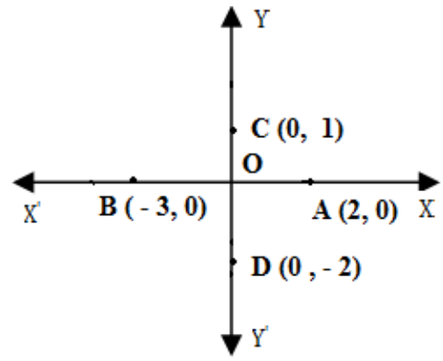
7.2 Distance formula

a. Consider the points A(2, 0) and B(– 3, 0). These two points lie on the x-axis. Now OA = 2 and OB = 3.

∴ AB = 2 + 3 = 5 (here 2 – (– 3) = 5).

∴ the difference between the x-coordinates gives the distance AB.

If P ≡ $(x_1, 0)$ and Q ≡ $(x_2, 0)$ where $x_1 < x_2$ are the given two points then the distance between P and Q is $= x_2 - x_1$.



b. Consider the points C(0, 1) and D(0, – 2). These two points lie on the y-axis. Now OC = 1 and OD = 2.

∴ CD = 1 + 2 = 3 (here 1 – (– 2) = 3).

∴ the difference between the y-coordinates gives the distance CD.

If R ≡ $(0, y_1)$ and S ≡ $(0, y_2)$ where $y_1 < y_2$, are the given two points then the distance between R and S is $= y_2 - y_1$.

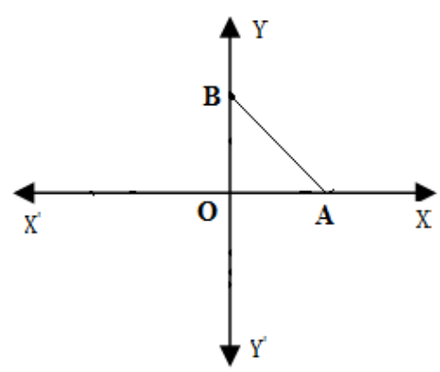
c. Consider the points A(1, 0) and B(0, 3). Since A lies on the x-axis and B lies on the y-axis, the ΔOAB is right angled at O.

∴ $AB^2 = OA^2 + OB^2 = 1^2 + 3^2 = 10$. ∴ $AB = \sqrt{10}$.

In general if A(a, 0) and B(0, b) are the given two points then the distance

AB = $\sqrt{OA^2 + OB^2} = \sqrt{a^2 + b^2}$.

By using this we can find the distance between a point on the x-axis and a point on the y-axis.

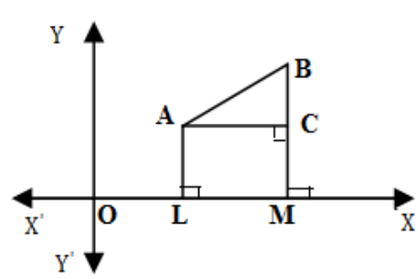


For example if C ≡ (0, 2) and D ≡ (– 4, 0) are the given two points then CD = $\sqrt{2^2 + 4^2} = \sqrt{20}$.

If E ≡ (–3, 0) and F ≡ (0, – 3) then EF = $\sqrt{3^2 + 3^2} = \sqrt{9 + 9}$ $= \sqrt{18} = 3\sqrt{2}$.

If G ≡ (0, – 4) and H ≡ (3, 0) then GH = $\sqrt{4^2 + 3^2} = \sqrt{16 + 9}$ $= \sqrt{25} = 5$.

d. By using the concepts explained above, we can find the distance between the points lying on the x-axis, the distance between the points lying on the y-axis or the distance between a point lying on the x-axis and a point lying on the y-axis. Now let us consider the most general case.



Let A(x_1, y_1) and B(x_2, y_2) be the given two points. Draw AL and BM perpendiculars to the x-axis. Draw AC ⊥ BM.

Here AC = LM = OM – OL = $x_2 - x_1$ and

BC = BM – CM = BM – AL = $y_2 - y_1$.

Now $AB^2 = AC^2 + BC^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

∴ AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Thus **the distance between the points A(x_1, y_1) and B(x_2, y_2) is** $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$**.**

This is called the distance formula.

