### KARNATAKA EXAMINATION AUTHORITY

**SYLLABUS: MATHEMATICS(2021)** 

# COMPETITIVE EXAMINATION FOR THE POST OF ASSISTANT

#### PROFFESSOR IN GOVERNMENT FIRST GRADE COLLEGES

#### UNIT -1

### Analysis

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem.

Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Power Series, Fourier Series.

Metric spaces, compactness, connectedness, completeness.

Lebesgue measure on the real line, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness.

# Linear Algebra

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations.

Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis.

Quadratic forms, reduction and classification of quadratic forms

#### Calculus

- Functions of two or more variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers;
- Double and Triple integrals and their applications to area, volume and surface area; Vector Calculus: gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

### Algebra

Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria.

Fields, finite fields, field extensions, Galois Theory.closed fields.

# Complex Analysis

Algebra of complex numbers, the complex plane, power series, transcendental functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

# Topology

Basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

#### **UNIT-3**

### Ordinary Differential equations

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

# **Partial Differential Equations**

Lagrange and Charpit'smethods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

### **Numerical Analysis**

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

#### **Mathematical Methods**

Calculus of Variations: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Laplace Transforms, Differentiation and integration of transforms, Convolution theorem, Inverse Laplace transform: Applications of Laplace transform in obtaining solutions of ordinary differential equations and integral equations.

Fourier Transforms: Fourier sine and cosine transforms, Inverse Fourier sine and cosine transforms, properties, Convolution theorem for Fourier transform, Parseval's identity, Fourier transform of derivatives, Applications to boundary value problems, Finite Fourier transform

#### Mechanics

Equilibrium of a particle, Equilibrium of a system of particles, Couples, Moment of a couple, Work and potential energy, Principle of virtual work. Centres of Gravity and Common Catenary, Simple harmonic motion (SHM), projectiles in a vertical plane and cycloidal motion. Central Orbits Equation of motion under a central forces, Kepler's laws of planetary motion.

# **Probability and Statistics**

Descriptive statistics, exploratory data analysis Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

# Linear Programming

Linear programming models, convex sets, graphical method, simplex method, alternate optima; Duality theory, Transportation problems, least cost method, north-west corner rule, Vogel's approximation method, Optimal solution, assignment problems, Hungarian method. Game Theory: Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear programming method for solving a game.

### Discrete Mathematics

Partially Ordered Sets, Order isomorphism, Hasse diagrams, Dual of a poset, Duality principle, Maximal and minimal elements, Least upper bound and greatest upper bound. Lattices, Sub lattices, modular and distributive lattices; Complemented, relatively complemented and sectionally complemented lattices. Boolean Algebras and Switching Circuits.

Graphs, types of graphs, Subgraphs, Eulerian circuits, Hamiltonian cycles, trees, Adjacency matrix, Weighted graph, Travellingsalesman problem, Shortest path and Dijkstra's algorithm.