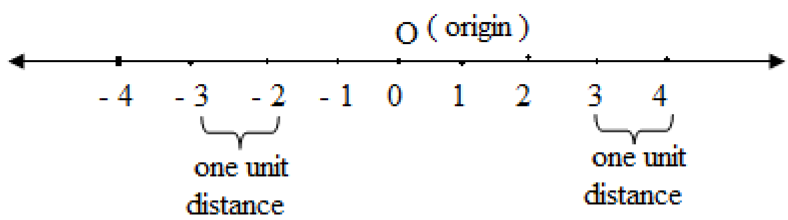
7. ANALYTICAL GEOMETRY

7.1 Cartesian coordinate system:

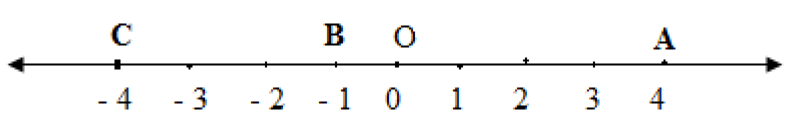
a. Number line

We have studied the **number line** in our lower classes. On the number line, distances from a fixed point (called the **origin** or the zero point) are marked in equal units. We mark the positive units on the right hand side of the origin (origin is usually denoted by O) and the negative units on the left hand side of the origin. On the right hand side of the origin, the consecutive markings from the origin are indicated by 1, 2, 3,... and -1, -2, -3,... on the left hand side. Here the distance between the two consecutive markings is taken as **one unit measure**.



Each unit can be divided into parts to represent the rational and irrational numbers.

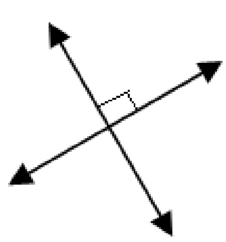
On the number line the position of the number 0 is at the origin. If one unit distance represents the number 1 then 2 units distance represents the number 2, 3 units distance represents the number 3 and so on.

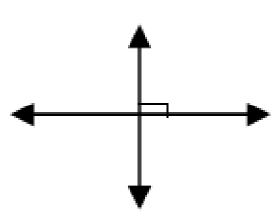


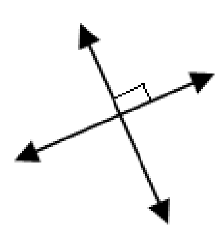
Observe the adjacent figure. Here A, B and C are three points on the number line as indicated. With reference to the point O, the mark A indicates the positive 4 units, the mark B indicates the negative 1 unit and the mark C indicates the negative 4 units. Also the width AB (distance between A and B) indicates 5 units, the width BC indicates 3 units and the width AC indicates 8 units.

b. Co-ordinate axes

Rene Descartes {(1596 - 1650), is a famous mathematician, who gave a new method of combining algebra and geometry} invented the method of placing 2 such number lines perpendicular to each other on a plane.



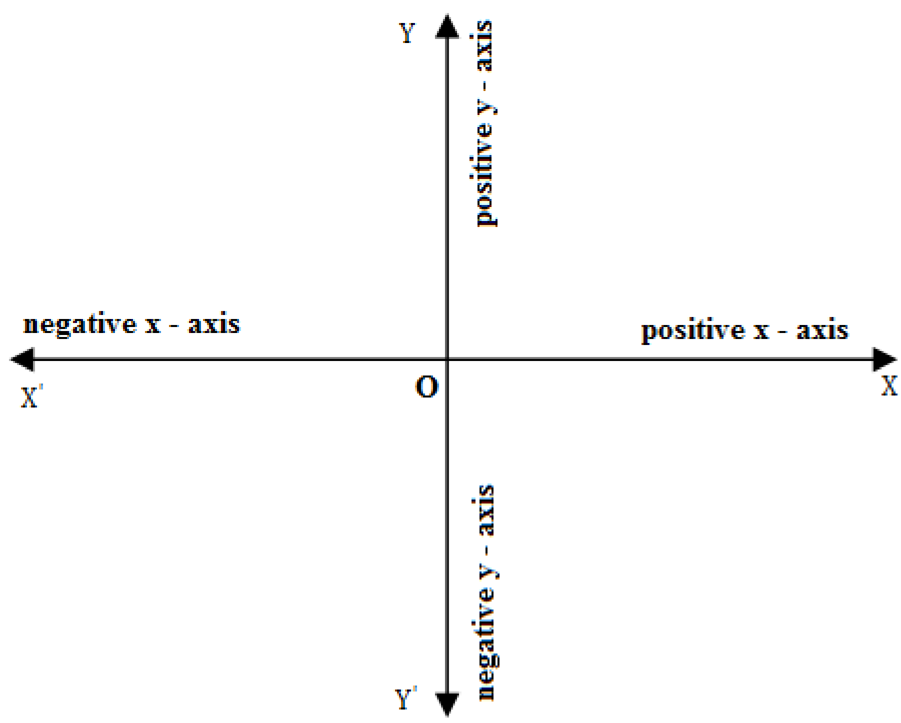




He gave the method of representing a point on the plane referring it to these 2 lines. The perpendicular lines may be in any direction such as in figure shown above. These two perpendicular lines are called the **reference axes**.

