1.2 GRAPHS AND THEIR USES

The graphs provide a convenient method to present pictorially basic information about a variety of events, such as, motion. For example, in a one-day cricket match, vertical bars often shown the runs scored by a team in each over (Fig-1). You are familiar with such bar graphs about which you have studied in mathematics.

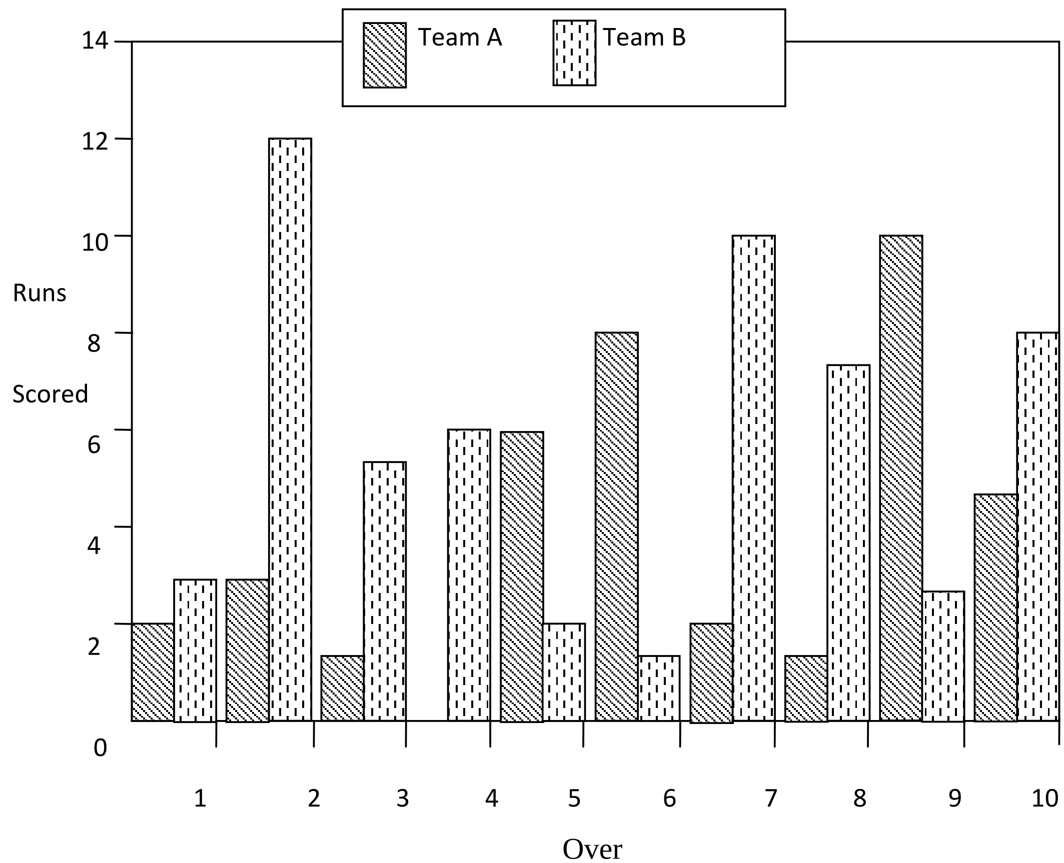

Fig-1. Bar graph showing comparison of runs scored in each over by two cricket teams

We shall now learn about another type of graph, which are known as line graph. Graphs of this type are most commonly used to show dependence of one quantity, say distance or velocity, on another quantity, such as, time.

To learn how to plot a graph, we shall consider a simple case of motion of a car moving along a straight road. We shall assume that the car is moving on a level road along a straight-line path. The distance moved by it in every twelve minutes is shown in table-1.

Time	0 min	12 min	24 min	36 min	48 min	60 min
Distance moved from the starting point	0 km	10 km	20 km	30 km	40 km	50 km

Table-1

To draw a line graph we first draw two perpendicular lines, as shown in fig-2a. The horizontal line $XOX^{|}$ is known as the x-axis and the vertical line YOY| as y-axis. The point of intersection of $XOX^{|}$ and $YOY^{|}$ is called the origin. It has been shown by point o in fig-2a. Let x-axis represent the time and y-axis the distance. The positive values of the quantities on the x-axis are shown along ox while those on the y-axis are shown along OY.

