Episode No.11
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Topic : VECTOR ALGEBRA

***If $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors such that $\vec{a}.\vec{b} = 0$ and $\vec{a} + \vec{b} = \vec{c}$**

(1) $|\vec{a}|^2 + |\vec{b}|^2 = |\vec{c}|^2$(2) $|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2$

(3) $|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2$(4) None of these

Correct Option: (1)

***If $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$ then**

(1) $|\vec{a}| = 1, |\vec{b}| = |\vec{c}|$ (2) $|\vec{c}| = 1, |\vec{a}| = 1$

(3) $|\vec{b}| = 2, |\vec{b}| = 2|\vec{a}|$ (4) $|\vec{b}| = 1, |\vec{c}| = |\vec{a}|$

Correct Option: (4)

***If $|\vec{a} + \vec{b}| > |\vec{a} - \vec{b}|$ then angle between $\vec{a}$ and $\vec{b}$ is**

(1) acute (2) obtuse (3) coplanar (4) none of these

Correct Option: (1)

***If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b} < 0$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, then angle between vectors $\vec{a}$ and $\vec{b}$ is**

(1) π (2) $\frac{7\pi}{4}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

Correct Option: (4)

***If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does not exceed**

(1) 4 (2) 9 (3) 8 (4) 6

Correct Option: (2)

***Let $\vec{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ if $\vec{b}$ is a vector such that $\vec{a}.\vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$ then $|\vec{b}| =$**

(1) 7(2) 14 (3) $\sqrt{7}$ (4) 21

Correct Option: (3)

***If $\vec{a}$ and $\overrightarrow{b}$ are two unit vectors inclined at an angle $\frac{\pi}{3}$, then the value of $|\vec{a} + \vec{b}|$ is**

(1) greater than 1 (2) less than 1 (3) equal to 1 (4) equal to 0

Correct Option: (1)

***If $\vec{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $|\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$ then the area of the triangle formed by these two vectors as two sides is**

(1) $\frac{15}{2}$ (2) 15 (3) $\frac{15}{4}$ (4) $\frac{15\sqrt{3}}{2}$

Correct Option: (3)

***If $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ each one of $\vec{a}, \vec{b}$ and $\vec{c}$ is perpendicular to the sum of the remaining then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to**

(1) $\frac{5}{\sqrt{2}}$ (2) $\frac{2}{\sqrt{5}}$ (3) $5\sqrt{2}$ (4) $\sqrt{5}$

Correct Option: (3)

***If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is**

(1) $\sqrt{18}$ (2) $\sqrt{72}$ (3) $\sqrt{33}$ (4) $\sqrt{45}$

Correct Option: (3)

***Let $\vec{a}, \vec{b}$ and $\vec{c}$ be four non-zero vectors such that $\vec{r} \cdot \vec{a} = 0, |\vec{r} \times \vec{b}| = |\vec{r}||\vec{b}|, |\vec{r} \times \vec{c}| = |\vec{r}||\vec{c}|$ then $[\vec{a}, \vec{b}, \vec{c}]$ is**

(1) -1 (2) 0 (3) 1 (4) 2

Correct Option: (2)

*** If $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{i} + 3\hat{k}$ and $\vec{a}$ is a unit vector then the maximum value of $[\vec{a}, \vec{b}, \vec{c}]$is**

(1)$\sqrt{59}$ (2) $\sqrt{69}$ (3) 3 (4) -1

Correct Option: (1)

*** If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors, then $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ if and only if**

(1) $\vec{b}$ and $\vec{c}$ are collinear (2) $\vec{a}$ and $\vec{c}$are collinear

(3) $\vec{a}$ and $\vec{b}$ are collinear (4) None of these

Correct Option: (2)

*** Which of the following is a true statement?**

(1) $(\vec{a} \times \vec{b}) \times \vec{c}$ is coplanar with $\vec{c}$ (2) $(\vec{a} \times \vec{b}) \times \vec{c}$ is perpendicular to $\vec{a}$

(3) $(\vec{a} \times \vec{b}) \times \vec{c}$ is perpendicular to $\vec{b}$ (4) $(\vec{a} \times \vec{b}) \times \vec{c}$ is perpendicular to $\vec{c}$

Correct Option: (4)

*** Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is**

(1) $\frac{2\sqrt{2}}{3}$ (2) $-\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{-2\sqrt{3}}{3}$

Correct Option: (1)

*** If $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$ then the angle between $\vec{a}$ and $\vec{b}$ is**

(1) $\frac{3\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

Correct Option: (4)

ADDITIONAL PROBLEMS:

* If $\lambda(3\hat{\imath} + 2\hat{\jmath} - 6\hat{k})$ is a unit Vector, then the value of λ are

(a) $\pm\frac{1}{7}$ (b) ± 7 (c) $\pm\sqrt{43}$ (d) $\pm\frac{1}{\sqrt{43}}$

Answer : a

* Three non-zero, non-collinear vector a, b & c are such that a + 3b is collinear with c, 3b + 2c is collinear with a, then a + 3b + 2c =

(a) 0 (b) 2a (c) 3b (d) 4c

Answer : a

* Area of the Rhombus is ..... where diagonals are

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ & $\vec{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$.

(a) $\sqrt{21.5}$ (b) $\sqrt{31.5}$ (c) $\sqrt{28.5}$ (d) $\sqrt{38.5}$

Answer : c

* Given p = $3\,\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$, a = $\hat{\imath} + \hat{\jmath}$, b = $\hat{\jmath} + \hat{k}$, c = $\hat{\imath} + \hat{k}$ and p=xa+yb+zc. Then x, y and z are respectively.

(a) $\frac{3}{2}, \frac{1}{2}, \frac{5}{2}$ (b) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ (c) $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}$ (d) $\frac{1}{2}, \frac{5}{2}, \frac{3}{2}$

Answer : b

* The three points whose position vectors are $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $3\,\hat{\imath} + 4\hat{\jmath} + 7\hat{k} 3\,\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$

(a) form an equilateral Δ^{le} (b) form a right angled Δ^{le}

(c) are collinear (d) form an isosceles Δ^{le}

Answer : c

* If $\vec{a}$ is a non – zero vector of Magnitude ‘a’ and ‘λ‘ a non - zero scalar, then $\lambda\vec{a}$ is a unit vector if

(a) λ = 1 (b) λ = -1 (c) a = |λ| (d) $a = \frac{1}{|\lambda|}$

Answer : d

* If $|\vec{a}|$ = 3, $|\vec{b}|$ = 4 and $|\vec{a} - \vec{b}|$ = 5, then $|\vec{a} + \vec{b}|$ =

(a) 6 (b) 5 (c) 4 (d) 3

Answer : b

* If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c}$ = 0

then $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} =$

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{-3}{2}$

Answer : d

* If i + j + $\hat{k}$, i – j, i + 2j + ak, are coplanar then a =

(a) $\frac{3}{2}$ (b) 3 (c) -3 (d) 0

Answer : a

* If $\vec{a}$ and $\vec{b}$ are unit vectors such that θ is the angle between them, $|\vec{a} - \vec{b}| =$

(a) $2\cos\theta$ (b) $2\sin\theta$ (c) $2\cos{}^{\theta}/_{2}$ (d) $2\sin{}^{\theta}/_{2}$

Answer : d

*$[\vec{a} + \vec{b}\vec{b} + \vec{c}\vec{c} + \vec{a}] =$

(a) $[\vec{a}\vec{b}\vec{c}]$ (b) $\sum(\vec{a}\,.\vec{b})\vec{c}$ (c) $2[\vec{a}\vec{b}\vec{c}]$ (d) $|\vec{a}||\vec{b}||\vec{c}|$

Answer : c

* The projection of $\vec{a} = 5\hat{\imath} - \hat{\jmath} + 3\hat{k}$ on $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ is

(a) 6 (b) $\sqrt{6}$ (c) $\sqrt{3}$ (d) None of these.