For the sake of simplicity we take one line along the horizontal direction and the other perpendicular to the horizontal. The horizontal and the vertical lines are combined so that the origin of the horizontal coincides with the origin of the vertical.

The horizontal line is denoted by XX' or $\mathbf{XOX'}$ and is called the horizontal axis or the **x-axis** and the vertical line is denoted by YY' or $\mathbf{YOY'}$ and is called the vertical axis or the **y-axis**.

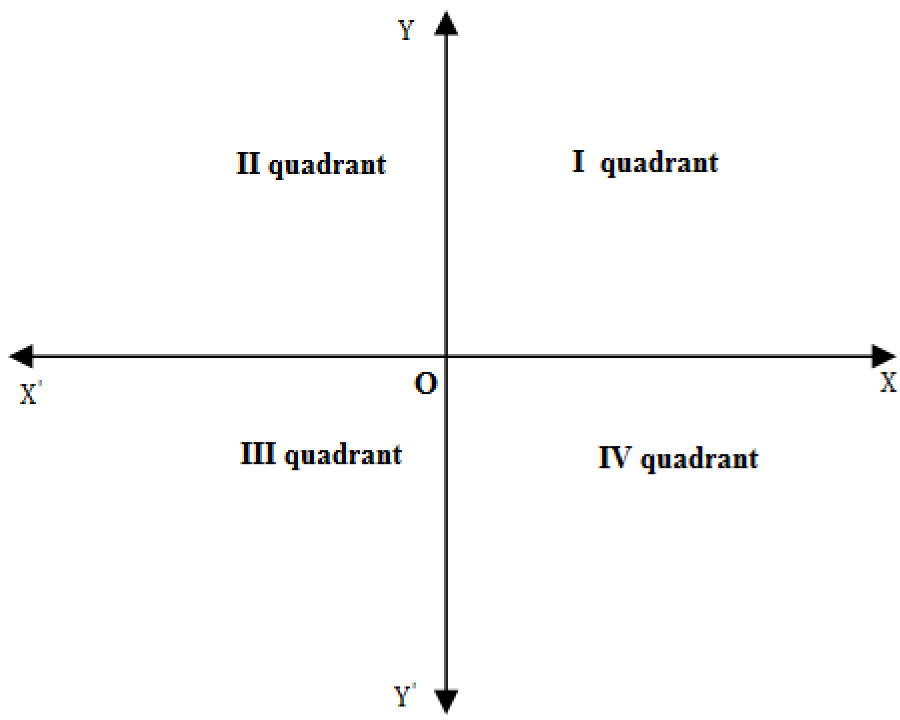


The positive units are marked on the direction OX (right side of O of the horizontal axis) and OY (above O of the vertical axis). OX is called the **positive direction** of the x-axis and OY is called the positive direction of the y-axis. The negative units are marked on the direction OX' (left side of O of the horizontal axis) and OY' (below O of the vertical axis). OX' is called the **negative direction** of the x-axis and OY' is called the negative direction of the y- axis.

c. Quadrants

The two mutually perpendicular axes divide the plane into 4 parts. Each part is called a quadrant (one fourth of the plane). The four quadrants are numbered I, II, III and IV in the anti-clockwise direction starting from top right quadrant.

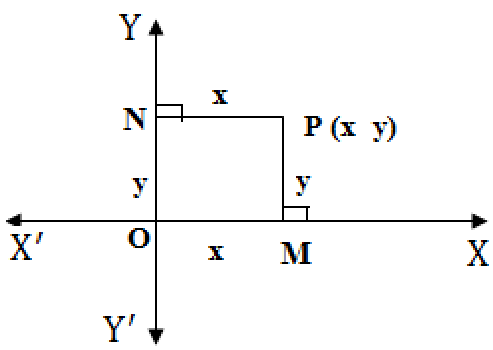
I quadrant is the region bounded by the positive x-axis and the positive y-axis. II quadrant is the region bounded by the negative x-axis and the positive y-axis. III quadrant is the region bounded by the negative x-axis and the negative y-axis. IV quadrant is the region bounded by the positive x-axis and the negative y-axis.