Now, we have to decide upon suitable scales to represent the two quantities on the graph. It is always advantageous to select a scale by carefully studying: (i) the total range of data i.e., the difference between the lowest and the highest value for each quantity and their intermediate values, and (ii) the size of the graph paper available. In the example we have taken (Table -1), the distance varies in a range of 50 km in steps of 10 km while time has a range of 60 minutes in steps of 12 minutes. Suppose we have a graph paper having a length 25 cm and width 15 cm.

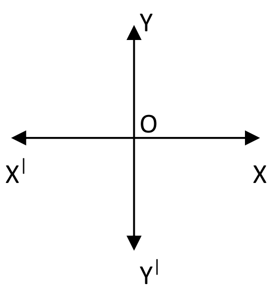

Fig -2a

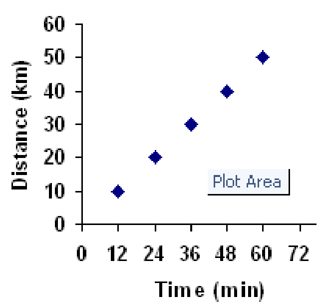

Fig- 2b

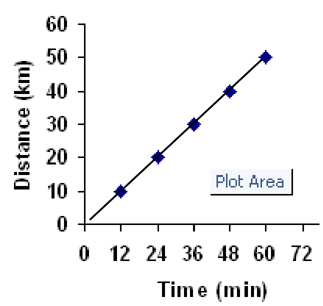

Fig-2c

One of the appropriate scales to accommodate the given data could be

distance 5 km = 1 cm time 6 min = 1 cm

One can, however, choose any other convenient scale to draw the graph. Once we have decided the scale, the next step is to plot the given data on the graph. We make a decision about the quantity to be shown along the x-axis and that along the y-axis. We begin with marking the points corresponding to values given for the two quantities – one on the x – axis and the other on y-axis. In the case of the data given in table-1, for example, points marked at every 2 cm on the x-axis would correspond to a time interval of 12 min. Similarly, points marked at every 2 cm on the y-axis would correspond to a distance of 10 km.

Next, we have to mark the point on the graph paper corresponding to each set of the two values, distance and time. Let us consider a particular value of time and corresponding value of distance. We note that at time 12 min, the distance moved is 10 km. we draw a line parallel to the distance axis (or perpendicular to x-axis) at the point that marks 12 min on the x-axis. Similarly, we draw a line parallel to the time axis at the point that marks 10 km on y-axis. The point where the two lines intersect each other gives the location of the point corresponding to the set of values under consideration. The points corresponding to other set of values are similarly marked on the graph (Fig-2b). Once all the points for a given data are marked on the graph paper, these are joined together by a smooth curve. In the present case, this curve is a straight line (Fig-2c). In general, as you will see later, the curve can be of any shape. Let us take a few more examples of motion and plot distance-time graphs for them.

Activity: The time of arrival and departure of a train at three stations A, B and C and their distance from the station A is given in the table-2. Plot the distance-time graph for the motion of this train assuming that its motion between two stations is uniform.

Station	Distance from Station	Time of Arrival	Time of Departure
A	0 km	08:00	08:15
B	120 km	11:15	11:30
C	180 km	13:00	13:15

Table-2

Adopting a convenient scale of your choice, plot the graphs between the distance and the time of arrival and departure. It is often convenient to use the lowest value as origin. Compare the shape of the graph obtained by you with that shown in Fig-3. Is the shape of the graph same or different from that shown in Fig-3.

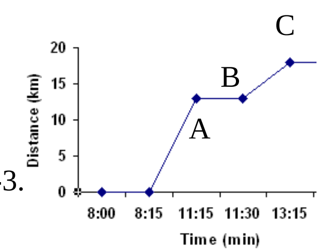

Fig-3

Activity: Amitabh and his sister Archana go to the school on their bicycles. Both of them start at the same time from their home but take different time to reach the school although they follow the same route. Table-3 shows the distances moved by them at different instants of time. Plot the distance-time graph for their motion on the same graph paper assuming a scale of your choice.