Here while deriving the above formula we considered the points lying in the first quadrant. This is not necessary. We can apply the same formula to find the distance between any two points A(x_1, y_1) and B(x_2, y_2). (try to establish this)

Since $(a - b)^2$ is same as $(b - a)^2$, we can also say that AB = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

For example distance between the two points A(2, – 3) and B(– 5, 1) is $= \sqrt{\left(2 - (-5)\right)^2 + \left(-3 - 1\right)^2} = \sqrt{49 + 16} = \sqrt{65}$ units.

e. Consider the points A(x_1, y_1) and B(x_2, y_1) where the y-coordinates are the same. The distance between the points A(x_1, y_1) and B(x_2, y_1) is

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_1)^2} = \sqrt{(x_1 - x_2)^2}$

$= x_1 - x_2$ if $x_1 > x_2$ or $= x_2 - x_1$ if $x_2 > x_1$.

Thus if the y-coordinates are the same then the distance between the two points is the difference between the x-coordinates (note that the difference a positive quantity).

For example the distance between the points A(2, 9) and B(8, 9) is $= 8 - 2 = 6$.

f. Consider the points C(x_1, y_1) and D(x_1, y_2) where the x-coordinates are the same. The distance between the points C(x_1, y_1) and D(x_1, y_2) is

$= \sqrt{(x_1 - x_1)^2 + (y_1 - y_2)^2} = \sqrt{(y_1 - y_2)^2}$

$= y_1 - y_2$ if $y_1 > y_2$ or $= y_2 - y_1$ if $y_2 > y_1$.

Thus if the x-coordinates are the same then the distance between the two points is the difference between the y-coordinates.

For example the distance between the points C(3, 7) and D(3, – 4) is $= 7 - (-4) = 11$.

g. Let P(x, y) be a point in the Cartesian coordinate system. The distance of the point P from the origin O(0, 0),

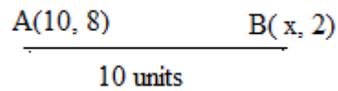
OP = $\sqrt{(x-0)^2+(y-0)^2} = \sqrt{x^2+y^2}$.

For example the distance of the point (– 3, 4) from the origin is $\sqrt{(-3)^2+4^2} = \sqrt{9+16} = 5$.

Worked examples

1. Find x if the distance between the points A(10, 8) and B(x, 2) is 10 units.

Solution: The given condition is AB = 10.



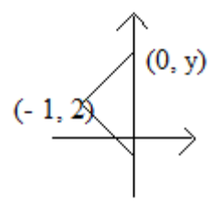
$\therefore$ $AB^2 = 100$. This implies

$(x-10)^2+(2-8)^2 = 100 \Rightarrow x^2+100-20x+36 = 100$

$\Rightarrow x^2-20x+36 = 0 \Rightarrow (x-18)(x-2) = 0 \Rightarrow x = 18$ or 2.

2. Find a point on the y-axis at a distance of 4 units from the point A(– 1, 2).

Solution: If a point lies on the y-axis then the x-coordinate of the point is zero. Let P(0, y) be the required point. Then PA = 4



$\Rightarrow \sqrt{(0-(-1))^2+(y-2)^2} = 4$

$\Rightarrow 1+(y-2)^2 = 16$

$\Rightarrow 1+y^2+4-4y = 16 \Rightarrow y^2-4y-11 = 0$

$\Rightarrow y = \frac{4 \pm \sqrt{16-4(-11)}}{2} = \frac{4 \pm \sqrt{60}}{2} = \frac{4 \pm 2\sqrt{15}}{2} = \frac{2(2 \pm 2\sqrt{15})}{2} = 2 \pm 2\sqrt{15}$.

Thus the required points are given by $(0,\ 2 \pm 2\sqrt{15})$.