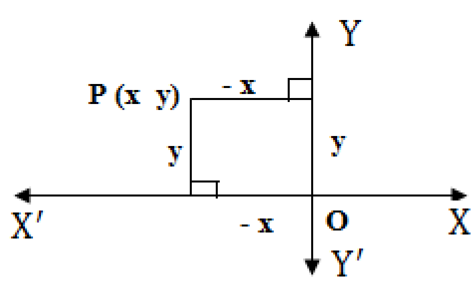


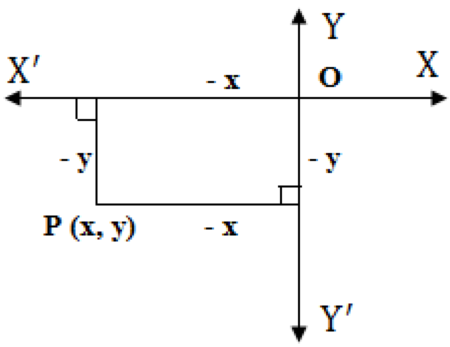
d. Co-ordinates of a point

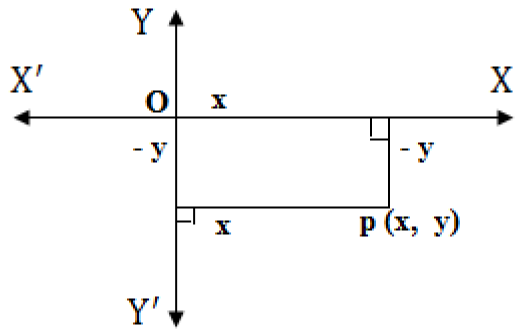
Let P be a point on the plane determined by the two perpendicular axes XOX' and YOY'. Draw PM perpendicular to the x-axis and PN perpendicular to y-axis. Here PM is the distance (also called the directed distance) of the point P from the x-axis and PN is the distance (also called the directed distance) of the point P from the y-axis. Here M is the foot of the perpendicular drawn from P to the x-axis and N is the foot of the perpendicular drawn from P to the y-axis. Here OMPN is a rectangle.

Let PN = x and PM = y. Then we write P ≡ (x, y). Here **x** is called the **x - coordinate of the point P** or the **abscissa** of the point and **y** is called the **y-coordinate** of the point or the **ordinate** of the point. The numbers x and y are called the **rectangular (Cartesian) coordinates** of the point P.









We have the following.

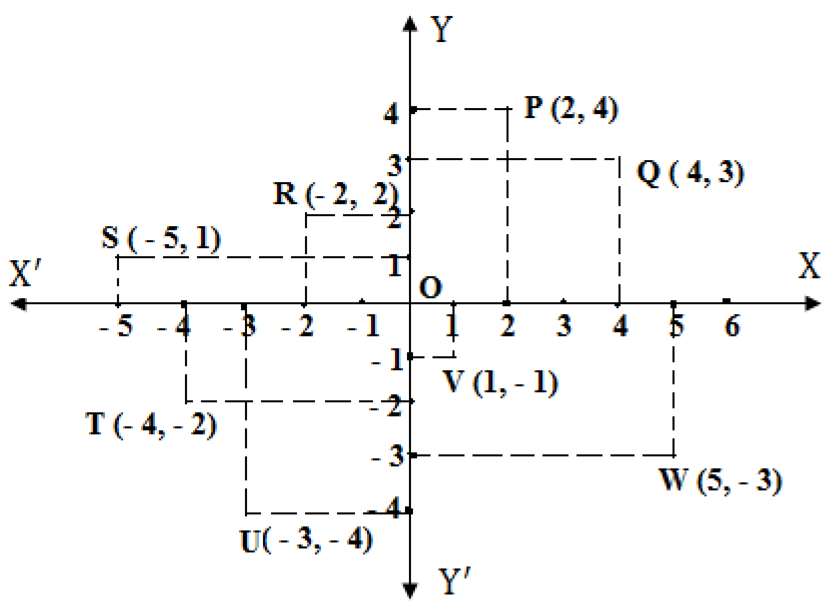
i) If $x > 0$ **and** $y > 0$ then the point P(x, y) lies in the **I quadrant** and conversely.

ii) If $x < 0$ **and** $y > 0$ then the point P(x, y) lies in the **II quadrant** and conversely.