Answer : b

* The Area of the triangle whose vertices are

A = (1, -1, 2), B = (2, 1, -1) and C = (3, -1, 2) is

(a) $4\sqrt{3}$ Sq. units (b) $2\sqrt{3}$ Sq. units

(c) $\sqrt{13}$ Sq. units (d) $\sqrt{15}$ Sq. Units

Answer : c

* If $\vec{a}$ and $\vec{b}$ are unit vectors and $|\vec{a} \times \vec{b}| = 1$ Then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) π

Answer : b

*$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{c} = 4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ Then$|\vec{a} \times (\vec{b} \times \vec{c})|$ =

(a) $\sqrt{10}$ (b) 1 (c) 2 (d) $\sqrt{5}$

Answer : d

* If the vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, $-2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$, $\lambda\hat{\imath} - \hat{\jmath} + 2\hat{k}$ are linearly dependent, then the value of λ is

(a) 0 (b) 1 (c) 2 (d) 3

Answer : a

* The point collinear with (1, -2, -3) and (2, 0, 0) among the following

(a) (0, 4, 6) (b) (0, -4, -5) (c) (0, -4, -6) (d) (0, -4, 6)

Answer : c

* If the vectors $\hat{\imath} - 2x\hat{\jmath} - 3y\hat{k}$ and $i + 3xj + 2y\hat{k}$ are orthogonal to each other, then the locus of the point (x, y) is

(a) circle (b) an ellipse (c) parabola (d) straight line.

Answer : a

* The position vectors of the vertices of a triangle are 2i – j + k, i – 3j – 5k, and 3i – 4j – 4k then it is

(a) equilateral Δ^{le} (b) isosceles Δ^{le}

(c) right angled Δ^{le} (d) right angled isoscelesΔ^{le}

Answer : c

* The ratio in which the line segment joining the points (2, 4, 5) & (3, 5, -4) is divided by the point (0, 2, 23) is

(a) 3 : 2 (b) -2 : 3 (c) 1 : 2 (d) -1 : 2

Answer : b

* A set of direction cosines of the vector which is equally inclined to co–ordinate axes is

(a) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Answer : d

* If $\propto$, β, ϑ are the angles made by a vector with the coordinate axes, then $\cos 2 \propto + \cos 2\beta + \cos 2\vartheta =$

(a) - 1 (b) 1 (c) 2 (d) - 2

Answer : a

* If i, j are unit vectors and $i \times j = k$, then $(i + j) \times (j - i) =$

(a) k (b) 2k (c) - k (d) - 2k

Answer : b

*$i\,.(j \times k) + j\,.(k \times i) + k\,.(i \times j) =$

(a) 0 (b) 1 (c) 3 (d) None of these.

Answer :

* If $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ then $|2\vec{a} - \vec{b}|$

(a) 6 (b) $\sqrt{3}$ (c) $3\sqrt{2}$ (d) 18

Answer : c

* If $\vec{a} = (3i - \hat{\jmath} + 2\hat{k})$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, then $\vec{a} \times (\vec{a}\,.\vec{b})$ is

(a) 0 (b) $3\vec{a}$ (c) $3\sqrt{14}$ (d) None of these.

Answer : d

* If $\vec{a}.\ \vec{b}.\ \vec{c}$ are mutually $\perp^{r}$ unit vectors then $|\vec{a} + \vec{b} + \vec{c}| =$

(a) $\sqrt{3}$ (b) 3 (c) 1 (d) 0

Answer : a

* If $\vec{a}.\vec{b}.\vec{c}$ are the position vectors of the vertices of an equilateral Δ^{le} whose orthocentre is at origin, then

(a) $\vec{a} + \vec{b} + \vec{c} = \hat{0}$ (b) $\vec{a}^2 = \vec{b}^2 + \vec{c}^2$

(c) $\vec{a} + \vec{b} = \vec{c}$ (d) None of these.

Answer : a

* If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, then

(a) $\vec{a} \parallel \vec{b}$ (b) $\vec{a} \perp \vec{b}$

(c) either $\vec{a}$ or $\vec{b}$ is a null vector (d) None of these.

Answer : c

*Let $\vec{a} = j - \hat{k}$ and $\vec{c} = i - j - \hat{k}$ then, the vector $\vec{b}$ satisfying $\hat{a} \times \hat{b} + \hat{c} = 0$ and $\vec{a} . \vec{b} = 3$ is

(a) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$ (b) $2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

(c) $\hat{\imath} - \hat{\jmath} - 2\hat{k}$ (d) $\hat{\imath} + \hat{\jmath} - 2\hat{k}$

Answer : a

* If 2a + 3b + c = 0 then $a \times b + c \times b + c \times a =$

(a) $6(b \times c)$ (b) $3(b \times c)$ (c) $2(b \times c)$ (d) 0

Answer : b