b) $\log_{10}72$ [Hint : $\log_{10}72 = \log_{10}(2^3 x 3^2)$
$= 3 \log_{10} 2 + 2 \log_{10} 3$]

c) $\log_{10}\left(\frac{27}{16}\right)$

d) $\log_{10}(10.8)$ [Hint : $\log_{10}(10.8) = \log_{10}(108/10)$

$= \log_{10}(2^2 x 3^3) - \log_{10}10$

$= 2\log_{10}2 + 3\log_{10}3 - 1$

Answers : 1) a) 3 b) -2 c) 8/5 d) 6 e) 100

2) a) $10^3 = 1000$ b) $4^4 = 256$ c) $7^3 = 343$ d) $4^{1/2} = 2$

3) a) $\log_2 32 = 5$ b) $\log_6 216 = 3$ c) $\log_{10} 10000 = 4$ d) $\log_{10} 0.001 = -3$

f) $\log_{1000} 10 = ½$ g) $\log_a y = x$

4) a) 2.4080 b) 1.8572 c) 0.2273 d) 1.0333

3.91 Surds

you have already studied irrational numbers and Surds in the earlier clases. So this is just a refresher course, as the operations of the surds and rationalisation are required in the higher classes.

Consider the following irrational numbers :-

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt[3]{4}, 2\sqrt{7}$

All these number have rational number under the root sign. So they are the irrational roots of a rational number .they are called surds.

Surd : - An irrational root of a rational number is called a Surd.

General form of a Surd : $\sqrt[n]{a}$ is called a surd of order n, where a is a +ve rational number and n is a +ve integer greater than 1

Example : - 1) $\sqrt{2}, \sqrt{3}, 2\sqrt{5}, 3\sqrt{7}$, are Surds of order 2

2) $\sqrt[3]{2}$, $4\sqrt[3]{3}$, $\sqrt[3]{5}$ $2\sqrt[3]{6}$, are Surds of order 3

Note : - 1) $\sqrt{4}$, $\sqrt[3]{8}$, $\sqrt[2]{\frac{25}{9}}$, are not surds as they are not irrational numbers

2) In a Surd the radicand should always be a rational number.
So $\sqrt{2+\sqrt{3}}$ and $\sqrt{1+\sqrt{5}}$ are not Surds, as they are roots of an irrational numbers.

3) All surds are irrational numbers, where as all irrationals are not Surds.
Exp : $\sqrt{2-\sqrt{3}}$ is irrational but not a surd.

Reduction of a surd : Writing a surd in its simplest form is called the reduction of the surd.

Exp : 1) $\sqrt{8} = \sqrt{4x2} = \sqrt{4}$ x $\sqrt{2} = 2\sqrt{2}$

2) $\sqrt{48} = \sqrt{16x3} = \sqrt{16}x\sqrt{3} = 4\sqrt{3}$

Pure surds : The surds having 1 as rational Coefficient, are called Pure surds.

Exp : $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{5}$, $\sqrt[4]{7}$, $\sqrt[5]{6}$ are Pure Surds.

Mixed Surd :- The Surds having rational Coefficient other then 1 are called Mixed Surds.

Exp : $2\sqrt[3]{5}$, $4\sqrt[2]{3}$, $-5\sqrt{6}$, $3\sqrt[4]{5}$, are mixed surds.

Like [similar] Surd :- Surds in their simplest formhaving same order and radicand are called Like surds.

Exp : $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $-5\sqrt{2}$, $\frac{3}{5}\sqrt{2}$, are Similar surds.

Unlike (dissimilar) Surds :- Surds which are not similar are called unlike surds.

Exp : 1) $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$
2) $\sqrt{3}$, $\sqrt[3]{5}$, $\sqrt[4]{5}$ are Unlike Surds.

Exp : Express the following in pure surd form :-
$2\sqrt{3} = \sqrt{4}\ \ \sqrt{3} = \sqrt{12}$

$\sqrt[3]{5} = \sqrt[3]{8} \text{ x } \sqrt[3]{5} = \sqrt[3]{8x5} = \sqrt[3]{40}$

2) Express the following in mixed surd form

$\sqrt[3]{54} = \sqrt[3]{27x2} = \sqrt[3]{27} \text{ x } \sqrt[3]{2} = 3\sqrt[3]{2}$

3.92 <u>Operations</u> of <u>Surds</u>

Addition and Subtraction : Similar Surds can be added or Subtracted. But dissimilar surds can be added or Subtracted , where as it can not be simplified-

Exp : 1) $2\sqrt{3} + 5\sqrt{3} - 3\sqrt{3} = 7\sqrt{3} - 3\sqrt{3} = 4\sqrt{3}$

2) $5\sqrt{a} - 3\sqrt{a} + 6\sqrt{a} = 2\sqrt{a} + 6\sqrt{a} = 8\sqrt{a}$

Multiplication of Surds :-

1) Surds of the same order can be Multiplied as $\sqrt[n]{a} \text{ x } \sqrt[n]{b} = \sqrt[n]{ab}$
2) Surds of different order can be multiplied by reducing them to the same order.