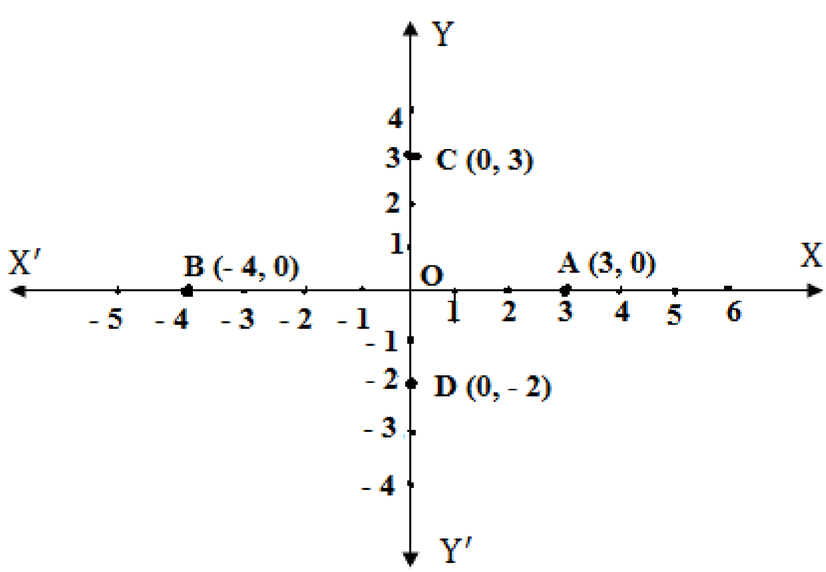
iii) If $x < 0$ **and** $y < 0$ then the point P(x, y) lies in the **III quadrant** and conversely.

iv) If $x > 0$ **and** $y < 0$ then the point P(x, y) lies in the **IV quadrant** and conversely.

Observe the following figure. Here P and Q are points in the first quadrant; R and S are points in the second quadrant; T and U are points in the third quadrant and V and W are points in the fourth quadrant.



e. Points on the x-axis and the y-axis



i) Let A be a point lying on the positive x-axis. Here distance of the point from the x-axis is zero. Therefore y-coordinate of the point A is zero. In general y-coordinate of any point lying on the x-axis is zero. Let OA = x_1.Then x-coordinate of the point A is x_1. Thus A ≡ $(x_1, 0)$.

For example if A is a point on the positive x-axis and if OA = 3 units then A ≡ (3, 0).

ii) Let B be a point lying on the negative x-axis. Let OB = x_2. Then x-coordinate of the point B is $-x_2$. Thus B ≡ $(-x_2, 0)$.

For example if B is a point on the negative x-axis and if OB = 4 units then B ≡ (– 4, 0).

iii) Let C be a point lying on the positive y-axis. Here distance of the point from the y-axis is zero. Therefore x-coordinate of the point C is zero. In general x-coordinate of any point lying on the y-axis is zero. Let OC = y_1.Then y-coordinate of the point C is y_1. Thus C ≡ $(0, y_1)$.

For example if C is a point on the positive y-axis and if OC = 3 units then C ≡ (0, 3).

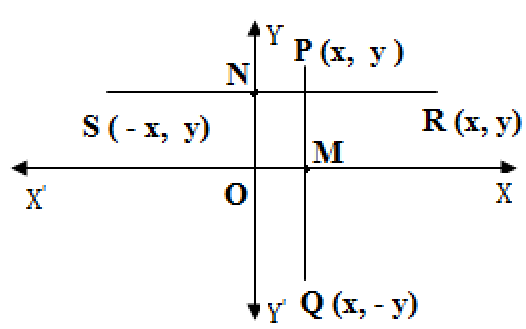
iv) Let D be a point lying on the negative y-axis. Let OD = y_2. Then y-coordinate of the point D is $-y_2$. Thus D ≡ $(0, -y_2)$.

For example if D is a point on the negative y-axis and if OD = 2 units then D ≡ (0, – 2).

v) Since origin is a point on both the x-axis and the y-axis, the origin O ≡ (0, 0).

f. Image of a point about the axes

i) Let P(x, y) be a point in the Cartesian co-ordinate system. Draw PM ⊥ to the x-axis and produce to the point Q such that PM = MQ. Then Q is the image or reflection of the point P about the x-axis.



If P is a point in the first quadrant then its image Q, about the x-axis is a point in the fourth quadrant. If P is a point in the second quadrant then its image Q, about the x-axis is a point in the third quadrant.

Thus y-coordinate of Q is negative of the y-coordinate of P. Here M is the midpoint of PQ. Since PQ is perpendicular to the x-axis, the x-coordinates of P and Q are equal (because OM gives the x-coordinate of P and Q).