Compare the distance-time graph obtained by you with that shown Fig-4. You will find that the graphs for a given set of data are similar in appearance even if the scale is changed.

Time	8:00 am	8:05 am	8:10 am	8: 15 am	8: 20 am	8:25 am
Distance moved by Amitabh	0 km	1.0 km	1.9 km	2.8 km	3.6 km	_
Distance moved by Archana	0 km	0.8 km	1.6 km	2.3 km	3.0 km	3.6 km

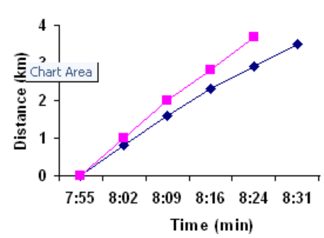

Fig - 4

Table – 3, Distance covered by Amitabh and Archana at different time on their bicycles

USES OF GRAPHS:

Distance-time graph can provide us information about the motion.

For example, Table-1, gives the position of the car only at definite instants of time. However, we can determine the position of the car at any intermediate time with the help of its distance-time graph shown Fig-5. Suppose we want to know how much distance the car has moved after 33 minutes. We first mark the point corresponding to 33rd minutes on the X-axis and drop a perpendicular from it on the graph at A. The distance, on the Y-axis, corresponding to the point where this perpendicular meets the graph gives us the distance moved by the car

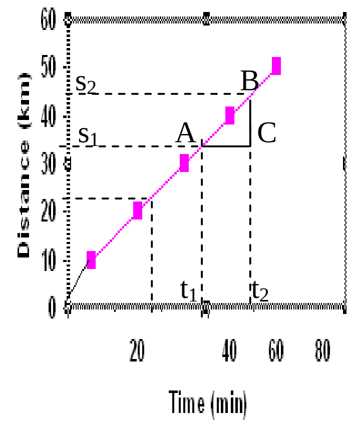

Fig-5

after 33 minute. You will find that the car has moved a distance of 27.5 km in that time. Similarly, we can determine the location of the train at any given time with the help of its distance-time graph shown in Fig-5.

We can also compute the velocity of a car from its distance-time graph. Let us consider a small part AB as shown by Fig-5. To find velocity of the car we draw a line parallel to the x-axis from point A and another line parallel to y-axis from point B. These two lines meet at point C to form a triangle ABC. Now on the graph, AC denotes a time interval (t_2- t_1) while BC corresponds to the distance ($s_2 - s_1$). It can be seen from the graph that as the car moves from the point A to B, it covers a distance ($s_2 - s_1$) in time ($t_2 - t_1$). The velocity of the car, is given by,

$$v = \frac{s_2 - s_1}{t_2 - t_1} \quad \text{i.e. velocity = slope of the curve} = \frac{BC}{AC}$$

We can also plot velocity-time graph to describe the motion of a body.

Let us first consider a body moving with a constant velocity, i.e., in uniform motion. Suppose we wish to plot the velocity-time graph for the motion of a car moving with a constant velocity of 40 km/h. That means the car will cover a distance of 40 km in 1 hour, 80 km in 2 hours, 120 km in 3 hours and so on (Fig-6) . It can be seen that the graph is a straight line parallel to the time axis. This is true for all velocity-time graphs if the motion is uniform.

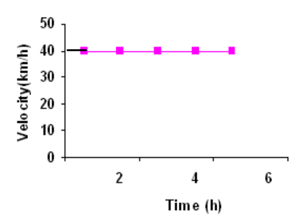

Fig-6

We can compute the distance moved by a body in a given time from its velocity-time graph. Suppose we want to know the distance the car has moved between time t_1 and t_2. We draw perpendiculars from points corresponding to the time t_1 and t_2 on the graph (Fig-7)

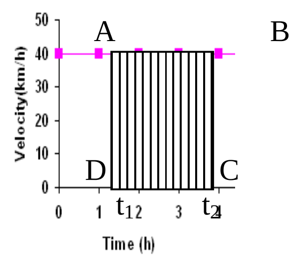

Fig- 7

The two perpendiculars enclose a rectangle ABCD between the graph and the time axis or the x-axis. Now, the sides AD and BC are equal to 40 km/h. We know that the distance, S, covered by a body moving a velocity, V, in time t, is given by $\boxed{S = Vt}$