<u>Method to convert the surds of different order to same order</u> :-

1) First take LCM of the orders of the given surds
2) Convert each surd into index form and express then as surds with LCM as the order.

<u>Division of surds</u> :- Apply same procedure as in the case of multiplication of Surds.

Example : 1) $\sqrt{6} \text{ x } \sqrt{5} = \sqrt{6x5} = \sqrt{30}$

2) $5\sqrt[3]{2} \text{ x } 3\sqrt[3]{6} = 15\sqrt[3]{2}\sqrt[3]{6} = 15\sqrt[3]{12}$

3) $\frac{2\sqrt{3}}{5\sqrt{5}} = \frac{2}{5}\sqrt{\frac{3}{5}}$

4) Find $\sqrt[2]{3} \text{ x } \sqrt[3]{4}$

$\sqrt[2]{3} \text{ x } \sqrt[3]{4} = 3^{½} \text{ x } 4^{1/3}$

L.C.M of 2 and 3 =6

$\sqrt[2]{3} \text{ x } \sqrt[3]{4} = 3^{3/6} \text{ x } 4^{2/6} = \sqrt[6]{3^3} \text{ x } \sqrt[6]{4^2} = \sqrt[6]{27x16} = \sqrt[6]{432}$

5) $\frac{\sqrt[2]{3}}{\sqrt[3]{4}} = \frac{3^{\frac{3}{6}}}{4^{\frac{2}{6}}} = \frac{\sqrt[6]{27}}{\sqrt[6]{16}} = \sqrt[6]{\frac{27}{16}}$

3.93 Rationalisation :-

Consider the following example :

$\sqrt{3} x \sqrt{3} = \sqrt{3x3} = 3$ (a rational number)

So when a surd $\sqrt{3}$ multiplied by another surd $\sqrt{3}$, then the product is a rational number. Thus $\sqrt{3}$ is called the rationalising factor of $\sqrt{3}$ and this process is called rationalisation.

Definition of rationalising factor and rationalisation:-

if the product of two surds is a rational number, then one surd is called as the rationalising factor of another and this process is called rationalisation.

Example :

S.N	Surd	Rationalising Factor
1	$\sqrt{5}$	$\sqrt{5}$
2	$3\sqrt{2}$	$\sqrt{2}$
3	$\sqrt{a}$	$\sqrt{a}$
4	$\sqrt{x+y}$	$\sqrt{x+y}$
5	$\sqrt[3]{a}$	$a^{2/3}$
6	$\sqrt[3]{5}$	$5^{2/3}$

Binomial Surd :- The sum of two surds or the sum of a surd and rational number is called a Binomial Surd.

example :- $2+\sqrt{3}$, $5-\sqrt{2}$, $\frac{1}{2}+\sqrt{5}$, $\sqrt{3}+\sqrt{2}$, $2\sqrt{3}-5\sqrt{2}$, $2\sqrt{x}+3\sqrt{y}$, $a\sqrt{x}+b\sqrt{y}$ are Binomial Surds

Conjugate surds :- $a+\sqrt{b}$, and $a-\sqrt{b}$ are called conjugate surds.

Note : - 1) For surd $a+\sqrt{b}$ its Conjugate is $a-\sqrt{b}$ and viceversa.
2) For surd $\sqrt{a}+\sqrt{b}$ its Conjugate is $\sqrt{a}-\sqrt{b}$ and viceversa.
3) the product of the surd and its conjugate is a rational number.
4) Conjugate of $a+\sqrt{b}$ is $a-\sqrt{b}$, but the rationalising factor of $a+\sqrt{b}$ is $a-\sqrt{b}$ - $a+\sqrt{b}$.

Example : -

1)

S.N	Surd	Rationalising Factor	Conjugate Surd
1	$a+\sqrt{b}$	$a-\sqrt{b}$ or $-a+\sqrt{b}$	$a-\sqrt{b}$
2	$a-\sqrt{b}$	$a+\sqrt{b}$ or $-a+\sqrt{b}$	$a+\sqrt{b}$
3	$\sqrt{a}+\sqrt{b}$	$\sqrt{a}-\sqrt{b}$ or $-\sqrt{a}+\sqrt{b}$	$\sqrt{a}-\sqrt{b}$
4	$\sqrt[3]{a}+\sqrt[3]{b}$	$a^{2/3}-a^{1/3}b^{1/3}+b^{2/3}$	-
5	$\sqrt[3]{a}-\sqrt[3]{b}$	$a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}$	-

2) Rationalise the denominator :

$$\frac{1}{\sqrt{2}+\sqrt{3}} = \frac{1}{\sqrt{2}+\sqrt{3}} \; \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$ by multiplying both Nr and Dr by the rationalising factor $\sqrt{2}-\sqrt{3}$

$$= \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2} = \frac{\sqrt{2}-\sqrt{3}}{2-3} = \frac{\sqrt{2}-\sqrt{3}}{-1} = \sqrt{3}-\sqrt{2}$$

