help in the analysis of the results.

The motion of the body can be conveniently represented graphically.

Time is plotted on x-axis and velocities or instantaneous position or acceleration on y-axis.

There are three types of graphs.

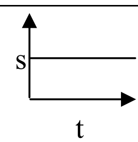
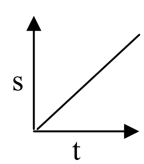
Position-Time graph:

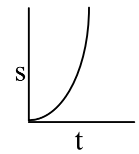
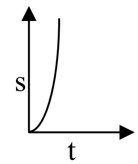
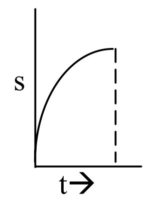
When instantaneous positions of a particle are plotted versus time, the resulting graph is known as position-time graph.

Note:

1. The graph gives instantaneous value of displacement at any instant.
2. The slope of tangent drawn to the graph at any instant of time gives the instantaneous velocity at that moment.
3. The s-t graph cannot make sharp turns.

Different cases of displacement-time graph

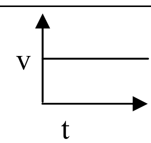
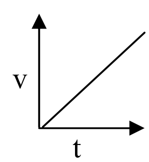
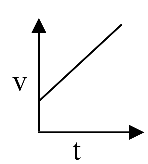
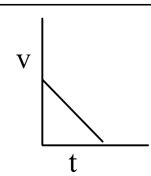
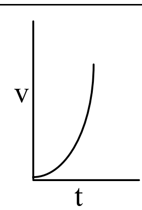
Sl No	Different cases	s-t Graph	Main Features
1	At Rest		Slope 'v' = 0
2	Uniform motion		θ = constant v = constant a = 0

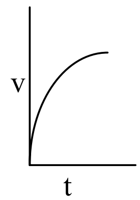
3	Uniform accelerated motion with u=0, s=0 at t=0		θ is increasing. So ‘v’ is increasing & ‘a’ is positive
4	Uniform accelerated motion with u≠0, but s=0 at t=0		Slope of s-t graph gradually goes on increasing
5	Uniformly retarded motion		θ is decreasing. So ‘v’ is decreasing & ‘a’ is negative

Velocity-Time Graph

1. Velocity-time graph gives the instantaneous value of velocity at any instant.
2. The slope of tangent drawn on graph gives instantaneous acceleration.
3. Area under v-t graph with time axis gives the value of displacement covered in given time.
4. The v-t curve cannot take sharp turns.

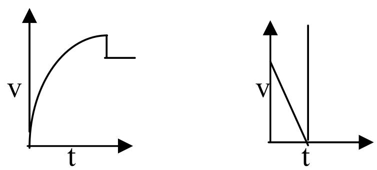
Different cases of Velocity-Time Graph

Sl No	Different cases	v-t Graph	Main Features
1	Uniform motion		$\theta = 0^0$ v = constant slope a = 0
2	Uniformly accelerated motion with u=0, s=0 at t=0		θ = constant a = constant v is increasing uniformly with time
3	Uniformly accelerated motion with u≠0, s=0 at t=0		Positive constant acceleration because θ is constant but the initial velocity of the particle is positive
4	Uniformly accelerated motion		Slope of v-t graph = -a (retardation)
5	Non-uniformly accelerated motion		Slope increases with time. That is ‘a’ increases

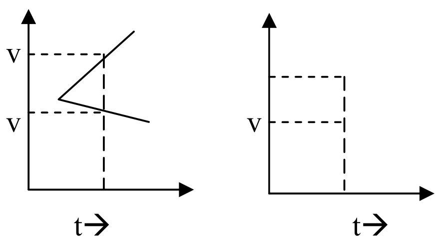
6	Non-uniformly accelerating motion		θ is decreasing. So acceleration is decreasing

Note:

1. Slope of v-t graph can never be infinite at any point, because infinite slope of v-t graph means infinite acceleration. Similarly, infinite slope of s-t graph means infinite velocity. Hence the following graphs are not possible.

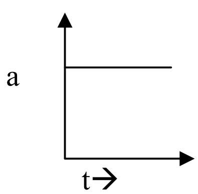


2. At one time, two values of velocity or displacement are not possible. Hence the following graphs are not acceptable.

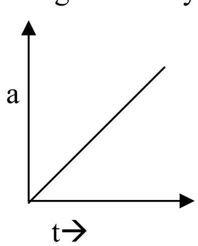


Acceleration-Time graph:

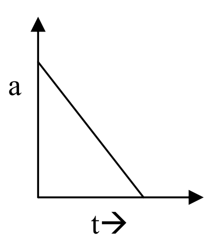
1) When a-t graph is a straight line parallel to the time axis, then acceleration ‘a’ is constant.



2) When a-t graph is a straight line passing through the origin, then acceleration of particle is increasing uniformly.



3) When a-t graph is a straight line of negative slope, then acceleration is decreasing uniformly.



4) Acceleration-time graph gives the instantaneous value of acceleration at any instant of time.

5) Area under a-t graph give value of change in velocity during the given time interval.

2.1 **Questions:**

1. Can the displacement of a particle be zero?
2. What does the speedometer in an automobile indicate average speed or instantaneous speed?
3. Can acceleration be negative?
4. What is the direction of ‘g’?
5. Is it possible to realize the motion of a body in which the velocity and acceleration are oppositely directed?
6. Can the velocity of a particle be zero but not its acceleration?
7. Distinguish between average speed and average velocity?
8. What does slope in a position-time graph represent?
9. What does area under a velocity-time graph represent?
10. Can a body be both at rest as well as in motion? Explain.
11. How will you find the displacement of an object in a given time interval from its velocity –time graph?.
12. Suppose we call the rate of change of acceleration as SLAP what are the unit and dimensions of SLAP.
13. A line on velocity –time graph parallel to time axis indicates that ------------ is zero.
14. An ant is crawling on the floor, state the types of motion.
15. The speed of a particle decreases from 30m/s to 10m/s in 5 sec what is the deceleration of the particle?

ANSWERS

2.1 Motion in one dimension

1. Yes, when body returns to its initial position its displacement is zero
2. Instantaneous speed.
3. yes
4. Vertically down words
5. Yes
6. Yes when the body is at the highest point
7. Speed is scalar but velocity is vector.
8. Slope in a P-T graph represents velocity of the body.
9. Area under the V-T graph represents distance covered by the body in a given intervals of time.
10. Rest and motion are relative .
11. Displacement =Area enclosed by V-T graph and T-axis.
12. [ms-3 , LT-3]
13. Acceleration
14. Two dimensional
15. 4 m/s^2