Therefore if P ≡ (x, y) then Q ≡ (x, – y).Thus **image of (x, y) about the x-axis is (x, – y)**. The image of a point about the x-axis is obtained by replacing y by – y.

For example, the image of P(2, 3) about the x-axis is Q(2, – 3) and the image of A(– 3, 5) about the x-axis is B(– 3, – 5).

ii) Let R(x, y) be a point in the Cartesian coordinate system. Draw RN ⊥ to the y-axis and produce to the point S such that RN = NS. Then S is the image (reflection) of R about the y-axis.

If R is a point in the first quadrant then its image S, about the y-axis is a point in the second quadrant. If R is a point in the fourth quadrant then its image S, about the y-axis is a point in the third quadrant.

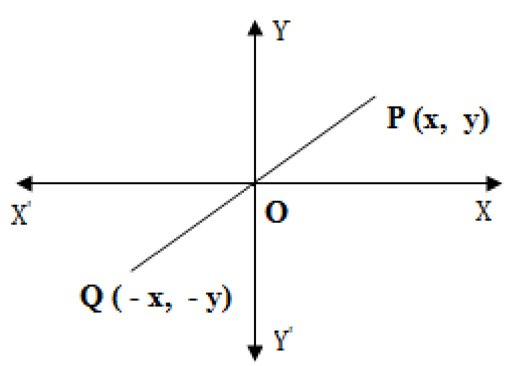
Thus x-coordinate of S is negative of the x-coordinate of R. Here N is the midpoint of RS. Since RS is perpendicular to the y-axis the y-coordinates of R and S are equal (because ON gives the y-coordinate of R and S).

Therefore if R ≡ (x, y) then S ≡ (– x, y). Thus **image of (x, y) about the y-axis is (– x, y)**. The image of a point about the y-axis is obtained by replacing x by – x.

For example, the image of R(2, 3) about the y-axis is S(– 2, 3) and the image of C(3, – 5) about the y-axis is D(– 3, – 5).

g. Image of a point about the origin

Let P(x, y) be the given point. Join P to the origin and produce to the point Q such that OP = OQ. Then Q is called the image of the point P about the origin.



The image of P(x, y) about the origin is Q(– x, – y).

The image of a point about the origin is obtained by changing the signs of the x- and y- coordinates.

For example, image of the point A(2, – 3) about the origin is B(– 2, 3) and image of C(– 3, 4) about the origin is D(3, – 4).

Exercise - 7.1

1. State the quadrant in which the following points lie

a. (2, 5) **b.** (– 3, 8) **c.** (– 5, – 1) **d.** (4, –2)

2. State the axes in which the following points lie

a. (2, 0) **b.** (– 3, 0) **c.** (0, – 1) **d.** (0, 2)

3. Write the distance of the following points from the x-axis and y-axis

a. (3, 5) **b.** (– 1, 8) **c.** (– 4, – 1) **d.** (6, –2)

4. Mark the following points and name the triangle formed;

a. (2, 0), (– 2, 0) and (0, 2)

b. (2, 3), (4, 3) and (2, 5)

5. Mark the following points and name the quadrilateral formed;

(– 4, 1), (2, 1), (4, – 2) and (– 2, – 2)

6. Fill up the blanks

a. If the abscissa of the point P(x, y) is three times the ordinate of the point then y : x =.....

b. If the abscissa of the point P(x, y) is square of its ordinate then x = ...

c. The point (3, 2) moves a distance of 5 units in the negative direction of the x-axis. From this new position it turns to the direction of the negative y-axis and moves a distance of 7 units. The coordinates of the point in the new position is .....

d. A point lies on the x-axis at a distance of 5 units from the point (– 2, 0). The position of the point is ..... or .....

7. Fill up the blanks

a. The image of (3, – 2) about the x-axis and y-axis respectively are ..... and .....

b. The image of (x, y) about the x-axis and y-axis respectively are (3, – 2) and (– 3, 2). Then x =.... and y =....

8. Write the distance of the point (2, – 1) from its reflection

i. about the x-axis and **ii.** about the y-axis.

Answers

1. a. I; b. II; c. III; d. IV.

2. a. positive x-axis; b. negative x-axis; c. negative y-axis; d. positive y-axis.

3. a. 5, 3; b. 8, 1; c. 1, 4; d. 2, 6.

4. a. isosceles triangle; b. right angled triangle.

5. Parallelogram.
6. a. 1 : 3; b. y^2; c. $(-2, -5)$; d. (3, 0) or $(-7$, 0).
7. a. (3, 2), (− 3, 2); b. (3, 2).
8. 2, 4.