3) Simplify :- $\frac{1}{2+\sqrt{3}} + \frac{1}{2-\sqrt{3}}$

$$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} + \frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{2-\sqrt{3}}{4-3} + \frac{2+\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} + \frac{2+\sqrt{3}}{1}$$

$$= 4$$

4) Rationalise the denominator :-

$$\frac{x}{1-\sqrt{1-x}} = \frac{x}{1-\sqrt{1-x}} \cdot \frac{1+\sqrt{1-x}}{1+\sqrt{1+x}} = \frac{x(1+\sqrt{1-x})}{1-(1-x)}$$

$$= \frac{x(1+\sqrt{1-x})}{x}$$

$$= 1+\sqrt{1-x}$$

Exescise -(3.8)

1) Test whether the following are similar or not

(a) $2\sqrt{3}, \sqrt{27}, \sqrt{48}, -5\sqrt{3}$ (b) $3\sqrt{2}, 4\sqrt{2}, \sqrt{8}, \sqrt{12}$,

2) Add the following surds:-

a) $\sqrt{75}$, $\sqrt{48}$, $7\sqrt{3}, -8\sqrt{3}$,

b) $3a\sqrt{x}, -4a\sqrt{x}, \sqrt{25a^2x}$

3) Simiplify the following :-

a) $\sqrt{2}x\sqrt{7}$, b) $\sqrt{3}x\sqrt[3]{2}$, c) $\sqrt{5}x\sqrt[4]{2}$, d) $\frac{\sqrt{3}}{\sqrt[3]{2}}$

4) Simplify by rationalising the denominator :-

a) $\frac{5}{\sqrt{3}}$ b) $\sqrt{\frac{3a}{5}}$ c) $\frac{2}{\sqrt{a}-\sqrt{b}}$ d) $\frac{\sqrt{m}}{\sqrt{m}+\sqrt{n}}$

Answers :- 1) a) Similar b) Not 2) a) $8\sqrt{3}$ b) $4a\sqrt{x}$

3) a) $\sqrt{14}$ b) $\sqrt[6]{108}$ c) $\sqrt[4]{50}$ d) $\sqrt[6]{\frac{27}{4}}$

4) a) $\frac{5\sqrt{3}}{3}$ b) $\frac{\sqrt{15a}}{5}$ c) $\frac{2(\sqrt{a}+\sqrt{b})}{a-b}$ d) $\frac{\sqrt{m}(\sqrt{m}-\sqrt{n})}{m-n}$

CHAPTER -5

5.1) SIMPLE INTEREST

When we deposit some money in a bank for a certain time, the bank will return some extra money apart from money invested.Similarly when we take a loan from a money lender or bank, we have to pay some extra money apart from the money taken. This extra money is called INTEREST.

Simple Interest :-

The interest is called simple interest ,when the interest is paid as it due. Here the interest will not earn interest.Simple interest will be calculated on the initial principal only.

Formula for Simple Interest :-the Simple interest (I) can be calculated by a simple formula

$$I = \frac{PTR}{100},$$

Where P is the principal amount

T is the period(in years)

R is the rate of interest per 100

Amount is the sum of the principal and interest and it is denoted by **A**

5.2) COMPOUND INTEREST

The interest is called compound,if the interest earned for a stipulated period is added to the principal and the total amount becomes the principal for the next period.

NOTE : 1) Compounding will be done yearly or half yearly or quarterly.

2) In case of compound interest , as the interest will earn interest , naturally the compound interest will be more than that of simple interest for the same period and same rate of interest .

3) In bank, if the interest on loan is not repaid in time , then that interest will be added to the principal and thus making it compound .

Example:- Aperson invests Rs 5000 in a bank for a period of two years. If the rate of interest is 10%, find the total amont he gets at the end of two years I) if the interest compounded yearly II) if the interest is not compounded III) conclude which is beneficial?

Solution: I) here the initial principal =Rs 5000

The interest at the end of I year = $\frac{P \times 1 \times R}{100} = \frac{5000 \times 1 \times 10}{100}$ = Rs 500

Amount = 5000+500 = Rs 5500

principal for the second year = Rs 5500

Interest at the end of II year = $\frac{5500 \times 1 \times 10}{100}$ = Rs 550

Total amount he gets at the end of two years = Rs5500+550= Rs 6050

II) Simple interest = $\frac{5000 \times 2 \times 10}{100}$ = Rs 1000

Total amount he gets at end of two years = Rs 5000 +Rs1000 = Rs 6000

III) Compound interest is more beneficial as he gets Rs 50 more

Note : I) In banks the compound interest will be calculated by using compound interest tables.

II) The amount for the principal P at the end of T years at the rate of interest R%, when the interest is compounded yearly is given by the formula

$$A = P\left(1 + \frac{R}{100}\right)^T$$

III) Compound interest = A – P

Where A is final amount

P is initial principal