3. Find 'a' if the point (a, 0) is equidistant from (2, 5) and (1, 3).

Solution: If P(a, 0) is equidistant from A(2, 5) and B(1, 3) then PA = PB. $\therefore \sqrt{(a-2)^2+(0-5)^2} = \sqrt{(a-1)^2+(0-3)^2}$

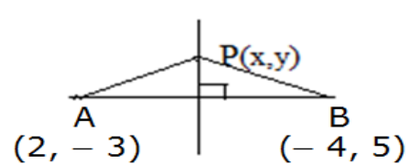
$\Rightarrow (a-2)^2+25 = (a-1)^2+9$

$\Rightarrow a^2+4-4a+25 = a^2+1-2a+9 \Rightarrow 2a-4a = 10-29$

$\Rightarrow -2a = -19 \Rightarrow 2a = 19 \Rightarrow a = \frac{19}{2}$.

4. If P(x, y) is a point on the perpendicular bisector of the line joining the points A(2, – 3) and B(– 4, 5) prove that 3x – 4y + 7 = 0.

Solution: If P(x, y) is a point on the perpendicular bisector of the line joining the two points A and B then PA = PB (why?).



Now PA = PB

$\Rightarrow \sqrt{(x-2)^2+(y-(-3))^2} = \sqrt{(x-(-4))^2+(y-5)^2}$.

By squaring we get $(x - 2)^2 + (y + 3)^2 = (x + 4)^2 + (y - 5)^2$

$\Rightarrow x^2 + 4 - 4x + y^2 + 9 + 6y = x^2 + 16 + 8x + y^2 + 25 - 10y$

$\Rightarrow -4x + 6y + 13 = 8x - 10y + 41$

$\Rightarrow -12x + 16y - 28 = 0.$

Dividing by -4 we get, $3x - 4y + 7 = 0.$

5. Verify whether the points A(2, – 4), B(4, – 2) and C(7, 1) are collinear or not.(note: If three points lie on a line then the points are called collinear points)

Solution: AB $= \sqrt{(2 - 4)^2 + (-4 - (-2))^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}.$

BC $= \sqrt{(4 - 7)^2 + (-2 - 1)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}.$

AC $= \sqrt{(2 - 7)^2 + (-4 - 1)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}.$

Now AB + BC $= 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$ = AC.

∴ The points are collinear.

6. Prove that the points A(– 2, 5), B(3, – 4) and C(7, 10) are the vertices of a right angled isosceles triangle. Also find its area.

Solution: $AB = \sqrt{(-2 - 3)^2 + (5 - (-4))^2} = \sqrt{25 + 81} = \sqrt{106}.$

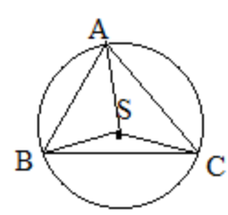
$BC = \sqrt{(3 - 7)^2 + (-4 - 10)^2} = \sqrt{16 + 196} = \sqrt{212}.$

$AC = \sqrt{(-2 - 7)^2 + (5 - 10)^2} = \sqrt{81 + 25} = \sqrt{106}.$

Here $AB^2 + AC^2 = 106 + 106 = 212 = BC^2.$

∴ the triangle ABC is right angled. Also AB = AC. ∴ the triangle ABC is isosceles. Here the angle A is 90°. Thus the area of the triangle ABC is $= \frac{1}{2}.AB.AC = \frac{1}{2}.\sqrt{106}.\sqrt{106} = \frac{106}{2} = 53$ sq. units.

7. Find the circumcenter of the triangle formed by the points A(– 2, 0), B(5, – 1) and C(2, 8). Find also the area of the circumcircle.



Solution: Let S(x, y) be the circumcenter of the triangle ABC. Then SA = SB = SC.

Now SA = SB

$\Rightarrow \sqrt{(x - (-2))^2 + (y - 0)^2} = \sqrt{(x - 5)^2 + (y - (-1))^2}.$

By squaring we get, $(x + 2)^2 + y^2 = (x - 5)^2 + (y + 1)^2$

$\Rightarrow x^2 + 4 + 4x + y^2 = x^2 + 25 - 10x + y^2 + 1 + 2y$

$\Rightarrow 4 + 4x = 26 - 10x + 2y$

$\Rightarrow 14x - 2y - 22 = 0.$

Dividing by 2, we get, $7x - y - 11 = 0$ … …(1)