So, S the distance moved by the car in time $(t_2 - t_1)$ would be

$S = [(40 \text{ km/h})(t_2 - t_1)\text{h}]$

$S = 40 (t_2 - t_1)$ km

S = the area of the rectangle ABCD

The area under the velocity-time graph, therefore, gives the distance traveled by the car. This holds true for any velocity-time graph, irrespective of whether the motion is uniform or non-uniform.

We can also study about uniformly accelerated motion by plotting its distance-time and the velocity-time graphs. In table-4, we have given the velocity of a car after every five seconds. The velocity-time graph for the motion of the car is shown in fig-8. For all uniformly accelerated motions, this graph is a straight line as shown in fig-8

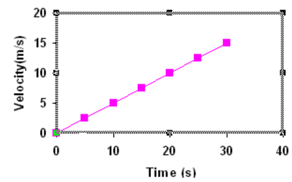

Fig-8

We can also plot the distance-time graph for the uniformly accelerated motion of the car in the above case. Table-4, shown the distance moved by the same car in an interval of 2 second.

Sl.No	Time (s)	Distance moved(m)
1	0	0
2	2	1
3	4	4
4	6	9
5	8	16
6	10	25
7	12	36

Table – 4 Distance moved by car at regular time intervals

The distance-time graph for this shape of this graph is different from the distance-time graph for uniform motion as shown in fig-9a. The curve shown by the graph in fig-9a is called a parabola.

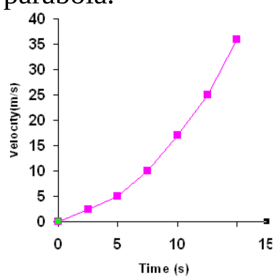

Fig – 9a

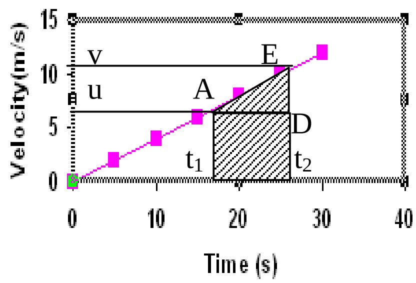

Fig-9b

Science and Technology:

We can also determine the distance moved by a body in a uniformly accelerated motion from its velocity-time graph. The area under the velocity-time graph gives the distance moved by a body in a given time interval. This is similar to what we have seen in the case of uniform motion earlier.

In the case of non-uniformly accelerated motion, the distance-time and velocity-time graphs can have any shape. Fig-10 shows the velocity-time graphs for a few different types of non-uniformly accelerated motion. Let us try to interpret the type of motion that is shown by the graph in Fig-10a. The shape of the graph shows that the velocity of body increases from v_0 to v_1 during time t_0 and t_1, it decreases to v_0 between t_1 and t_2. You may like to interpret the types of motion shown by the other two graphs shown in fig-10b and fig-10c.

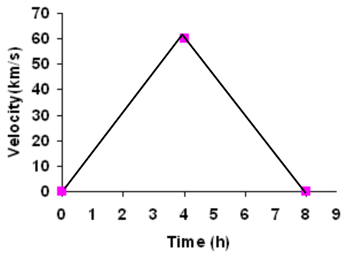

Fig-10a

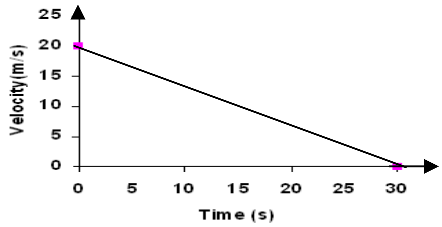

Fig-10b

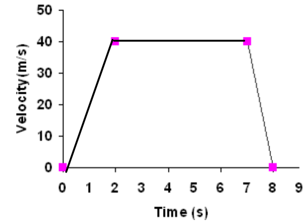

Fig-10c