Also SB = SC $\Rightarrow \sqrt{(x - 5)^2 + (y - (-1))^2} = \sqrt{(x - 2)^2 + (y - 8)^2}.$

By squaring we get, $(x - 5)^2 + (y + 1)^2 = (x - 2)^2 + (y - 8)^2$

$\Rightarrow x^2 + 25 - 10x + y^2 + 1 + 2y = x^2 + 4 - 4x + y^2 + 64 - 16y$

$\Rightarrow 26 - 10x + 2y = 68 - 4x - 16y$

$\Rightarrow -6x + 18y - 42 = 0.$

Dividing by -6, we get, $x - 3y + 7 = 0$ ... ...(2)

Solving (1) and (2), we get, x = 2 and y = 3.

Thus the circumcenter is S(2, 3).

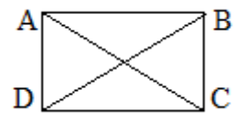
The radius of the circumcircle is = SA

$= \sqrt{(2-(-2))^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5.$

The area of the circumcircle is = $\pi.5^2 = 25\pi.$

8. Show that the points A(2, – 2), B(8, 4), C(5, 7) and D(– 1, 1) are the vertices of a rectangle. Also find the area of the rectangle.

Solution: A quadrilateral ABCD is a rectangle if the opposite sides are equal and the diagonals are equal.



AB = $\sqrt{(2-8)^2 + (-2-4)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}.$

BC = $\sqrt{(8-5)^2 + (4-7)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$

CD = $\sqrt{(5-(-1))^2 + (7-1)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}.$

DA = $\sqrt{(-1-2)^2 + (1-(-2))^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$

Here AB = CD and BC = DA.

∴ opposite sides of the quadrilateral are equal. ∴ ABCD is a parallelogram.

Also AC = $\sqrt{(2-5)^2 + (-2-7)^2} = \sqrt{9+81} = \sqrt{90}.$

BD = $\sqrt{(8-(-1))^2 + (4-1)^2} = \sqrt{81+9} = \sqrt{90}.$

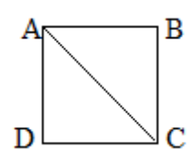
Thus the diagonals of the parallelogram ABCD are equal.

∴ ABCD is a rectangle.

The area of the rectangle is = AB.BC = $6\sqrt{2}.3\sqrt{2} = 36$ sq units.

9. If A(3, 4) and C(2, – 3) are the two opposite vertices of a square find the length of its sides.

Solution: Here AC is a diagonal of the square. If x is the length of the side of the square then by the Pythagorus theorem $x^2 + x^2 = AC^2$. ∴ $2x^2 = AC^2$.



Now AC = $\sqrt{(3-2)^2 + (4-(-3))^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}.$

∴ $2x^2 = AC^2 = 50.$ ∴ $x^2 = 25.$ ∴ $x = 5.$

Exercise 7.2

1. Write the distance between the points

a. (3, – 2) and (– 4, 1) ; **b.** (2, 3) and (– 2, – 3)

2. Write the distance between the points

a. (0, – 2) and (0, 1) ; **b.** (2, 0) and (– 2, 0) **;**

c. (5, – 3) and (5, 1) ; **d.** (7, 8) and (– 2, 8)

3. Write the distance between the points

a. (5, 0) and (0, – 9) ; **b.** (– 2, 0) and (0, 7)

4. Write the distance of the following points from the origin

a. (5, 0) **b.** (0, – 9) **c.** (3, – 4) **d.** (– 2, 6)

e. (2cos x, – 2sin x) **f.** (a, – 2a), $a > 0$

5. Fill up the blanks

a. If P(5, 5) and Q(– 7, – 7) then OP : OQ is = ...

b. The points lying at a distance of 4 units from the point (3, – 4) satisfies the condition ... ...

Answers

1.a. $\sqrt{58}$, $\sqrt{52}$. 2.a. 3, b. 4, c. 4, d. 9.

3.a. $\sqrt{106}$, $\sqrt{53}$. 4.a. 5, b. 9, c. 5, d. $\sqrt{40}$, e. 2, f. $a\sqrt{5}$.

5.a. 5 : 7, b. $x^2 + y^2 - 6x + 8y + 9 = 0$.